STRANGE PARTICLES FREEZE-OUT CONDITIONS
FROM A NEW HADRONIC EQUATION OF STATE

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ABSTRACT

We propose a thermodynamically consistent equation of state (EOS) for a hot and dense hadron gas (HG) incorporating the hard-core finite-size correction for baryons which result in a Van-der Waals excluded volume effect. We develop a complete description within a thermal hadron gas model to determine the production of strange particles in relativistic nucleus-nucleus collisions and calculate the strange particle ratios as function of baryon chemical potential μ_B and temperature T . The results are compared with other existing thermodynamically consistent EOS. Furthermore we extract the thermal freeze-out parameters for different strange particles using the latest data obtained by the CERN experiments.

I. INTRODUCTION

High-energy heavy-ion collisions offer the possibility of creating a high-density nuclear matter in the initial stage of the collision. We believe that a colour deconfined matter may be produced for a very brief time, such a matter is called as a quark-gluon plasma (QGP). Recently many theoretical and experimental papers have appeared on the properties and signals of QGP [1-4]. Strangeness production has been a much studied possible signature for the identification of QGP formation in relativistic nucleus-nucleus collisions [5-8]. It has been suggested that in the baryon-rich QGP phase there would be an enhanced strange quark production followed by its speedy equilibration which will result in a large equilibrium strangeness density. However, models based on purely hadronic scenarios also show an enhancement of strangeness. In order to have a reliable prediction regarding strangeness enhancement, knowledge of proper equation of state (EOS) for a hot and dense hadron gas (HG) is essential. An ideal gas approximation for HG with point-like baryons and mesons has been found unsuitable because the point-like baryons can be compressed to an infinitely large density. Moreover, it has also been demonstrated that without a repulsive interaction in the hadronic EOS, one cannot construct a first order quark-hadron phase transition using Gibbs' criteria in the whole temperature T -chemical potential μ plane. A phenomenological solution to this difficulty has been suggested by introducing a short range

and repulsive hard-core interactions between a pair of baryons or antibaryons in the form of an excluded volume effect [9-14] which is akin to the well-known Van-der Waals method in molecular physics. However, we notice that the original suggestion of Cleymans et al [9] as well as of Kuono and Takagi [10] were not thermodynamically consistent because the thermodynamical quantities cannot be derived from a thermodynamical potential or a partition function. Some recent approaches [13,14] have taken into consideration the effect of finite-size correction in a thermodynamically consistent manner. However, these methods involved cumbersome solutions and numerical evaluations of various thermodynamical quantities pose a problem. Here we propose a thermodynamically consistent equation of state (EOS). We will attempt to calculate the various strange particle ratios as function of baryon chemical μ_B and temperature T and the results are compared with the existing thermodynamically consistent EOS [13,14]. Finally we will extract the thermal freeze-out parameters for various hadrons using CERN experimental data obtained by NA35 [15] and others [16,17].

One of the central problems in the study of high energy heavy-ion collisions lies in deducing the state of the system by determining temperature and chemical potential formed in the process from the observed final particle properties. The freeze-out stage of the system when the particles fly towards the detectors without further interactions, is directly connected to the observed multiplicity distributions of the

particles. We first connect the thermodynamical properties of the system to an appropriate EOS for the hot and dense hadron gas. Finally we deduce the freeze-out conditions of the thermal HG fireball formed in the heavy-ion collisions. Recently many attempts have been made to determine the freeze-out conditions of the fireball [18-21]. On the other hand strangeness enhancement has also been explained in the conventional hadronic quark gluon string model and relativistic quantum molecular dynamics (RQMD) type of transport approach [22-23] where thermalized fireball concept has completely been ignored. It is indeed encouraging to notice that simple thermal models [18-21] can explain the data on the strangeness enhancement. However, the whole exercise crucially depends on the EOS used to determine the thermal and chemical parameters at the freeze-out of the fireball of a hot and dense HG. The purpose of this paper is to obtain a new thermodynamically consistent EOS and compare its predictions for the freeze-out conditions with those of the other approaches.

