Exclusive Hadronic $B$-Decays\footnote{Plenary talk presented at the b20 Symposium: Twenty Beautiful Years of Bottom Physics, Chicago, June 29-July 2, 1997}

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Abstract

ExCLUSIVE non-leptonic two-body decays are discussed on the basis of a generalized factorization approach which also includes non-factorizeable contributions. Numerous decay processes can be described satisfactorily. The success of the method makes possible the determination of decay constants from non-leptonic decays. In particular, we obtain $f_{D_s} = (234\pm25)\text{ MeV}$ and $f_{D_s^*} = (271\pm33)\text{ MeV}$. The observed constructive and destructive interference pattern in charged $B$- and $D$-decays, respectively, can be understood in terms of the different $\alpha_s$-values governing the interaction among the quarks. The running of $\alpha_s$ is also the cause of the observed strong increase of the amplitude of lowest isospin when going to low energy transitions.
1 Introduction

Since we celebrate today 20 years of beauty physics it may be appropriate to start the discussion of hadronic weak interactions by briefly recalling what was known about this subject in the seventies. In spite of many years of intense research on $K$- and hyperon decays, there was no coherent understanding of non-leptonic decays. For example, the empirically found dominance of $|\Delta I| = 1/2$ transitions over $|\Delta I| = 3/2$ transitions by a factor 500 was a complete mystery. Moreover, the strongest of all weak decay amplitudes - the $K \to 2\pi$ amplitude - was found to have to vanish in the $SU3$ symmetry limit (Gell-Mann’s theorem) and no close relation between $K$-decays and hyperon decays could be seen. In 1974 an important step forward was made: the construction of an effective Hamiltonian which incorporates the effects of hard gluon exchange processes\cite{1}. Still, a factor 20 out of the factor 500 could not be explained, nor could the specific pattern of hyperon decays. The physics at this time dealing with $u, d$ and $s$-quarks was not rich enough. In the corresponding decay processes too few fundamentally different decay channels are open.

The discovery of open charm in 1976 brought hope for enlightenment. Many decay channels could now be studied. But also new puzzles showed up. Unexpectedly, the non-leptonic widths of $D^0$ and $D^+$ turned out to differ by a factor 3 and a strong destructive amplitude interference in exclusive decays was found. While $D$-decays occur in a resonance region of the final particles which complicates the analysis, the discovery of beauty precisely 20 years ago gave us particles – the $B$-mesons – which are ideally suited for the study of non-leptonic decays. Again, new interesting effects showed up, in particular and contrary to the case in $D$-decays, a constructive amplitude interference in charged $B$-decays. Recent results\cite{2} of large Penguin-type contributions and sizeable transitions to the $\eta'$ particle have still to be understood. Moreover, $B$-meson decays give the first realistic possibility to find CP-violating effects outside the $K$-system.

The dramatic effects observed in hadronic weak decays gave rise to many speculations. It was a great challenge to find the correct explanation. Today we know that the strong confining colour forces among the quarks are the decisive factor. These forces are enormously effective in low energy processes and still sizeable even in energetic $B$-decays. Although a strict theoretical treatment of the intricate interplay of weak and strong forces is not yet possible, a semi-quantitative understanding of exclusive two-body decays from $K$-decays to $D$- and $B$-decays has been achieved. The consequences of the QCD-modified weak Hamiltonian can be explored by relating the complicated matrix elements of 4-quark operators to better known objects, to form factors and decay constants.
In the present talk I will describe the generalized factorization method developed recently [3], which also takes non-factorizable contributions into account and has been quite successful so far. It allows the prediction of many exclusive $B$-decays. I will also show that the interesting and so far puzzling pattern of amplitude interference in $B$, $D$- and $K$-decays is caused by the different values of $\alpha_s$ acting in these cases.

