Cosmic shear and biasing

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Abstract

The correlation between cosmic shear as measured by the image distortion of high-redshift galaxies and the number counts of foreground galaxies is calculated. For a given power spectrum of the cosmic density fluctuations, this correlation is proportional to the bias factor, which can thus directly be measured. In addition, this correlation provides a first-order measure of cosmic shear and is therefore easier to observe than quadratic measures hitherto proposed. Analytic approximations show that the expected signal-to-noise ratio of the correlation is large, so that a significant detection is possible with a moderate amount of data; in particular, it is predicted that the ongoing ESO Imaging Survey (EIS) will be able to detect this correlation on scales of $\sim 10'$ at a 3-$\sigma$ level, and at with higher significance on smaller angular scales.

1 Introduction

The distortion of high-redshift galaxy images by the (tidal) gravitational field of intervening matter inhomogeneities (often called ‘cosmic shear’) can be used to study the intervening mass distribution. In particular, if the large-scale structure of the (dark) matter is considered, the observable image distortions constrain the statistical properties of the cosmic matter distribution. This method of determining the power spectrum of cosmic density fluctuations has been investigated recently in considerable detail (Blandford et al. 1991; Miralda-Escudé 1991; Kaiser 1992, 1996, hereafter K92, K96, respectively; Villumsen 1996; van Waerbeke, Bernardeau & Mellier 1997; Jain & Seljak 1997; Sanz, Martínez-González & Benítez 1997; Schneider et al. 1997a, hereafter SvWJK). A first significant detection of cosmic shear has been reported in Schneider et al. (1997b).

Cosmic shear probes the statistical properties of the projected density fluctuations, where the projection takes into account the redshift distribution of the source galaxies and geometric factors. Similarly, the surface number density of galaxies is obtained by a projection of the three-dimensional galaxy distribution. Provided galaxies trace the underlying dark matter distribution, these two projected fields are correlated.

In this letter, this correlation is investigated, aiming at a method to constrain the bias factor $b$ which relates the number density fluctuations of galaxies to those of the
underlying dark matter distribution.\footnote{A different method to obtain the bias factor directly lensing uses the magnification effect which causes a correlation between foreground galaxies and high-redshift QSOs (e.g., Bartelmann 1995, Sanz et al. 1997, Dolag & Bartelmann 1997) and changes the angular two-point correlation function of high-redshift galaxies (e.g., Moessner, Jain & Villumsen 1997).} A brief summary of the aperture mass statistics as a measure for cosmic shear is presented in Sect. 2, and an analogous statistics is introduced for the galaxy number counts. The general expression for the correlation between these two measures is derived in Sect. 3, and practical estimators are considered in Sect. 4. Focusing on large angular scales, linear theory presents a useful approximation for the growth of cosmic density fluctuations; in Sect. 5, the general expressions will be evaluated in this approximation. In particular, it is shown that the signal-to-noise ratio for the correlation coefficient is of order unity even in a single field. Therefore, this correlation should be easily detectable in currently conducted wide-field surveys, such as the EIS (Renzini & da Costa 1997). In particular, the significant verification of this correlation is probably the easiest way to detect cosmic shear.

2 Aperture mass and number counts

In this section we briefly summarize the $M_{ap}$-statistics for cosmic shear, and introduce a similar statistics for the number counts of (foreground) galaxies, following the notation of SvWJK.

Light propagation through a slightly inhomogeneous Universe can be described by an equivalent single-plane gravitational lens equation, to first order in the Newtonian gravitational potential (see SvWJK for a detailed discussion of this point). For sources with a redshift probability density $p_z(z) dz = p_w(w) dw$, where $w$ is the comoving distance out to redshift $z$, the dimensionless surface mass density at angular position $\theta$ of this single-plane lens is

$$\kappa(\theta) = \frac{3}{2} \left( \frac{H_0}{c} \right)^2 \Omega_d \int_0^{w_H} dw \ g(w) f_K(w) \frac{\delta(f_K(w)\theta, w)}{a(w)},$$  \hspace{1cm} (1)

where $\delta$ is the density contrast, $a = (1+z)^{-1}$ the cosmic expansion factor, $\Omega_d$ the density parameter in dust at present, $g(w) := \int_w^{w_H} dw' p_w(w') f_K(w' - w) / f_K(w')$ is the source-averaged distance ratio $D_{ds}/D_s$ for a density fluctuation at distance $w$, $f_K(w)$ is the comoving angular diameter distance to comoving distance $w$, and $w_H$ is the comoving distance to the horizon.

