New Mechanisms of Dynamical Supersymmetry Breaking and Direct Gauge Mediation

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Abstract

We construct supersymmetric gauge theories with new mechanisms of dynamical supersymmetry breaking. The models have flat directions at the classical level, and different mechanisms lift these flat directions in different regions of the classical moduli space. In one branch of the moduli space, supersymmetry is broken by confinement in a novel manner. The models contain only dimensionless couplings and have large groups of unbroken global symmetries, making them potentially interesting for model-building. As an illustrative application, we couple the standard model gauge group to a model with an SU(5) global symmetry, resulting in a model with composite messengers and a non-minimal spectrum of superpartner masses.

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1 Introduction

The last few years have seen a revival of interest in models in which supersymmetry is broken at low energy scales [1, 2]. In this work, there has been a fruitful interplay between theoretical progress in understanding dynamical supersymmetry breaking [3, 4, 5, 6, 7] and model-building (for recent progress in gauge-mediated model-building, see e.g. Refs. [8, 9, 10, 11, 12]).

In this paper, we construct a class of models that exhibit a new mechanism of supersymmetry breaking. In these models, there is a classical flat direction that can be parameterized by a composite “baryon” chiral superfield \( B \sim Q^N \), where \( Q \) is an elementary chiral superfield. This field gets a dynamical superpotential

\[
W_{\text{dyn}} \sim B^p \sim Q^{Np}.
\]  

(1.1)

For large \( Q \), the Kähler potential is approximately canonical in \( Q \), so if \( Np > 1 \) the potential for \( B \) slopes toward \( B = 0 \). For small \( B \), the models exhibit smooth confinement (“s-confinement”) [13, 14], and the Kähler potential is smooth in \( B \). In this case, if \( p < 1 \) the potential for \( B \) slopes away from \( B = 0 \). Since the vacuum energy does not vanish for any value of \( B \), supersymmetry is broken with \( \langle B \rangle \neq 0 \).

The models considered here have additional classical flat directions as well as large groups of global symmetries. We are able to obtain a great deal of information about the location of the global minimum in the field space, but some important properties of the ground state depend on non-calculable strong dynamics.

We then use the models constructed above as building blocks for realistic models of gauge-mediated supersymmetry breaking. We construct an illustrative example by gauging a global symmetry with the standard-model gauge group. The resulting model has composite fermions that are charged under the standard-model gauge group, and we add additional interactions so that the composite fermions obtain a Dirac mass with elementary fields. This model can be realistic, and gives rise to interesting phenomenology. (For the model to work, we must make some assumptions about the signs of non-calculable Kähler terms, and the supersymmetry-breaking masses are also non-calculable.)

This paper is organized as follows. In Section 2, we describe models that realize the supersymmetry-breaking mechanism described above. In Section 3, we construct gauge-mediated supersymmetry breaking models. Section 4 contains our conclusions. Some additional supersymmetry-breaking models related to the models discussed in

\[1\] The model of Ref. [6] also breaks supersymmetry by confinement, but that model has a linear potential at the origin.
Section 2 are analyzed in the Appendix. These models also have classical flat directions and break supersymmetry through novel mechanisms.

2 $Sp(2N) \times SU(2N - 1)$ models

In this Section, we analyze models with gauge and global symmetry group

$$G = Sp(2N) \times SU(2N - 1) \times [SU(2N - 1) \times U(1) \times U(1)_R],$$

(2.1)

where the global symmetries are written in brackets. The matter content is

$$Q \sim (\square, \square) \times (1; 1, 1),$$

$$L \sim (\square, 1) \times (\square, -1, -\frac{3}{2N - 1}),$$

$$\bar{U} \sim (1, \square) \times (\square, 0, \frac{2N + 2}{2N - 1}),$$

$$\bar{D} \sim (1, \square) \times (1; -6, -4N),$$

(2.2)

and there is a tree-level superpotential

$$W = \lambda QL\bar{U}.$$

(2.3)

The field content and superpotential of this model are reminiscent of the “3–2” model of dynamical supersymmetry breaking [4]. (In fact we will see that the dynamics is similar to that of the 3–2 model in one branch of the moduli space.) If we turn off the $Sp(2N)$ gauge coupling and the superpotential, $SU(2N - 1)$ s-confines for any $N \geq 2$ [13]. If we turn off the $SU(2N - 1)$ gauge coupling and the superpotential, $Sp(2N)$ is in a non-Abelian Coulomb phase for $N \geq 6$, it has a weakly-coupled dual description for $N = 4, 5$, it s-confines for $N = 3$, and confines with a quantum-deformed moduli space for $N = 2$ [13, 15].