2 The effective Hamiltonian

At the tree level non-leptonic weak decays are mediated by single $W$-exchange. Hard gluon exchange between the quarks can be accounted for by using the renormalization group technique. One obtains an effective Hamiltonian incorporating gluon exchange processes down to a scale $\mu$ of the order of the heavy quark mass. For the case of $b \to c \bar{d} u$ transitions, e.g., the effective Hamiltonian is

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} V^*_{cd} \left\{ c_1(\mu)(\bar{d}u)(\bar{c}b) + c_2(\mu)(\bar{c}u)(\bar{d}b) \right\}$$

(1)

where $\langle \bar{d}u \rangle = (\bar{d} \gamma^\mu (1 - \gamma_5) u)$ etc. are left-handed, colour singlet quark currents. $c_1(\mu)$ and $c_2(\mu)$ are scale-dependent QCD coefficients known up to next-to-leading order [4]. Depending on the process considered, specific forms of the four-quark operators in the effective Hamiltonian can be adopted. Using Fierz identities one can put together those quark fields which match the constituents of one of the hadrons in the final state of the decay process. Let us consider, as an example, the decays $B \to D \pi$. The corresponding amplitudes are – apart from a common factor –

$$\mathcal{A}_{B^0 \to D^+ \pi^-} = \left( c_1 + \frac{c_2}{N_c} \right) \langle \bar{D}^+ \pi^- | (\bar{d}u)(\bar{c}b) | B^0 \rangle,$$

$$+ c_2 \langle \bar{D}^+ \pi^- | \frac{1}{2} (\bar{d} \sigma^\mu \pi^\mu u)(\bar{c} \sigma^\mu b) | B^0 \rangle$$

$$\mathcal{A}_{B^0 \to D^0 \pi^0} = \left( c_2 + \frac{c_1}{N_c} \right) \langle \bar{D}^0 \pi^0 | (\bar{d} u)(\bar{c} b) | B^0 \rangle$$

$$+ c_1 \langle \bar{D}^0 \pi^0 | \frac{1}{2} (\bar{d} \sigma^\mu \pi^\mu u)(\bar{c} \sigma^\mu b) | B^0 \rangle$$

$$\mathcal{A}_{B^- \to D^- \pi^-} = \mathcal{A}_{B^0 \to D^+ \pi^-} - \sqrt{2} \mathcal{A}_{B^0 \to D^0 \pi^0}.$$

(2)

$N_c$ denotes the number of quark colours and $t^a$ the Gell-Mann colour $SU(3)$ matrices. The last relation in (2) follows from isospin symmetry of the strong interactions. The three classes of decays illustrated in eq. (2) are referred to as class I, class II, and class III respectively.
3 Generalized Factorization

How shall we deal with the complicated and scale-dependent four-quark operators? Because the $(\bar{d}u)$ and the $(\bar{c}u)$ currents in (2) can generate the $\pi^-$ and $D^0$ mesons, respectively, the above amplitudes contain the scale-independent factorizable parts

$$\mathcal{F}_{(BD)\pi} = \langle \pi^- | (\bar{d}u) | 0 \rangle \langle D^+ | (\bar{c}b) | \bar{B}^0 \rangle,$$
$$\mathcal{F}_{(B\pi)_D} = \langle D^0 | (\bar{c}u) | 0 \rangle \langle \pi^0 | (\bar{d}b) | B^0 \rangle$$  \hspace{1cm} (3)

which can be expressed in terms of the decay constants $f_\pi$ and $f_D$, and the single current transition form factors $B \to D$ and $B \to \pi$, respectively. For the non-factorizable contributions we define hadronic parameters $\epsilon_1(\mu)$ and $\epsilon_8(\mu)$ such that the amplitudes (2) take the form[5, 3]

$$A_{B^0\to D^+\pi^-} = a_1 \mathcal{F}_{(BD)\pi},
A_{B^0\to D^0\pi} = a_2 \mathcal{F}_{(B\pi)_D},
a_1 = (c_1(\mu) + \frac{c_2(\mu)}{N_c})(1 + \epsilon_1^{(BD)\pi}(\mu)) + c_2(\mu)\epsilon_8^{(BD)\pi},
a_2 = (c_2(\mu) + \frac{c_1(\mu)}{N_c})(1 + \epsilon_1^{(B\pi)_D}(\mu)) + c_1(\mu)\epsilon_8^{(B\pi)_D} .$$  \hspace{1cm} (4)