The Jacobi matrix, which describes the locally linearized lens mapping, reads $A_{ij}(\theta) = \delta_{ij} - \psi_{,ij}(\theta)$, where indices preceded by a comma denote partial derivatives with respect to the components of $\theta$, and $\psi$ is related to $\kappa$ via the Poisson equation, $\nabla^2 \psi = 2\kappa$. The two components of the shear, here written in complex notation, are derived from the traceless part of $A$, $\gamma(\theta) = (\psi_{,11} - \psi_{,22})/2 + i\psi_{,12}$. The shear therefore describes the tidal part of the deflection potential which causes the distortion of images.
Provided the density contrast $\delta$ is a homogeneous and isotropic random field, so is the projected density $\kappa$. The power spectrum $P_\kappa(s)$ of $\kappa$ is related to the power spectrum $P(k)$ of the density fluctuations $\delta$ through

$$P_\kappa(s) = \frac{9}{4} \left( \frac{H_0}{c} \right)^4 \Omega_d^2 \int_0^{w_H} dw \frac{g^2(w)}{a^2(w)} P \left( \frac{s}{f_K(w)}; w \right);$$

(2)

see K92 and K96 for a derivation of (2). The second argument of $P$ indicates that the power spectrum evolves with redshift. Several sample power spectra $P_\kappa$ are plotted in Fig. 1 of SvWJK.

In SvWJK, the aperture mass

$$M_{ap}(\theta) := \int d^2 \vartheta \ U(|\vartheta|) \kappa(\vartheta)$$

(3)

was introduced as a statistics for measuring cosmic shear. Similar quantities had previously been considered in somewhat different contexts (e.g., Fahlman et al. 1994; Kaiser 1995; Kaiser et al. 1994; Schneider 1996). Here, $U(\vartheta)$ is a compensated filter function, i.e., $\int_0^\theta d\vartheta \ \partial^2 U(\vartheta) = 0$, which vanishes for $\vartheta > \theta$. The definition (3) is particularly useful since $M_{ap}$ can directly be expressed in terms of the shear,

$$M_{ap}(\theta) = \int d^2 \vartheta \ Q(|\vartheta|) \gamma_t(\vartheta),$$

(4)

where $Q(\vartheta) = (2/\vartheta^2) \int_0^\vartheta d\vartheta' \ \vartheta' U(\vartheta') - U(\vartheta)$, and the tangential component of the shear at a position $\vartheta = (\vartheta \cos \varphi, \vartheta \sin \varphi)$ is $\gamma_t(\vartheta) = -\Re \left( \gamma(\vartheta) e^{-2i\varphi} \right)$. Hence, on the one hand, $M_{ap}$ yields a spatially filtered version of the projected density field, and on the other hand, it can be expressed simply in terms of the shear. Since in the weak lensing regime, the observed galaxy ellipticities provide an unbiased estimate of the local shear, $M_{ap}$ is directly related to observables. The dispersion of $M_{ap}$ is related to the power spectrum $P_\kappa(s)$ by

$$\langle M_{ap}^2(\theta) \rangle = 2\pi \int_0^\infty ds \ s P_\kappa(s) [I(s\theta)]^2,$$

(5)

where $I(\eta) = \int_0^1 dx \ x u(x) J_0(\eta x)$; here, we have written $U(\vartheta) = u(\vartheta/\theta)/\theta^2$, and $J_n(x)$ denotes the Bessel function of first kind. Hence, $M_{ap}(\theta)$ provides a filtered version of the projected power spectrum, and the width of the filter, here expressed by $I^2$, depends on the choice of $u$.