II. FORMULATION OF THE MODEL

The HG models describe the hadronic matter as a gas of hadrons with different masses where individual mass states are populated according to equilibrium Bose-Einstein or Fermi-Dirac distributions. This is formulated within the framework of grand canonical ensemble by introducing baryon and strange chemical potentials μ_B and μ_S , which are required to guarantee the baryon number and strangeness conservation in the strongly interacting

hadronic matter. When applying these HG models to situations of high temperatures and large baryon chemical potentials, one must introduce a short range, hard-core repulsive interaction between a pair of baryons or antibaryons. The repulsive part has usually been parametrized in phenomenological models by assigning the hadrons a geometric hard-core volume which results into an excluded volume effect [9-11]. However, the main drawback of these models is their lack of consistency with the basic thermodynamical relations obtainable from a partition function. Rischke et al [13] suggested a novel method for restoring the thermodynamical consistency by replacing the chemical potential $\tilde{\mu}_j = \mu_j - V_j p^{\text{ex}}(T, \mu_j)$ where $p^{\text{ex}}(T, \mu_j)$ is the pressure of the HG after excluded volume correction, and V_j is the eigenvolume of the j^{th} type of hadrons, we finally get a transcendental solution for $p^{\text{ex}}(T, \mu_j)$ as :

$$p^{\text{ex}}(T, \mu_j) = \sum_j p_j^0(T, \mu_j - V_j p^{\text{ex}}) \quad (1)$$

where $p_j^0(T, \mu)$ is the pressure due to j^{th} type of hadrons with modified chemical potential. Although the method of incorporating the excluded volume effect in Rischke model rests on a mathematically sound technique, it still lacks a clear physical interpretation. Moreover, the transcendental solution one gets from Rischke model makes the numerical evaluation of the problem more difficult.

Recently Uddin and Singh [14] proposed a thermodynamically consistent model in which the grand canonical partition function Z is modified as follows :

$$\ln Z_i = (g_i/6\pi^2 T) \int_{V_i}^{V - \sum_j N_j V_j} dV' \int_0^\infty \left\{ k^4 / (k^2 + m_i^2)^{1/2} \right\} \left[1 / \{ \lambda_i^{-1} \exp(E_i/T) + 1 \} \right] dk \quad (2)$$

Here V is the total volume of the system, g_i is the spin-isospin degeneracy factor, m_i is the mass and λ_i is the fugacity for the i^{th} species of baryons, k is the momentum of hadron and N_j is the number of j^{th} type of baryons. Using Boltzmann approximation, we get :

$$\ln Z_i = V(1 - \sum_j n_j V_j) I_i \lambda_i \quad (3)$$

where

$$I_i = (g_i/2\pi^2) m_i^2 T^{-3/2} (m_i/T)$$

Using the expression for number density of i^{th} baryon species as

$$n_i = (\lambda_i/V) (\partial \ln Z_i / \partial \lambda_i) \Big|_{T,V}, \text{ we get from (3)}$$

$$n_i = (1 - \sum_j n_j V_j) I_i \lambda_i - I_i \lambda_i^2 \{ \partial (\sum_j n_j V_j) / \partial \lambda_i \} \quad (4)$$

Defining $R = \sum_j n_j V_j$ which indicates the fraction of the occupied volume in the total volume of the system, we get a differential equation :

$$\sum_j I_j V_j \lambda_j^2 (\partial R / \partial \lambda_j) + (\sum_j I_j V_j \lambda_j + 1) R - \sum_j I_j V_j \lambda_j = 0 \quad (5)$$

Using the method of 'parametric line' one can define the fugacities in the parametric space as :

$$\lambda_j = 1/(a_j + I_j V_j t) \quad (6)$$

We finally get the solution of (5) as :

$$R = 1 - \frac{\int_0^\infty \{\exp(-t')/G(t')\} dt'}{t \{\exp(-t)/G(t)\}} \quad (7)$$

where

$$G(t) = t \prod_{j=2}^p (a_j + I_j V_j t)$$

Here p gives the total number of baryon species. The parameter t is fixed by setting $a_1 = 0$ so that $t = 1/I_1 V_1 \lambda_1$. If one knows R , one can obtain $\partial R / \partial \lambda_i$ numerically. Obviously the solution (7) cannot be a unique one because one of the constants a_1 has been arbitrarily fixed as zero.

Instead of solving the differential equation (4), in the parametric space, one can get simple solution provided one substitutes $\partial(\sum_j n_j V_j) / \partial \lambda_i = (\partial n_i / \partial \lambda_i) V_i$. Here we assume that n_i depends only on the fugacity λ_i of the i^{th} species in the hadron gas consisting of multicomponent baryons.