The effective coefficients $a_1$ and $a_2$ are scale-independent. $\epsilon_1$ and $\epsilon_8$ obey renormalization-group equations and their scale dependence compensates the scale dependence of the QCD coefficients $c_1$ and $c_2$ [3]. $a_1$ and $a_2$ are process-dependent quantities because of the process dependence of the hadronic parameters $\epsilon_1$ and $\epsilon_8$. So far, then, Eq. (4) provides a parametrization of the amplitudes only and allows no predictions to be made. To get predictions, non-trivial properties of QCD have to be taken into account. We employ at this point the $1/N_c$ expansion of QCD. The large $N_c$ counting rules tell us that $\epsilon_1 = O(1/N_c^2)$ and $\epsilon_8 = O(1/N_c)$. Thus one obtains for $a_1$ and $a_2$ in (4)

$$a_1 = c_1(\mu) + c_2(\mu)\left(\frac{1}{N_c} + \epsilon_8^{(BD)\pi}(\mu)\right) + O(1/N_c^2),
a_2 = c_2(\mu) + c_1(\mu)\left(\frac{1}{N_c} + \epsilon_8^{(B\pi)_D}(\mu)\right) + O(1/N_c^2) .$$  \hspace{1cm} (5)

For $B$-decays using $c_1(m_b) = 1 + O(1/N_c^2)$ and $c_2(m_b) = O(1/N_c)$ one finally gets[3]
\[ a_1 = c_1(m_b) + O(1/N_c^2) \]
\[ a_2 = c_2(m_b) + \zeta^B c_1(m_b) + O(1/N_c^2) \]  

with \[ c_1(m_b) \approx 1 \quad \text{and} \quad \zeta^B = \frac{1}{N_c} + \epsilon_8^{(B\pi)_D}(m_b) \].

Now, neglecting \( O(1/N_c^2) \) terms, we are left with a single parameter (\( \zeta^B \)) only. It should be emphasized that putting this parameter equal to \( 1/N_c \) does not correspond to any consistent limit of QCD. For \( a_2 \) the more general expression (6) must be used[7, 6].

\( \zeta^B \) is a dynamical parameter: In general, it will take different values for different decay channels. To deal with this, let us introduce a process-dependent factorization scale \( \mu_f \) defined by \( \epsilon_8(\mu_f) = 0 \). The renormalization-group equation then gives[3]

\[ \epsilon_8(\mu) = -\frac{4\alpha_s}{3\pi} \ln \frac{\mu}{\mu_f} + O(\alpha_s^2) \]  

(7)

For different processes the variation of the factorization scale \( \mu_f \) is expected to scale with the energy release to the outgoing hadrons in the decay process. With \( \mu_f \approx O(m_b) \) one gets from (6), (7)

\[ \Delta \zeta^B \approx \frac{4\alpha_s}{3\pi} \frac{\Delta \mu_f}{m_b} \approx \text{few \%} \]  

(8)

Thus, the process dependence of \( \zeta^B \) is expected to be very mild. To a good approximation a single value appears sufficient for the description of two-body \( B \)-decays. One finds (see section 4) \( \zeta^B = 0.45 \pm 0.05 \).

A similar discussion also holds for \( D \)-decays. There one is led to[3]

\[ a_1 \approx c_1(m_c) + \zeta'^D c_2(m_c) \]
\[ a_2 \approx c_2(m_c) + \zeta'^D c_1(m_c) \]
\[ \zeta'^D \approx \zeta^D \]  

(9)

and again expects only a mild process dependence of \( \zeta^D \). Indeed, the corresponding description of exclusive \( D \)-decays brought reasonable success. \( \zeta^D \) turned out to be very small or zero. There is also theoretical support (using QCD sum rule methods) for a partial or full cancellation of the \( 1/N_c \) term by non-factorizable contributions[8]. On the other hand, the corresponding calculation of \( \zeta^B \) is more involved[9] and was so far not successful.
4 Determination of $a_1$ and $a_2$