Provided that galaxies are biased tracers of the underlying (dark) matter distribution, the expected number density of galaxies in the direction $\theta$ is given by (cf. Bartelmann 1995; Dolag & Bartelmann 1997; Sanz et al. 1997)

$$N(\theta) = \bar{N} \left[ 1 + b \int dw \ p_g(w) \delta (f_K(w) \theta, w) \right],$$

(6)

where $p_g(w)$ is the probability distribution of the galaxies in comoving distance (or, equivalently, redshift), which depends on the selection criteria of the galaxy sample (such
as limiting magnitude, color, etc.), $N$ is the mean number density, and $b$ is the average bias factor for this galaxy sample. In analogy to the aperture mass, we define the aperture number counts

$$N(\theta) = \int d^2\vartheta \ U(|\vartheta|) N(\vartheta) ,$$

with the same function $U$ as in (3).

### 3 Density – shear correlations

The correlation between $M_{\text{ap}}(\theta)$ and $N(\theta)$ is measured by

$$C(\theta) := \langle M_{\text{ap}}(\theta) N(\theta) \rangle ,$$

where the angular brackets denote the ensemble average. Inserting the explicit expressions for $M_{\text{ap}}$ and $N$, and using the same method as in K96 to calculate the resulting projection of the correlator $\langle \delta \delta \rangle$, a few manipulations similar to those in SvWJK yield

$$C(\theta) = 3 \pi \left( \frac{H_0}{c} \right)^2 \Omega_d b \bar{N} \int dw \frac{p_g(w) g(w)}{a(w) f_K(w)} \int ds \ s \ P \left( \frac{s}{f_K(w)}, w \right) [I(\theta s)]^2 .$$

Hence, the correlation $C$ depends on the cosmological model, the redshift distributions of the galaxies which are used to estimate the shear (which we shall call ‘background’ galaxies in the following, though this should not imply that all these galaxies are lying behind those from which $N$ is measured) and those with which $N$ is estimated (‘foreground’ galaxies). In particular, $C$ is proportional to the bias factor $b$. In Sect. 5 below, we shall calculate $C$ for a simple model of the power spectrum of cosmic density fluctuations, but first we turn to practical estimators of $C$.

### 4 Practical estimators

Assume that in a circular aperture of radius $\theta$ there are $N_b$ galaxies at positions $\vartheta_i$ whose ellipticity is measured and which are thus used to estimate the shear, and that $N_f$ galaxies with positions $\varphi_j$ are used for measuring $N$. An estimator for $M_{\text{ap}}$ is

$$\hat{M}_{\text{ap}}(\theta) = \frac{\pi \theta^2}{N_b} \sum_{i=1}^{N_b} Q(|\vartheta_i|) \epsilon_{t,i} ,$$

where $\epsilon_{t,i}$ is the tangential component of the image ellipticity, defined in analogy to $\gamma_t$. In the limit of weak lensing, to be considered here, the relation $\epsilon = \epsilon^{(s)} + \gamma$ between image ellipticity $\epsilon$ and intrinsic source ellipticity $\epsilon^{(s)}$ holds, so that $\epsilon_i$ is an unbiased estimate of $\gamma(\vartheta_i)$ because the intrinsic orientations of the source galaxies are assumed to be random.

An estimator for $N$ is
\[ \hat{N}(\theta) = \sum_{k=1}^{N_f} U(|\varphi_j|) , \]  

so that an estimator for \( C \) reads \( \hat{C}(\theta) = \hat{M}_{ap}(\theta) \hat{N}(\theta) \). To obtain the expectation value \( E(\hat{C}) \), several averages have to be taken. First, one has to average over the intrinsic source ellipticity distribution. Denoting this operator by \( \mathbf{A} \), one finds \( \mathbf{A} \left( \hat{C}(\theta) \right) = \mathbf{A} \left( \hat{M}_{ap}(\theta) \right) \hat{N}(\theta) \), since \( \hat{N} \) is unaffected by \( \mathbf{A} \). The next average has to be taken over the galaxy positions. The corresponding operator \( \mathbf{P} \) factorizes into two operators \( \mathbf{P}_1 \) and \( \mathbf{P}_2 \), where

\[ \hat{C}(\theta) = \hat{M}_{ap}(\theta) \hat{N}(\theta) \]

which has been assumed that the ‘background’ galaxies are distributed randomly in angle, and the probability density \( p(\varphi_k) \) for finding a ‘foreground’ galaxy at \( \varphi_k \) is

\[ p(\varphi_k) = N(\varphi_k) \left[ \int d^2 \varphi \ N(\varphi) \right]^{-1} \approx N(\varphi_k)/N_f. \]