Thus we get a modified differential equation as :

$$\frac{\partial n_i}{\partial \lambda_i} + n_i \left\{ \left(\frac{1}{I_i V_i \lambda_i^2} \right) + \left(\frac{1}{\lambda_i} \right) \right\} = \left(\frac{1 - \sum_{j \neq i} n_j V_j}{V_i \lambda_i} \right) \quad (8)$$

The solution of equation (8) can be obtained in a straight forward manner :

$$n_i = \frac{(1 - \sum_{j \neq i} n_j V_j) Q_i}{V_i \lambda_i \exp(-1/I_i V_i \lambda_i)} \quad (9)$$

where

$$Q_i = \int_0^{\lambda_i} \exp(-1/I_i V_i \lambda_i) d\lambda_i$$

In this model one can obtain R as follows :

$$R = \sum_j n_j V_j = X/(1 + x) \quad (10)$$

where

$$X = \sum_j Q_j / \{ \lambda_j \exp(-1/I_j V_j \lambda_j) - Q_j \}$$

One can interpret X as the ratio of occupied and the available volume. Finally the number density of i^{th} baryon species can be written as :

$$n_i = \{ (1-R)/V_i \} [Q_i / \{ \lambda_i \exp(-1/I_i V_i \lambda_i) - Q_i \}] \quad (11)$$

It is obvious from (11) that we have obtained an easy and simple solution of eq. (4). There are no parameters in the theory and thus it can be regarded as a unique solution. However, this still depends crucially on the assumption that the number density n_i of the i^{th} species of baryons is a function of the fugacity λ_i alone and it is completely independent of the fugacities of other kinds of baryons. Without this assumption one may be able to find other solution. Moreover, we notice that the calculation of all the thermodynamical quantities become easier in the present model.

The entire hadron gas spectrum upto the mass of 1400 MeV can be described in terms of two chemical potentials μ_B and μ_S which give baryon number and strangeness conservations, respectively. The two sets of chemical potentials for the quarks and hadrons are related by $\mu_B = 3\mu_q$ and $\mu_S = \mu_q - \mu_s$. In a strangeness neutral QGP fireball μ_s is always exactly zero and hence $\mu_S = \mu_B/3$. Since the net strangeness of the colliding nuclei in the initial state is zero, strangeness conservation dictates that the net strangeness of the fireball should remain close to zero. However, the strangeness distribution among hadrons of different masses modifies the strangeness chemical potential μ_S far different than $\mu_B/3$ where μ_B is the baryon chemical potential. The condition for vanishing total strangeness takes the form :

$$n_{\text{strange meson}} - n_{\text{antistrange meson}} + n_{\text{strange baryon}} - n_{\text{antistrange baryon}} = 0 \quad (12)$$

where n is the number density. The case of exact strangeness neutrality in a HG in absolute chemical equilibrium has been explored in detail by Cleymans and Satz [24].

III. RESULTS AND DISCUSSION

In the recent past several efforts [18-21] have been made to explain the experimental results in terms of the simple framework of a thermal fireball model. Such approaches have explained the strangeness abundances in terms of a few macroscopic parameters eg. temperature T , chemical potentials μ_B

and μ_s etc. The values of these parameters at the freeze-out point yield a complete thermodynamical description of the ratios of the observed particles. In our model, we have used the multicomponent HG consisting of seven baryonic states and eight mesonic states as shown in Table 1. The proper volume of the baryons (antibaryons) is obtained by employing the bag model considerations. Taking the hard-core radius of a nucleon (antinucleon) as $r_N = 0.6$ fm and mass M_N as 940 MeV, we can get the bag constant B from $4B = M_N/V_N$ where V_N is the eigenvolume of a nucleon ($V_N = 4\pi r_N^3/3$). The proper volume of other baryons is obtained by using $V_i = M_j/4B$.

In Fig. 1, we have shown the variation of the ratio $\bar{\Lambda}/\Lambda$ with baryon chemical potential μ_B at $T = 200$ MeV. We have also shown the curves obtained in the earlier Uddin-Singh approach and Rischke model as well as in the pointlike HG model. We find that our present calculation yields result which overlaps with the prediction of Uddin-Singh model. However, our values are a little larger than the predictions of the pointlike HG model and hence we can infer that the use of excluded volume correction yields the values of $\bar{\Lambda}/\Lambda$ different by a small amount than the pointlike HG model which has been very often used in such kind of descriptions. We can emphasize that the particle ratio is by construction independent of the specific relation between the baryon chemical potential and the baryon density. Therefore, the repulsion introduced in the form of an excluded volume effect cancels out to some extent in the ratio of $\bar{\Lambda}$ and Λ . This explains why our result lies close to that of pointlike case.

