**Constraining the scales of supersymmetric left-right models**

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**Abstract.** We’ll review our study of the constraints on the scales in the supersymmetric left-right model (SUSYLR). The conservation of color and electromagnetism in the ground state of the theory implies a relation between right-handed gauge boson mass and soft squark mass. Furthermore, in general for heavy $W_R$, $\tan \alpha$ is larger than one, and the right-handed sneutrino VEV, responsible for spontaneous $R$-parity breaking, is at most of the order $M_{SUSSY}/h_{\Delta R}$, where $M_{SUSSY}$ is supersymmetry breaking scale and $h_{\Delta R}$ is the Yukawa coupling in Majorana mass term for right-handed neutrinos.

**INTRODUCTION**

The conservation of baryon (B) and lepton (L) numbers, which is automatic in the Standard Model, is not apparent in low energy supersymmetric models. In MSSM, baryon and lepton number violation can occur at tree level with fast proton decay unless the corresponding couplings are very small. The most common way to eliminate these tree level B and L violating terms is to impose so-called R-parity, $R_p = (-1)^{3(B-L)+2S}$, where $S$ is the spin of the particle. However, R-parity conservation is not required for the internal consistency of the minimal supersymmetric standard model.

It would be more appealing to have a supersymmetric theory, where R-parity is related to a gauge symmetry, and its conservation is automatic because of the invariance of the underlying theory under an extended gauge symmetry. Indeed $R_p$ conservation follows automatically in certain theories with gauged (B-L). It has been noted by several authors \[1,2\] that if the gauge symmetry of MSSM is extended to $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$ or $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the theory becomes automatically R-parity conserving. Such a left-right supersymmetric theory (SUSYLR) solves the problems of explicit B and L violation of MSSM and has received much attention recently \[3-7\].

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2) Permanent address
It has been found in a wide class of such models [1] that R-parity must be spontaneously broken [3] because of the form of the scalar potential. Thus in the model we have several different scales, namely the \( SU(2)_R \) breaking scale, R-parity breaking scale, SUSY breaking scale and finally the weak scale. For phenomenological reasons, it would be desirable to relate these scales with each other.

It has been shown [4] in the minimal SUSYLR model that the mass \( m_{W_R} \) of the right-handed gauge boson \( W_R \) has an upper limit related to the SUSY breaking scale, i.e., \( m_{W_R} \leq g M_{SU SY}/h_{\Delta R} \), where \( g \) is the weak gauge coupling and \( h_{\Delta R} \) is the Yukawa coupling of the right-handed neutrinos with the triplet Higgs fields. Here we review some further constraints on the scales [10] following from the conservation of electric charge and color by the ground state of the theory.

**SUPERSYMMETRIC LEFT-RIGHT MODEL**

The quark and lepton superfields are denoted by \( Q(2,1,1/3); Q^c(1,2,-1/3); L(2,1,-1); L^c(1,2,1), \) and the Higgs superfields by \( \Delta_L(3,1,-2); \Delta_R(1,3,-2); \delta_L(3,1,2); \delta_R(1,3,2); \Phi(2,2,0); \chi(2,2,0). \) The numbers in the parantheses denote the representation content of the fields under the gauge group \( SU(2)_L \times SU(2)_R \times U(1)_{B-L}. \) The model is described by the superpotential

\[
W = h_{\delta Q} Q^T \tau_2 \Phi Q^e + h_{\lambda Q} Q^T i\tau_2 \chi Q^e + h_{\delta L} L^T i\tau_2 \Phi L^e + h_{\chi L} L^T i\tau_2 \chi L^e + h_{\delta L} L^T i\tau_2 \delta_L L + h_{\Delta L} L^T c^* i\tau_2 \Delta_R L^e + \mu_1 \text{Tr}(i\tau_2 \Phi^T i\tau_2 \chi) + \mu_1' \text{Tr}(i\tau_2 \Phi^T i\tau_2 \Phi) + \mu_2 \text{Tr}(i\tau_2 \chi^T i\tau_2 \chi) + \text{Tr}(\mu_2 L \Delta_L \delta_L + \mu_2 R \Delta_R \delta_R). \tag{1}
\]