Thus, we neglect deviations of the total number of ‘foreground’ galaxies in the circular aperture from the expected number; these deviations are of minor importance only, provided \( N_f \gg 1 \). Then, by performing both averages, one finds that the expectation value of \( \hat{C}(\theta) \) is indeed \( C(\theta) \),

\[ E(\hat{C}(\theta)) \equiv \langle \mathbf{P}(\mathbf{A}(\hat{C}(\theta))) \rangle = C(\theta). \]

We next consider the dispersion of \( C \) for the case that the ‘foreground’ galaxies are unrelated to the matter distribution which distorts the background galaxies, or in other words, that \( M_{ap} \) and \( N \) are uncorrelated. In that case, the expectation value of \( \hat{C} \) vanishes, and

\[ \sigma_0^2 := E \left( \hat{C}^2 \right) = E \left( \hat{M}_{ap}^2 \right) E \left( \hat{N}^2 \right). \]

For the first of these factors, one finds with the same methods as used in SvWJK that

\[ E(\hat{M}_{ap}^2) = \langle M_{ap}^2(\theta) \rangle + \frac{\sigma_\epsilon^2 G}{2N_b}, \]

where \( G = \pi \theta^2 \int d^2 \varphi \ Q^2(|\varphi|), \) \( \sigma_\epsilon \) is the dispersion of the intrinsic galaxy ellipticities, and a subdominant ‘shot-noise’ term has been dropped. For the second term in (4.9), one finds

\[ E(\hat{N}^2) = \langle N^2(\theta) \rangle + \frac{\hat{N}}{\pi \theta^2} \hat{G}, \]

where \( \hat{G} = \pi \theta^2 \int d^2 \varphi \ U^2(|\varphi|). \) The dispersion of \( N \) can be obtained either directly from observations, or can be calculated following the biasing hypothesis. With steps very similar to those used for deriving (9), one obtains

\[ \langle N^2(\theta) \rangle = 2\pi b^2 \hat{N}^2 \int dw \ \frac{p_s^2(w)}{f_K(w)} \int ds \ s \ P \left( \frac{s}{f_K(w)}, w \right) I^2(\theta s). \]
5 Analytical estimates

This section presents analytical estimates for $C$ and the corresponding signal-to-noise ratio, which are obtained after several simplifications. First, we shall assume an Einstein-de Sitter (EdS) cosmological model. Second, only angular scales larger than \( \sim 10 \text{ arcminutes} \) are considered; for an estimate on such large angular scales, the power spectrum of the density fluctuations can be assumed to evolve linearly, so that \( P(k, w) = a^2(w) P(k, 0) \equiv a^2(w) P_0(k) \). Third, since by a convenient choice of the function \( U(\vartheta) \), the resulting filter function \( I^2 \) is quite narrow (see Fig. 2 of SVwJK), only a small range of wavenumbers contribute to the integrals over the power spectrum; hence, we can locally approximate the power spectrum by a power law in \( k \). And finally, we shall choose the redshift distributions of sources and ‘foreground’ galaxies to be very localized.

Thus, let \( P_0(k) = A k^n \), with a slope \( n \), and an amplitude which shall be determined from the rms fluctuations \( \sigma_8 \) in a sphere of radius \( R = 8 h^{-1} \text{ Mpc} \). Then, \( A = 2\pi^2 \zeta_1^{-1} \sigma_8^2 R^{(3+n)} \), with \( \zeta_1 = 9 \int_0^\infty dx \ x^{-4} [\sin(x) - x \cos(x)]^2 \). For EdS, one finds \( f_K(w) = w = (2c/H_0)[1 - (1 + z)^{-1/2}] \), \( w_H = 2c/H_0 \). We assume that the background sources are all at the same redshift \( z_s \), so that \( p_s(z) = \delta_D(z - z_s) \), and \( g(w) = (w_s - w)/w_s \) for \( w < w_s \) and zero otherwise, and \( w_s \) is the comoving distance out to redshift \( z_s \). Similarly, we assume that the foreground galaxies are well localized around the redshift \( z_g \), corresponding to comoving distance \( w_g \). Except for the calculation of \( \langle N^2 \rangle \) below, we shall approximate \( p_g \) by a delta ‘function’.