However, in Rischke model the repulsive force has been considered in an indirect way by modifying the baryon chemical potential by a term proportional to the pressure and hence the cancellation appears to be the least effective. Therefore, the values in the model of Rischke et al are the largest of all. If we compare the experimental value of $\bar{\Lambda}/\Lambda = 0.18 \pm 0.05$ we notice that it corresponds to $\mu_B = 240$ MeV in the pointlike HG model, $\mu_B = 260$ MeV in the present as well as Uddin-Singh models and $\mu_B = 280$ MeV in the model of Rischke et al at a fixed value of $T = 200$ MeV. In order to see the effect of the eigenvolume of the baryons on the ratio $\bar{\Lambda}/\Lambda$, we have also shown in Fig. 1, the curve obtained in the present model for the nucleon radius as 0.8 fm. We find that the ratio increases by a significant amount. We can emphasize that the present model, Uddin-Singh approach as well as Rischke model are all thermodynamically consistent ones and are extendible to include more particle degrees of freedom. However, in comparison the present model is easier to handle in numerical calculations and does not require the solution of any transcendental type of equations as is the case in Rischke model.

In Fig. 2, we have demonstrated the variation of the ratio $\bar{E}/E$ with μ_B at $T = 200$ MeV. At small μ_B (< 250 MeV), the difference between the curves is almost negligible. However, the predictions of the thermodynamically consistent models differ much from that of the pointlike case at larger values of μ_B . Again the present calculation yields a curve which almost coincides with the curve obtained in Uddin-Singh approach.

However, the trend is reversed in this case and the curve for pointlike HG lies at the top of all the curves for the larger values of μ_B . The curve obtained in Rischke model lies at the bottom and almost coincides with the curve obtained in the present model for the nucleon radius 0.8 fm. We can compare our results with the WA85 experimental value of $\bar{E}/E = 0.45 \pm 0.05$ in S-W collisions in the p_T interval $1.2 < p_T < 3.0$ GeV [17] at the same energy. We find that this value now corresponds to a lower value of μ_B (=200 MeV) at the same value of temperature $T = 200$ MeV in almost all the above models. This indicates that the chemical freeze-out conditions in S-W collision experiment differ slightly from those obtained in S-S collisions. The smaller value of μ_B in S-W collision than in the S-S collisions is a surprising result because S-S is a smaller system, but still the nuclear stopping appears to be more effective in this case than in the S-W collisions. It can, however, arise due to the averaging over rapidity when using 4π particle yields [25].

In Fig. 3 we have presented the results of our model for the ratios $\bar{E}^+/\bar{\Lambda}$ and E^-/Λ and compared them with those obtained in other models. We again find that our present curve almost coincides with that of our earlier model for the ratio $\bar{E}^+/\bar{\Lambda}$. However, the two curves differ slightly in the ratio E^-/Λ . Moreover, our present calculation yields a curve which is much different than the one obtained in Rischke model. We can compare our results with the experimental results for $\bar{E}^+/\bar{\Lambda} = 0.21 \pm 0.02$ and $E^-/\Lambda = 0.095 \pm 0.006$ in S-W experiment [17]. We find that the former ratio indicates a value of $\mu_B = 300$ MeV at $T = 200$

MeV in the present calculation and $\mu_B = 500$ MeV at $T = 200$ MeV in the Rischke model. On the contrary, we notice that the experimental ratio for Ξ^-/Λ lies far below the theoretical curves at $T = 200$ MeV. It looks possible to find out a different temperature at which all these experimental ratios can be fitted in one model at a given μ_B . However, seeing the nature of curves vis-a-vis experimental points, the possibility for such a coincidence appears to be small. In view of the above fact, the introduction of some parameter is essential into the theory of the thermal models. We call this parameter as strangeness saturation factor γ_s ($0 < \gamma_s \leq 1$) which allows us to parametrize the incomplete chemical equilibration of the strange particles so that all the ratios can be fitted simultaneously for the same values of μ_B and T [18]. We can define the effective fugacity of each strange quark as $\gamma_s \lambda_s$ and that of strange antiquark as $\gamma_s \lambda_s^{-1}$. Since the value of μ_B is exactly zero for a QGP fireball, we expect γ_s to be equal to one. In the conventional HG picture, we hope that $\gamma_s \ll 1$ as one gets in p-p case [18,25]. A full strangeness saturation $\gamma_s = 1$ in the HG picture would reveal a surprisingly short strangeness equilibration time-scale in S-S collisions.