The vacuum expectation values of various scalar fields which preserve electric charge can be written as

\[
\langle \Phi \rangle = \begin{pmatrix} \kappa_1 & 0 \\ 0 & e^{i\varphi_2} \kappa_1' \end{pmatrix}, \quad \langle \chi \rangle = \begin{pmatrix} e^{i\varphi_2} \kappa_2' & 0 \\ 0 & \kappa_2 \end{pmatrix},
\]

\[
\langle \Delta_L \rangle = \begin{pmatrix} 0 & v_{\Delta L} \\ 0 & 0 \end{pmatrix}, \quad \langle \delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_{\delta L} & 0 \end{pmatrix},
\]

\[
\langle \Delta_R \rangle = \begin{pmatrix} 0 & v_{\Delta R} \\ 0 & 0 \end{pmatrix}, \quad \langle \delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_{\delta R} & 0 \end{pmatrix},
\]

\[
\langle L \rangle = \begin{pmatrix} \sigma_L \\ 0 \end{pmatrix}, \quad \langle L^c \rangle = \begin{pmatrix} 0 \\ \sigma_R \end{pmatrix}. \tag{2}
\]

The phases \( \varphi_1 \) and \( \varphi_2 \) are ignored in the following, although this does not affect the final conclusion. Due to the tiny mixing between the charged gauge bosons, \( \kappa_1' \) and \( \kappa_2' \) are taken to be much smaller than \( \kappa_1 \) and \( \kappa_2 \). Furthermore, since the electroweak \( \rho \)-parameter is close to unity, \( \rho = 1.0002 \pm 0.0013 \pm 0.0018 \), the triplet vacuum expectation values \( \langle \Delta_L \rangle \) and \( \langle \delta_L \rangle \) must be small. For definiteness, we shall take \( v_{\Delta R} \sim v_{\delta R} \sim v_R \); the generic scale of the right-handed symmetry breaking.
It has been shown on general grounds \cite{4} and explicitly \cite{7} that due to the $U(1)_{em}$ symmetry of the ground state of the model, the spontaneous breakdown of R-parity is inevitable in this class of models. Thus at least one of the sneutrinos has VEV in the minimum of the potential. We shall assume that $\sigma_L$ and $\sigma_R$ are non-zero. In electric charge preserving ground state $\sigma_R$ is necessarily at least of the order of the typical SUSY breaking scale $M_{SUSY}$ or the right-handed breaking scale $v_R$, whichever is lower \cite{9}.

**CONSTRAINTS FROM SQUARK MASSES**

It is straightforward to find the squark masses from the scalar potential of the model \cite{10}. We’ll consider the up- and down-squark mass matrices for the lightest generation (ignoring the intergenerational mixing) which gives the tightest constraint in our case. The part of the potential containing the squark mass terms can be written as

$$V_{\text{squark}} = \left( \begin{array}{cc} U_L^* & U_R^* \end{array} \right) \tilde{M}_U \left( \begin{array}{c} U_L \\ U_R \end{array} \right) + \left( \begin{array}{cc} D_L^* & D_R^* \end{array} \right) \tilde{M}_D \left( \begin{array}{c} D_L \\ D_R \end{array} \right).$$ \hspace{1cm} (3)