The compensated filter function \( U(\vartheta) \) will be the same as in SvWJK, namely \(^2 u(x) = (9/\pi)(1 - x^2)(1/3 - x^2) \) for which the corresponding function \( Q(\vartheta) = q(\vartheta/\theta)/\theta^2 \) is \( q(x) = (6/\pi) x^2 (1 - x^2) \). Then, \( I(\eta) = (12/\pi) (J_4(\eta)/\eta^2) \), and \( G = \hat{G} = 6/5 \).

With these assumptions and simplifications, we now estimate \( C(\theta) \) according to (9): Defining dimensionless comoving distances by \( \hat{w} \equiv w H_0/c \), (9) becomes

\[
C(\theta) = \frac{6\pi^3 \zeta_2 r^{3+n}}{\zeta_1} b \bar{N} \sigma_8^2 \frac{a(w_g) (\hat{w}_s - \hat{w}_g)}{\hat{w}_s \hat{w}_g^{(1+n)}} \theta^{-(2+n)},
\]

where \( r = RH_0/c = (8/3) \times 10^{-3} \) and \( \zeta_2 = \int_0^\infty dx \ x^{1+n} I^2(x) \). We shall write (16) in the following convenient form,

\[
C(\theta) = 1.822 \times 10^{-3} \zeta(n) b \bar{N} \sigma_8^2 \frac{a(w_g) (\hat{w}_s - \hat{w}_g)}{\hat{w}_s \hat{w}_g^{(1+n)}} \left( \frac{\theta}{10} \right)^{-(n+2)},
\]

where a good approximation of \( \zeta(n) \) is \( \zeta(n) \approx 1 + 1.04(n + 3/2) + 0.275(n + 3/2)^2 \), which is accurate to better than 0.2% for \( n \in [-2, -1] \).

Next we calculate \( \langle N^2 \rangle \) from (15) with the assumptions listed above. However, the occurrence of the factor \( p_g^2(w) \) in the integrand of (15) shows that we cannot simply use

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\(^2\) This choice corresponds to \( \ell = 1 \) in SvWJK
a delta-function distribution of the redshifts of the ‘foreground’ galaxies. The reason for this problem lies is the derivation of (15): it is valid only if the functions which are projected vary on a much larger scale than the largest wavelength on which the power spectrum has appreciable amplitude. This condition is obviously violated if the ‘foreground’ galaxies are very sharply localized. Therefore, we assume that $p_s$ is constant in an interval of width $\nu w_g$ around $w_g$, with $\nu \ll 1$. Then,

$$\langle N^2(\theta) \rangle = \frac{4\pi^3 \zeta_2 r^{3+n}}{\zeta_1} b^2 \hat{N}^2 \sigma_s^2 \frac{a^2(w_g)}{\nu \hat{w}_g^{n+3}} \theta^{-(n+2)}$$

$$= 1.215 \times 10^{-3} \zeta(n) b^2 \hat{N}^2 \sigma_s^2 \frac{a^2(w_g)}{\nu \hat{w}_g^{n+3}} \left( \frac{\theta}{10'} \right)^{-(n+2)}. \quad (18)$$

We compare the two terms in (14) to see which one dominates. Therefore, consider the ratio

$$\frac{1}{\lambda_1} := \frac{\langle N^2(\theta) \rangle \pi \theta^2}{\hat{N} \hat{G}} = 1.590 \zeta(n) \frac{b^2 \sigma_s^2 a^2(w_g)}{\nu \hat{w}_g^{n+3}} \left( \frac{\hat{N}}{5\text{arcmin}^{-2}} \right) \left( \frac{\theta}{10'} \right)^{-n}. \quad (19)$$

We thus see that $\lambda_1$ is typically smaller than unity in a situation when $\nu \ll 1$ and $\hat{w}_g < 1$ (note that $\hat{w}_g = 1/3$ corresponds to $z_g = 0.44$).