Fig. 3 is also significant because it reveals the influence of the proper volume correction in the hadronic equation of state. As pointed out earlier, the excluded volume effects for Λ and $\bar{\Lambda}$ yield almost a similar factor. Similarly when we go from p-W to S-W collisions, Λ and $\bar{\Lambda}$ productions increase by a similar factor and thus $\bar{\Lambda}/\Lambda$ ratio remains

unaltered [1]. However, the case becomes different when we consider the ratios like Ξ/Λ or $\bar{\Xi}/\bar{\Lambda}$ involving two different particles. Fig. 3 shows that the excluded volume correction becomes more significant for larger μ_B or baryon density. At a large μ_B , all the thermodynamically consistent approaches start differing in their predictions and thus we hope the above ratios are most suitable in drawing a line of distinction between the models. Moreover, these ratios crucially depend upon the value of γ_s because each of them is proportional to γ_s . In comparison, $\bar{\Lambda}/\Lambda$ and $\bar{\Xi}/\Xi$ do not depend at all on γ_s . Thus the experimental results for these ratios yield the information about γ_s or strangeness phase space saturation factor.

It is worth emphasizing here that the values of the ratios $\bar{\Xi}/\Xi$, $\bar{\Lambda}/\Lambda$ and $\bar{\Xi}/\bar{\Lambda}$ depend mostly on the relation of μ_S with μ_B at a fixed value of temperature. This relation is obtained by using the strangeness conservation constraints in hadron gas model. For the HG resulting from the hadronization of QGP, we get $\mu_S = \mu_B/3$. However, for a HG phase without any QGP formation, the strangeness is distributed among mesons and baryons of much different masses, and one usually gets a completely different relation from the above. This is a significant point because it avoids misinterpretations when evaluating the experimental data on the strangeness production in all these models. In all the thermodynamically consistent models with excluded-volume correction, the value of μ_S first increases as μ_B increases from zero but after a certain value of μ_B , μ_S becomes constant [20].

In Fig. 4, we have shown the variation of the ratio K^+/K^- with μ_B at $T = 200$ MeV. We find that the predictions of the thermodynamically consistent models with excluded volume correction differ quite significantly from those of the pointlike HG model at large μ_B . This again stresses the role of excluded volume correction at large μ_B or baryon density. In the HG model consisting of point-like particles, the ratio K^+/K^- exponentially increases whereas in the thermodynamically consistent approaches, it increases in the beginning but slowly it becomes constant of μ_B . However, this ratio is again a function of μ_B and μ_S at a constant T and thus depends on the relation between μ_B and μ_S . We find that the predictions of the present model almost overlap with our old model but are far larger than those of Rischke model. Again we notice that our curve for nucleon radius as 0.8 fm almost overlaps with the curve of Rischke model for $r_N = 0.6$ fm.

The variation of the ratio $\Lambda/(p-\bar{p})$ with μ_B is depicted in Fig. 5 at $T = 200$ MeV. Obviously this ratio is a measure of strangeness enhancement similar to the ratio K^+/π^+ as suggested by some authors [18]. We notice that the present model gives a larger contribution to $\Lambda/(p-\bar{p})$ than the predictions in other approaches. It can be interjected here that the value of the parameter γ_S is most sensitive to the experimental ratio of $\Lambda/(p-\bar{p})$ and it significantly controls the value of the ratio. We find that without using γ_S , we cannot fit the experimental value of $\Lambda/(p-\bar{p}) \approx 0.41 \pm 0.08$ by NA35 on the curves in Fig. 5 if the temperature is $T = 200$ MeV. In Fig. 5, we find that the ratio

$\Lambda/(p-\bar{p})$ is also more sensitive to the value of r_N . For $r_N = 0.8$ fm, we notice that the ratio increases still further by a large amount.