The diagonal mass matrix elements for the up-type squarks ($g_L, g_R, g_{B-L}$ are the gauge couplings and $\tilde{m}_Q^2 pr \tilde{m}_{Qe}^2$ are the soft squark masses) are given by

$$(\tilde{M}_U)_{UL} = \tilde{m}_Q^2 + m_u^2 + \frac{1}{4} g_L^2 (\omega_L^2 - 2\omega_L^2) + \frac{1}{6} g_{B-L}^2 (\omega_L^2 - \omega_R^2),$$

$$(\tilde{M}_U)_{UR} = \tilde{m}_{Qe}^2 + m_u^2 + \frac{1}{4} g_R^2 (\omega_R^2 - 2\omega_R^2) + \frac{1}{6} g_{B-L}^2 (\omega_R^2 - \omega_L^2),$$ \hspace{1cm} (4)

and for down-type squarks

$$(\tilde{M}_D)_{DL} = \tilde{m}_Q^2 + m_d^2 - \frac{1}{4} g_L^2 (\omega_L^2 - 2\omega_L^2) + \frac{1}{6} g_{B-L}^2 (\omega_L^2 - \omega_R^2),$$

$$(\tilde{M}_D)_{DR} = \tilde{m}_{Qe}^2 + m_d^2 - \frac{1}{4} g_R^2 (\omega_R^2 - 2\omega_R^2) + \frac{1}{6} g_{B-L}^2 (\omega_R^2 - \omega_L^2),$$ \hspace{1cm} (5)

where

$$m_u = h_{\phi Q} \kappa_1 + h_{\chi Q} \kappa_2, \quad m_d = h_{\phi Q} \kappa_1 + h_{\chi Q} \kappa_2.'$$ \hspace{1cm} (6)

and

$$\omega_L^2 = v_{\Delta L}^2 - v_{\Delta L}^2 - \frac{1}{2} \sigma_L^2, \quad \omega_R^2 = v_{\Delta R}^2 - v_{\Delta R}^2 - \frac{1}{2} \sigma_R^2, \quad \omega_{\kappa}^2 = \kappa_1^2 + \kappa_2^2 - \kappa_1^2 - \kappa_2^2. \hspace{1cm} (7)$$

In order not to break electromagnetism or color, none of the physical squared masses of squarks can be negative. Thus necessarily all the diagonal elements of the squark mass matrices are non-negative. Next we define an angle $\alpha$ by
\[ \tan^2 \alpha = \left( v_{\delta R}^2 + \frac{1}{2} \sigma_R^2 \right) / v_{\Delta R}^2 \]  

(8)

and write \( \tilde{m}_Q^2 = \tilde{m}_{Q'}^2 \equiv \tilde{m}^2 \). We then recall that the right-handed gauge boson mass is given by (ignoring weak scale effects) [4]

\[ m_{W_R}^2 = g_R^2 (v_{\Delta R}^2 + v_{\delta R}^2 + \frac{1}{2} \sigma_R^2) = g_R^2 v_{\Delta R}^2 (1 + \tan^2 \alpha). \]  

(9)

Then combining the diagonal elements of the mass matrices \( \tilde{M}_U \) and \( \tilde{M}_D \), and ignoring terms of the order of the weak scale or smaller, it follows that

\[ m_{W_R}^2 |\cos 2\alpha| \leq 4 \tilde{m}^2. \]  

(10)

If the \( W_R \) boson is lighter than twice the soft squark mass \( \tilde{m} \), Eq.(10) is fulfilled for any \( \tan \alpha \). If \( \tilde{m} \sim M_{SUSY} \sim 1 \) TeV as is commonly assumed, \( m_{W_R} \) cannot be much less, since experimentally \( m_{W_R} > 420 \) GeV [12]. On the other hand, if \( m_{W_R} \) is much larger than \( 2 \tilde{m} \), \( \tan \alpha \) has to be close to one, e.g. for \( m_{W_R} = 10 \) TeV and \( \tilde{m} = 1 \) TeV, one would need \( 0.96 \leq \tan \alpha \leq 1.04 \). It is interesting to note in this context that the vanishing of \( D \)-terms implies \( \tan \alpha = 1 \). To translate the limit for \( \tan \alpha \) to an upper bound for the VEV \( \langle \Delta_0 R \rangle \), one needs a lower limit for \( g_R \). This was found in [13] from \( \sin^2 \theta_W = \frac{e^2}{g_L^2} = 0.23 \), namely \( g_R \geq 0.55 g_L \). Consequently \( v_{\Delta R} \leq m_{W_R} |\cos \alpha|/(0.55 g_L) \), e.g. in our example \( v_{\Delta R} \lesssim 20 \) TeV.