The most direct way to determine the effective constant $a_1$ consists in comparing non-leptonic decay rates with the corresponding differential semi-leptonic rates at momentum transfers equal to the masses of the current generated particles\cite{10}. One gets, for example,

$$\Gamma (\bar{B}^0 \to D^{(*)}+\rho^-) \over d\Gamma (\bar{B}^0 \to D^{(*)}+\ell^-\nu) / dq^2 |q^2=m_{\rho}^2| = 6\pi^2|V_{ud}|^2 f_{\rho}^2 |a_1|^2 .$$  \hspace{1cm} (10)

Because the generated particle is a vector particle like the lepton pair, the form factor combinations occurring in the nominator and denominator cancel precisely. Thus, the ratio (10) is solely determined by $|a_1|$ and the $\rho$-meson decay constant $f_{\rho}$. Taking by convention $a_1$ real and positive, the measured rates\cite{11} give $a_1=1.09\pm0.13$ in agreement with the expectation (6). $a_1$ values obtained from several other processes are in full agreement with the above number. In transition to pseudoscalar particles the form factor combinations in equations replacing (10) do not cancel. But for $B \to D, D^*$ matrix elements all form factors are well determined using experimental data and the heavy quark effective theory\cite{12}. The latter relates in particular longitudinal form factors to the transversal ones.

Values for $|a_2|$ can be obtained from the analysis of class II transitions. The decays $\bar{B}^0 \to D^{(*)}h^0$ ($h^0: \pi^0, \rho^0, a_1^0$) have not yet been observed, but the branching ratios for $\bar{B} \to K^{(*)}J/\psi$ and $\bar{B} \to K^{(*)}\psi(2S)$ are available\cite{11}. The analysis requires model estimates for the heavy-to-light form factors, which enter here. We use the NRSX model\cite{13} which is based on the extrapolation of the BSW form factors\cite{6} at $q^2=0$ by appropriate pole and dipole formulae. Where available, more sophisticated calculations agree with these results. (See. e.g. Ref. 14). We find\cite{3} $|a_2|=0.21\pm0.01\pm0.04$, where the second error accounts for the model dependence.

The relative phase between $a_2$ and $a_1$ together with the magnitude of $a_2$ can be obtained from the decays $B^- \to D^{(*)}h^-$ where, as seen from (2) and (4), the two amplitudes interfere. The data for the ratios $\Gamma (B^- \to D^{(*)}h^-) / \Gamma (\bar{B}^0 \to D^{(*)}+h^-)$ give conclusive evidence for constructive interference\cite{11}. Taking $a_2$ to be a real number (vanishing final state interaction), we find\cite{3} $a_2/a_1=+0.21\pm0.05\pm0.04$. Combined with the value for $a_1$ this gives $a_2=+0.23\pm0.05\pm0.04$. The nice agreement between the two determinations of $|a_2|$ shows that the process dependence of this quantity cannot be large. There is no evidence for it. An analysis with an alternative and very simple form factor model gives slightly larger values for $a_2$ but the results from different processes are again consistent with each other\cite{3}.
The positive value for \( a_2/a_1 \) in exclusive \( B \)-decays is remarkable. It is different from the value of the same ratio in exclusive \( D \)-decays. There \( a_2/a_1 \) is negative causing a sizeable destructive amplitude interference. The change of \( a_2/a_1 \) by going from \( B \)- to \( D \)-and \( K \)-decays will be discussed in section 6.