It is certainly an interesting programme to search the existence of a thermalized system in an ultra-relativistic heavy-ion collision. It leads us to discard a model based on the uncorrelated superposition of nucleon-nucleon scattering. The temperature calculated from the particle ratios depends on the chemical freeze-out conditions and the temperature extracted from the slope of the p_T -distribution of the particles should yield the information regarding the thermal freeze-out when the particles decouple from the fireball. Since the chemical freeze-out occurs earlier than the thermal freeze-out, one expects a lower value of temperature from the p_T -distribution than that obtained from the particle ratio. However, presence of a collective flow can enhance the thermal freeze-out temperature. For example, taking a mean $\beta = 0.3$ for the transverse flow, one gets an effective temperature (as derived from the p_T -distribution) from the relation [27] :

$$T_{\text{eff}} = T \left[\frac{1 + \beta}{1 - \beta} \right]^{1/2} \quad (13)$$

Taking T as 175 MeV approximately, one gets from (13) $T_{\text{eff}} = 238$ MeV. This compares well with a temperature of $T = 228 \pm 9$ MeV as derived from the p_T distribution for Λ particles by NA36 collaborators in S-Pb collisions. This exercise demonstrates that the maximum temperature of the fireball source can be around 175 MeV if the blue shifting effect generated by

transverse flow is also considered.

It is worthwhile to investigate the chemical freeze-out conditions in heavy-ion collisions of S with S at 200 GeV/A at CERN SPS. We use the strangeness phase space saturation factor γ_s in order to get a good fit to the various experimental ratios simultaneously. In Table 2, we have summarized the values of the chemical freeze-out parameters T , μ_B and μ_s obtained in the present model and these are compared with those obtained in the earlier Uddin-Singh model [26]. The experimental information used for extracting the parameters are given in Table 3. Here we have utilized the relation for the quark fugacity $\lambda_q = \exp(\mu_q/T)$ with $\mu_B = 3\mu_q$ and the strange quark fugacity $\lambda_s = \exp(\mu_s/T)$ where $\mu_s = -\mu_q + \mu_B/3$. The thermodynamical state of the fireball is given by the parameters $T = 175$ MeV, $\mu_B = 235$ MeV, $\mu_s = 10$ MeV and $\gamma_s = 0.94$. The corresponding values in Uddin-Singh model are $T = 200$ MeV, $\mu_B = 280$ MeV, $\mu_s = 15$ MeV and $\gamma_s = 0.78$. These values can also be compared with those obtained by Sollfrank et al [18] : $T = 191$ MeV, $\mu_B = 221$ MeV and $\mu_s = 16.5$ MeV and $\gamma_s = 0.58$. They correspond to a thermal HG model consisting of pointlike particles. The most surprising result is the value of γ_s which is close to one in the present model. It means that unlike all other models, the present model indicates the saturation of the strangeness phase space in the hadron gas formed in S-S collision. Consequently we infer that the strangeness equilibrium is almost achieved in the heavy ion collisions according to the present EOS used here. Moreover, we notice a large difference in the values of the parameters in

these models, although we find a little difference in the ratios $\bar{\Lambda}/\Lambda$ and $\bar{E}/E$ as shown in Figs. 1 and 2.

In Table 4, we have listed the multiplicities of various hadrons obtained in our thermal hadron gas models using the chemical freeze-out conditions obtained above. These values are consistent with the experimental data obtained at CERN experiments with S-S collisions. We compare our results also with those obtained in Uddin-Singh model. The knowledge of total hadron multiplicities in particular can be used to calculate the extensive parameters of the system because we can extract the number densities of the hadrons from the thermodynamical potential or the partition function in our model. Thus the 'thermal' volume of the system at chemical freeze-out can be estimated using the meson as well as baryon multiplicities. We can obtain the volume accessible to mesons and baryons separately at freeze-out. The values of k^+ and $p-\bar{p}$ multiplicities have been used to fix the freeze-out volumes for the mesons and baryons separately. In our present calculation, freeze-out volume accessible to baryons V_b is 478 fm^3 which is far larger than $V_b = 217 \text{ fm}^3$ obtained in Uddin-Singh model. All these values are larger than the volume of a sulphur nuclei $V_b = 170 \text{ fm}^3$. Thus the present calculation again indicates the presence of a collective expansion even before the chemical freeze-out conditions are achieved. We also find that the fraction of the total occupied volume is 16% according to the present calculation which is smaller than the value 35% as obtained in Uddin-Singh model. It can be interjected that we

have calculated μ_s from the condition of strangeness neutrality as given by the densities of the particles in Eq. (12). However, as an important consistency check, we have also employed the constraint of strangeness conservation given by the total strange and antistrange particles contained in the total volume of the fireball and we find that this condition is also satisfied. For a larger r_N ($=0.8$ fm), we find that the volume $V_b = 565 \text{ fm}^3$ in the present model and $V_b = 303 \text{ fm}^3$ in the Uddin-Singh model. This shows that the change in the eigenvolume of the baryons alters the freeze-out baryonic volume by a smaller fraction in the present calculation than what we get in the old model.