For the same model, $\langle M_{\text{ap}}^2(\theta) \rangle$ can be calculated. One finds:

$$\langle M_{\text{ap}}^2(\theta) \rangle = 2.733 \times 10^{-3} \zeta(n) \sigma_s^2 \hat{w}_s^{1-n} Z(n) \left( \frac{\theta}{10'} \right)^{(2+n)}, \quad (20)$$

where $Z(n) = \int_0^1 dx (1 - x)^2 x^{-n} = 2/[(1 - n)(2 - n)(3 - n)]$. Comparing the two terms in (13), we define the ratio

$$\frac{1}{\lambda_2} := \frac{\langle M_{\text{ap}}^2(\theta) \rangle}{\sigma_e^2 G} 2 N_b$$

$$= 35.8 \zeta(n) \sigma_s^2 \hat{w}_s^{1-n} \left( \frac{Z(n)}{0.05} \right) \left( \frac{\theta}{10'} \right)^{-n} \left( \frac{n_g}{20 \text{arcmin}^{-2}} \right) \left( \frac{\sigma_e}{0.2} \right)^2, \quad (21)$$

where $Z(-1.5)$ was taken as a fiducial value. We therefore conclude that the first term in (13) dominates in all cases of interest here (i.e., for angular scales larger than $\sim 10'$), $\lambda_2 \ll 1$. With (18) and (20), the signal-to-noise ratio becomes

$$\frac{C}{\sigma_0} = \left( 1 - \frac{w_g}{w_s} \right) \left( \frac{w_g}{w_s} \right)^{\frac{1-n}{2}} \sqrt{\frac{\nu}{Z(n)}} \frac{1}{\sqrt{(1 + \lambda_1)(1 + \lambda_2)}}, \quad (22)$$

with $\lambda_1 < 1$, $\lambda_2 \ll 1$ in typical situations. This ratio is indeed encouragingly large: Consider, for example, foreground and background galaxies with distance ratio $w_g/w_s = 1/2$; then, for a width parameter $\nu = 0.2$ and spectral index $n = -1.5$, the signal-to-noise ratio is 0.42 times the $\lambda$-dependent terms. Note that the dependence of the signal-to-noise ratio on the number density of foreground and background galaxies enters only through the $\lambda_i$-factors; as long as $\lambda_i \ll 1$, $C/\sigma_0$ is independent of these densities.
6 Discussion

In this letter, a statistical measure for the correlation $C$ of cosmic shear with the number density of ‘foreground’ galaxies was defined and calculated in terms of the power spectrum of cosmic density fluctuations. A practical unbiased estimator for this correlation was defined, and its dispersion calculated. On large angular scales, linear theory yields an accurate estimate for the power spectrum of cosmic density fluctuations; in the framework of this approximation, $C$ and the corresponding signal-to-noise ratio were evaluated explicitly.

A measurement of $C$ would yield, for each cosmological model and initial power spectrum $P(k)$, a direct estimate of the bias factor $b$ averaged over angular scale $\theta$. The method allows to probe the scale and redshift dependence of the bias parameter. On a short term, the detection of cosmic shear via the correlation $C$ is perhaps more useful: assuming the validity of the biasing hypothesis, $C$ is a first-order measure of the cosmic shear. The large signal-to-noise ratio (22) per single field shows that it should be much easier to get a significant detection of $C$ than for the previously proposed quadratic estimators of the shear.

Taking the EIS as an example, with ‘foreground’ galaxies chosen to have $I \leq 21$, and ‘background’ galaxies with $22 \leq I \leq 23.5$, the number densities will be approximately 3 and 7 per arcmin$^2$, and the characteristic redshifts of the two galaxy populations will be $\sim 0.3$ and $\sim 0.8$, respectively (e.g., Lilly et al. 1995). Hence, even in this case the $\lambda_i$ are smaller than unity, and the signal-to-noise ratio per field will be of order 0.3. Thus, with $\sim 100$ fields taken from the EIS, a 3-$\sigma$ detection of the correlation on angular scales larger than $\sim 10'$ should be possible, provided the data are of sufficient image quality.

In a future publication, $C$ will be calculated on smaller angular scales, using the fully non-linear evolution of the power spectrum (e.g., Peacock & Dodds 1996), and for different cosmological models. It is expected that the signal-to-noise ratio is not strongly affected by the non-linear evolution, and that an accurate measurement of $C(\theta)$ on small angular scales ($\sim$ few arcminutes) will be possible with a moderate amount of high-quality image data.

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