5 Tests and Results

The \( B \)-meson, because of its large mass, has many decay channels. We learned from important examples the values of \( a_1 \) and \( a_2 \) and their near process-independence in energetic two-body decays. Thus numerous tests and predictions for branching ratios and for the polarizations of the outgoing particles can be made. I will be very brief here and simply refer to Ref. 3 for the compilation of branching ratios in tables, for a detailed discussion and for comparison with the data. Also discussed there is the possible influence of final state interactions. Limits on the relative phases of isospin amplitudes are given. In contrast to \( D \)-decays final state interactions do not seem to play an important role for the dominant exclusive \( B \)-decay modes. For the much weaker Penguin-induced transitions, \( \bar{B} \to K^{(*)}\pi \) for example, this statement does not hold. Small amplitudes can get an additional contribution from stronger decay channels[6, 15]. In the \( \bar{B} \to K^{(*)}\pi \) case the decay can proceed via virtual intermediate \( D^{(*)}\bar{D}^{(*)} \) like channels generated by the \( b \to c\bar{c}s \) interaction. The colour octet \( c\bar{c} \) pair, if at low invariant mass, may then turn into a pair of light quarks by gluon exchange. This gives rise to a ”long range Penguin” contribution[15] in addition to the short distance Penguin amplitude. In future application of our generalized factorization method to rare decays this should be kept in mind. Here, however, I will not discuss this subject further.

Non-leptonic decays to two spin-1 particles also need a separate discussion. Here one has 3 invariant amplitudes corresponding to outgoing \( S \), \( P \), and \( D \)-waves. Non-factorizeable contributions to these amplitudes may, in general, have an amplitude composition different from the factorizeable one which cannot be dealt by introducing effective \( a_1 \) parameters. Whether or not and to what extent factorization also holds in these more complicated circumstances can be learned from the polarization of the final particles. In class I decays the factorization approximation predicts a polarization identical to the one occurring in the corresponding semi-leptonic decays at the appropriate \( q^2 \) value. For \( B \to D^*V \) decays the theoretical predictions have very small errors only[3]. Another case of particular interest is the polarization of the \( J/\psi \) particle in the decay \( B \to K^*J/\psi \). Form factor models predict a longitudinal polarization of around 40\%. A recent CLEO measurement[16] gives \((52 \pm 7 \pm 4)\% \). It can be shown[17] that small changes of the ratios of form factors obtained
in the NRSX model at $q^2 = m^2_{J/\psi}$ are sufficient to get full agreement with the measurements of the longitudinal as well as both transverse polarizations. At present, even with respect to polarization measurements, the generalized factorization approximation is in agreement with the data.

Because of its success, the generalized factorization method, besides allowing many predictions for yet unmeasured decays, can also be used to determine unknown decay constants. A case in point is the determination of the decay constant of the $D_s$ and $D_s^*$ particles. Comparing non-leptonic decays to $D_s, D_s^*$ with those to light mesons, we find\[ \text{(11)} \]

$$f_{D_s} = (234 \pm 25) \text{ MeV}, \quad f_{D_s^*} = (271 \pm 33) \text{ MeV}.$$ 

In this determination $a_1$ cancels and, presumably, also some of the experimental systematic errors. The value for $f_{D_s}$ is in excellent agreement with the value $f_{D_s} = (241 \pm 37) \text{ MeV}$ obtained from the leptonic decay of the $D_s$ meson\[ \text{[18]} \]. There are several other decay constants which can be measured this way. Of particular interest are the decay constants of $P$-wave mesons like the $a_0, a_1, K^0, K_1$ particles.