A crucial question here is how many of the observed resonances we should include in the calculation. However, in our thermodynamically consistent EOS for the HG, the inclusion of higher mass resonances in the HG spectrum yield some changes in the result as obtained here. For example if we include all the known mesonic and baryonic resonances in the HG spectrum upto 1900 MeV, we find that the freeze-out baryonic volume $V_b = 137 \text{ fm}^3$ which is smaller than the sulphur nucleus and hence is absurd. Moreover, we get $\gamma_s \approx 0.98$ and it does not exceed one as obtained by Sollfrank et al [18] in the pointlike case. This is a significant consequence of using the thermodynamically consistent EOS here. We also notice that this still gives an approximate fit to all the ratios, but in some ratios, eg., $\bar{E}/\bar{\Lambda}$, the deviation from the experimental result is large enough.

Strongly interacting matter has hitherto only been

observed in hadronic form in which elementary constituents are confined in meson and baryon clusters. On the other hand, we believe that a phase transition will occur to a phase in which quarks can propagate almost freely over distances larger compared with typical hadronic radii, if the density and/or temperature are sufficiently high. However, when hadrons form a dense system and are very close to each other at very high temperature or density, the force between two hadrons is also quite large. The entire analysis performed here would be well outside the region of validity of the hadronic equation of state, provided the hadrons really undergo a phase transition at the temperature and chemical potentials derived in this paper. The simple way of illustrating this point will be by drawing a phase diagram which will also involve an equation of state for QGP phase as well. Usually one considers QGP as an ideal gas of quarks and gluons in a confining bag with a bag constant B . This simple picture gives $T_c \approx 150$ MeV at $\mu_c = 0$. However, the description of this transition region is very complicated and it requires a considerable deepening of our intuition regarding strong interaction phenomena in order to be able to identify viable simple approximation schemes. Thus we conclude that neither the ideal HG or a HG with an excluded volume approximation nor an ideal QGP approximation can work under such circumstances. For example, putting a second-order perturbative interaction term proportional to strong coupling α_s in the EOS of QGP yields a value of $T_c \sim 200$ MeV for $\alpha_s = 0.6$ at $\mu_c = 0$ [7]. Similarly the consideration of a finite-size statistical

system in the EOS of HG will also reduce the freeze-out temperature further [21] by a significant amount. Therefore, the derivation of a consistency check for the thermal HG picture by drawing a phase diagram does not appear worth-while unless we get a precise method for obtaining the same. Moreover, we have assumed here that even in a hot and dense situation like the one obtained above, the HG does not undergo any phase transition so that the entire exercise as outlined above remains valid at the temperatures and chemical potentials obtained in this paper.

The very small value of μ_s and almost unit value for γ_s obtained here suggest that the predictions of a thermal hadron gas model do not differ much from those of a strangeness neutral QGP. This is a significant information which can find use in the diagnostics of QGP formation in ultra-relativistic heavy-ion collisions. If the collision history in such collisions is a thermally and chemically equilibrated hadron gas, then we observe the particle ratios when the particles decouple from such a system. Under the circumstances, these ratios cannot yield any information whether any deconfined QGP phase existed earlier. Enhanced strangeness production was originally proposed as a signal for QGP formation. But the exercise in this paper reveals that a similar enhancement is obtained from a dense and hot hadron gas in equilibrium by employing a suitable equation of state. Thus we conclude that the strangeness production in heavy-ion collisions does test the onset of equilibration and hence it constitutes an important tool in the study of a hot and dense hadron gas produced in such collisions.

In conclusion we have presented a model for the EOS of a dense and hot HG incorporating the excluded volume correction in a thermodynamically consistent way. The implications of the model for the strange particle production in the heavy-ion collisions are explored. Our calculations reveal that the experimental data at CERN yield a value of the freeze out temperature $T \approx 175$ MeV and indicates that the state of strangeness equilibrium has almost reached in these experiments. Chemical equilibration of strangeness has a far reaching implication on the search of a QGP signal. The observed temperature is found to be much smaller than apparent temperature obtained from the inverse slope of the observed m_T -spectra which thus reveals a considerable blue-shifting effect due to collective expansion. Our value of T is smaller than what we get in pointlike hadron gas [18] and Uddin-Singh HG model. These results thus amply demonstrate the wide applicability of a statistical 'thermal fireball' model in the analysis of ultra-relativistic heavy ion collisions. To conclude we can emphasize that the abundance of multistrange hadrons observed in CERN experiments does not constitute convincing evidence for deconfinement as it can easily be interpreted in the hadron gas scenario.