**CONSTRAINTS FROM DOUBLY CHARGED HIGGS MASSES**

In the case of large \( m_{W_R} \), we note that if the right-handed scale and R-parity breaking scale differ from each other, one has \( v_{\Delta R}, v_{\delta R} > \sigma_R \), since \( \tan \alpha \sim 1 \). We recall then the doubly charged Higgs mass matrix [4] given by (ignoring terms suppressed by \( \sigma_R/v_{\Delta R} \) or \( \sigma_R/v_{\delta R} \))

\[ M_{\Delta \pm \delta \pm}^2 = \begin{pmatrix} m_{\Delta \delta}^2 v_{\Delta}^2 & -4 h_{\Delta}^2 \sigma_R^2 & -m_{\Delta \delta}^2 \sigma_R^2 \\ -h_{\Delta}^2 \sigma_R^2 & m_{\Delta \delta}^2 v_{\Delta}^2 / v_{\delta}^2 & 2 g_R^2 \omega_R^2 \\ -m_{\Delta \delta}^2 \sigma_R^2 & 2 g_R^2 \omega_R^2 & m_{\Delta \delta}^2 v_{\Delta}^2 / v_{\delta}^2 \end{pmatrix}, \]  

(11)

where \( m_{\Delta \delta} \) is the soft parameter mixing right-handed Higgs triplets. The two eigenvalues of the mass matrix need to be real and non-negative in order not to break \( U(1)_{em} \). This leads to two conditions:

\[ h_{\Delta R}^2 \sigma_R^2 \leq \frac{1}{4} m_{\Delta \delta}^2 \left( \frac{v_{\delta R}}{v_{\Delta R}} + \frac{v_{\Delta R}}{v_{\delta R}} \right) \]  

(12)

and

\[-2 m_{\Delta \delta}^2 g_R^2 \omega_R^2 \left( \frac{v_{\delta R}}{v_{\Delta R}} - \frac{v_{\Delta R}}{v_{\delta R}} \right) + 4 g_R^4 \omega_R^4 + 4 m_{\Delta \delta}^2 h_{\Delta R}^2 \sigma_R^2 \frac{v_{\Delta R}}{v_{\delta R}} + 8 h_{\Delta R}^2 \sigma_R^2 g_R^2 \omega_R^2 \leq 0 \]  

(13)
From (12) we see that \( h_{\Delta R} \sigma_R \) can be at most of the order of \( m_{\Delta \delta} \sim M_{\text{SUSY}} \). To fulfill the inequality (13) we can consider two cases: \( v_{\Delta R} < v_{\delta R} \) and \( v_{\delta R} < v_{\Delta R} \). In both cases one must have \( \omega^2_R \leq 0 \) or equivalently \( \tan \alpha \geq 1 \). The equality can hold for \( \sigma_R = 0 \) and \( v_{\delta R} = v_{\Delta R} \).

**SUMMARY**

In this talk we discussed the constraints on scales in SUSY left-right model. The \( W_R \) mass and the soft squark mass are related by (10), which implies that either the scale of the right-handed gauge symmetry breaking must be close to the SUSY breaking scale, or \( \tan \alpha \sim 1 \) corresponding to vanishing \( D \)-terms. In general, for large \( m_{W_R} \), the right-handed sneutrino VEV is constrained to be at most of the order \( M_{\text{SUSY}}/h_{\Delta R} \), and \( \tan \alpha \) is larger than one.

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**REFERENCES**