6 From $B$- to $D$- to $K$-Decays

The process dependence of the coefficients $a_1$ and $a_2$ governing exclusive $B$-decays turned out to be very mild. In fact, it is not seen within the errors of the present data. But $a_1$ and $a_2$ change strongly by going from $B$-decays to $D$-decays or even down to $K$-decays. In the generalized factorization scheme this is expected because of the different factorization scales and the corresponding $\alpha_s(\mu_f)$ values controlling the strength of the colour forces between the quarks. In Fig. 1 the ratio $a_2/a_1$ is plotted as a function of $\alpha_s(\mu_f)$. We used for the Wilson coefficients the renormalization group invariant definitions of Ref. 19. It appears appropriate for describing the changes of the scale-independent coefficients $a_1$ and $a_2$ with changing the particle energy. As seen from the figure the positive value of $a_2/a_1$ found for exclusive $B$-decays indicates that here small values of $\alpha_s$ govern the colour forces in the first instant of the decay process. This is an impressive manifestation of the colour transparency argument put forward by Bjorken\[ \text{[10]} \]. In $D$-decays the stronger gluon interactions redistribute the quarks: the induced neutral current interaction is already sizeable. We took the corresponding values of $a_1$ and $a_2$ from the measured isospin amplitudes. They are less affected by final state interactions than the individual amplitudes. The ratio $|A_{1/2}/A_{3/2}|$ is already rather large ($\approx 4$) leading to $a_2/a_1 \approx -0.45$. According to the figure this corresponds to an effective value $\alpha_s \approx 0.7$. The negative value of $a_2$, and the corresponding destructive amplitude interference in charged $D$-decays, has been known for many years\[6, 20\].
Figure 1: The ratio $a_2/a_1$ as a function of the running coupling constant evaluated at the factorization scale. The bands indicate the phenomenological values of $a_2/a_1$ extracted from $\bar{B} \rightarrow D\pi$ and $D \rightarrow K\pi$ decays.

Since the bulk of $D$-decays are two-body or quasi two-body decays, it is the main cause for the lifetime difference of $D^+$ and $D^0$ in full accord with estimates of the relevant partial inclusive decay rates[21].

Because of the onset of non-perturbative effects one cannot extent Fig. 1 down to larger $\alpha_s$ values. However, the trend to smaller and smaller values of the ratio of the Wilson coefficients $c_+(\mu_f)/c_-(\mu_f)$, which is already down to $\approx 0.17$ for $D$-decays, is visible. It indicates a strong and, presumably, non-perturbative force in the colour $3^*$ channel of two quarks, i.e. in the scalar diquark channel[22]. In $K$-decays one is very close to the limiting case $a_2/a_1 = -1$ for which the $|\Delta f| = 1/2$ rule would hold strictly.

7 Conclusions

The matrix elements of non-leptonic exclusive decays are notoriously difficult to calculate. Factorization provides for a connection with better known objects. If combined with the $1/N_c$ expansion method and properly applied and interpreted, it turns out to be very useful, at least for energetic $B$-decays, and has passed many tests. Thus it enables reliable predictions for many decay channels to be made and also permits the determination of decay constants which are difficult to measure otherwise. Factorization does not necessarily hold to the same degree for transitions to two vector particles. These are more sensitive to non-factorizeable contributions and final state interactions.

The constant $a_1$ is predicted to be one apart from $1/N_c^2$ corrections in exclusive $B$-decays and to be practically process-independent. The analysis confirmed these expectations. The particularly interesting parameter $a_2$, within
errors, also does not show a process dependence. The positive value of $a_2/a_1$ extracted from exclusive $B$-decays is remarkable. The obvious interpretation is that a fast-moving colour singlet quark pair interacts little with soft gluons. The constructive interference in energetic two-body $B^-$-decays does not imply that the lifetime of the $B^-$-meson should be shorter than the lifetime of the $B^0$ meson: The majority of transitions proceed into multi-body final states. For these the relevant scale may be lower than $m_b$ leading to destructive interference. Also, there are many decay channels for which interference cannot occur. The running of $a_1$ and $a_2$ with $\alpha_s(\mu_f)$, which in turn depends on the energy release to the final particles, is very interesting. It causes the change from constructive amplitude interference in $B^-$-decays to strong destructive interferences in $D$- and $K$-decays. Since exclusive two-body and quasi two-body decays are dominant in $D$-decays this destructive interference is the main cause of the lifetime difference between $D^0$ and $D^+$. By going to low energies the lowest isospin amplitude is seen to become more and more dominant. Strange particle decays are the most spectacular manifestation of the dramatic changes occuring when the effective $\alpha_s$ gets large. A unified picture of exclusive non-leptonic decays emerges which ranges from very low scales to the large energy scales relevant for $B$-decays.

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References