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FIGURE CAPTIONS

- Fig. 1. The variation of $\bar{\Lambda}/\Lambda$ ratio with baryon chemical potential at temperature $T = 200$ MeV is calculated in point like HG model and compared with present model as well as with Rischke and Uddin-Singh model.
- Fig. 2. The variation of $\bar{\Xi}/\Xi$ ratio with baryon chemical potential at $T = 200$ MeV is compared in different HG models.
- Fig. 3. The variation of ratios $\bar{\Xi}^+/\bar{\Lambda}$ and Ξ^-/Λ with baryon chemical potential at $T = 200$ MeV is compared in different HG models.
- Fig. 4. The variation of ratio K^+/K^- with baryon chemical potential at $T = 200$ MeV is compared in different HG models.
- Fig. 5. The variation of ratio $\Lambda/(p-\bar{p})$ with baryon chemical potential at $T = 200$ MeV is compared in different HG models.

TABLE CAPTIONS

Table 1. List of particles and antiparticles used in the calculation.

Table 2. Calculated thermal parameters and the freeze-out particle ratios obtained in the present model as well as in Uddin-Singh model.

Table 3. List of the experimental total particle multiplicities and the particle ratios in the full phase space for the central S-S collisions at 20 GeV/A [15] used for the determination of thermal freeze-out parameters.

Table 4. Total particle multiplicities obtained from the calculations in the present model and in Uddin-Singh model. The quantities with "therm" in parentheses are the multiplicities of pure thermal origin i.e. without any decay contributions from higher resonances. Other multiplicities signify the abundances of hadronic resonances before their strong decay but include the possible feed-down contributions from the higher resonances. The quantity N_B is the net baryon content of the fireball. The total freeze-out volumes accessible to the mesons and the baryons are indicated by V_m and V_b respectively.

TABLE 1

Particles and antiparticles used in the calculation

Particle	Mass (MeV)	Strangeness	Degeneracy factor (g_i)
π	140	0	3
K	490	1	2
η	548	0	1
ρ	770	0	9
ω	785	0	3
K^*	890	1	6
η'	958	0	1
ϕ	1015	0	3
N	940	0	4
Λ	1120	-1	2
Σ	1190	-1	6
Δ	1236	0	16
Ξ	1320	-2	4
Σ^*	1385	-1	12
Λ^*	1405	-1	2

TABLE 2

	Present Model	Uddin-Singh Model
λ_q	1.565	1.595
λ_s	1.06	1.08
γ_s	0.94	0.78
T (MeV)	175	200
$\bar{\Lambda}/\Lambda$	0.194	0.189
κ^+/κ^-	1.99	1.92
$\Lambda/(p-\bar{p})$	0.39	0.39
$\bar{\Xi}/\Xi$	0.475	0.404
$\bar{\Xi}^+/\bar{\Lambda}$	0.24	0.183
Ξ^-/Λ	0.099	0.086

TABLE 3

K^+	12.5 ± 0.4
K^-	6.9 ± 0.4
Λ	8.2 ± 0.9
$\bar{\Lambda}$	1.5 ± 0.4
$p-\bar{p}$	20 ± 3
h^-	98 ± 5
$\bar{\Lambda}/\Lambda$	0.18 ± 0.05
K^+/K^-	1.8 ± 0.1
$\Lambda/(p-\bar{p})$	0.41 ± 0.08

TABLE 4

Hadrons	Present Model	Uddin-Singh Model
K^+	12.5	12.5
K^-	6.27	6.5
Λ	7.8	7.82
$\bar{\Lambda}$	1.5	1.48
p - $\bar{p}$	20.0	20.0
h^-	55.0	63.0
η	4.9	5.54
η'	0.74	1.03
ϕ	1.7	2.44
ω	7.0	9.63
ρ^0	5.4	6.64
π^-	46.7	54.4
p	21.8	22.16
$\bar{p}$	1.8	2.06
Δ	22.2	24.32
$\bar{\Delta}$	1.8	2.31
Ξ	1.6	1.34
$\bar{\Xi}$	0.74	0.54
K^+ (therm)	6.96	5.90
K^- (therm)	3.19	2.70
Λ (therm)	3.1	2.7
p (therm)	10.6	10.0
π^- (therm)	14.8	12.5
N_B	53.8	53.5
V_m (fm ³)	227	125.0
V_b (fm ³)	478	217

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5