If we include the effects of the tree-level superpotential, this theory has a classical moduli space that can be parameterized by the gauge-invariants

$$M_{LL} = LL \sim (\square, -2, -\frac{6}{2N - 1}),$$

$$\bar{B}_U = U^{2N - 2}\bar{D} \sim (\square; -6, -\frac{4(N^2 - N + 1)}{2N - 1}),$$

$$\bar{B}_D = U^{2N - 1} \sim (1; 0, 2N + 2),$$

(2.4)

subject to the constraints

$$(M_{LL})^{jk}(\bar{B}_U)^\ell \epsilon_{\ell k \ell_1 \cdots \ell_{2N - 3}} = 0, \quad (M_{LL})^{jk}\bar{B}_D = 0.$$

(2.5)

\footnote{In our conventions, the fundamental representation of $Sp(2N)$ has dimension $2N$.}
These constraints split the moduli space into two branches: on one of them \( M_{LL} = 0 \) and \( \bar{B}_U, \bar{B}_D \neq 0 \), and on the other \( M_{LL} \neq 0 \) and \( \bar{B}_U, \bar{B}_D = 0 \).

### 2.1 The “Baryon” Branch

We first consider the branch where \( \bar{B}_U, \bar{B}_D \neq 0 \). In terms of the elementary fields, this corresponds to the vacuum expectation values (up to gauge and flavor transformations)

\[
\langle \bar{U} \rangle = \begin{pmatrix} v \cos \theta \\ v \mathbf{1}_{2N-2} \end{pmatrix}, \quad \langle \bar{D} \rangle = \begin{pmatrix} v \sin \theta \\ 0 \\ \vdots \\ 0 \end{pmatrix},
\]

where \( \mathbf{1}_{2N-2} \) is the \((2N - 2)\)-dimensional identity matrix. Far out along this flat direction, the \( SU(2N-1) \) gauge group is completely broken, and the fields \( Q \) and \( L \) get masses of order \( \lambda v \) (for \( \cos \theta \neq 0 \)). Below the scale \( \lambda v \), the effective theory is \( Sp(2N) \) super Yang–Mills, and gaugino condensation in this theory gives rise to the dynamical superpotential

\[
W_{\text{eff}} \simeq \frac{\Lambda_{\text{Sp}}^3}{16\pi^2} \left( \frac{4\pi \lambda \bar{U}}{\Lambda_{\text{Sp}}} \right)^{(2N-1)/(N+1)}. \tag{2.7}
\]

For \( v \gg \Lambda_{SU} \), the Kähler potential is approximately canonical in \( \bar{U} \), and so the potential for \( \bar{U} \) slopes toward \( \bar{U} = 0 \) for \( N > 2 \). (The special case \( N = 2 \) will be considered separately below.) However, if \( \bar{U} \) becomes small, we must reconsider the analysis.\(^3\)

The physics for small field values depends on the relative strength of the two gauge groups. We first consider \( \Lambda_{SU} \gg \Lambda_{Sp} \). (This is the situation that would arise if the two groups were unified at a higher scale.) In this case, the analysis above breaks down for \( v \lesssim \Lambda_{SU}/(4\pi) \), the scale at which the massive \( SU(2N-1) \) gauge bosons have mass \( g_{SU} v \sim \Lambda_{SU} \) according to “naïve dimensional analysis” [16]. For small values of \( \langle \bar{B}_D \rangle \), we can use a description where \( SU(2N-1) \) s-confines, and we obtain an effective theory (after integrating out states with mass \( \sim \lambda \Lambda_{SU}/(4\pi) \)) with symmetry group

\[
G_{\text{eff}} = Sp(2N) \times [SU(2N-1) \times U(1) \times U(1)_R], \tag{2.8}
\]

\(^3\)The analysis for the case \( \cos \theta = 0 \) is somewhat different. In that case, the \( Sp(2N) \) theory has one light flavor that would run away if there were no other interactions. However, the runaway direction is not \( D \) flat, and so there is no supersymmetric vacuum with \( \cos \theta = 0 \).
matter content

\[ M_{QD} = Q \bar{D} \sim \Box \times (1; -5, -4N + 1), \]

\[ B_Q = Q^{2N-1} \sim \Box \times (1; 2N - 1, 2N - 1), \]

\[ \bar{B}_U = \bar{U}^{2N-2} \bar{D} \sim (\Box; -6, -\frac{4N^2 - N + 1}{2N - 1}), \]

\[ \bar{B}_D = \bar{U}^{2N-1} \sim (1; 0, 2N + 2), \]

and an effective superpotential

\[ W_{\text{eff}} = B_Q M_{QD} \bar{B}_D. \] (2.10)

If this were an elementary theory, the \( Sp(2N) \) dynamics would force \( \bar{B}_D \) to run away. This can again be described by a superpotential of the form of Eq. (2.7), but in the regime we are now considering the Kähler potential is smooth in the field \( \bar{B}_D \).

Because the field \( \bar{B}_D \) is composite, we know that if \( \langle \bar{B}_D \rangle \) is large compared to \( \Lambda_{SU} \), we should use the previous analysis in terms of the elementary degrees of freedom. But this analysis shows that there is no supersymmetric vacuum for large field values, and we conclude that supersymmetry is broken. We see that this model realizes the mechanism of supersymmetry breaking described in the Introduction.

Note that the considerations above imply that there must be at least a local supersymmetry-breaking minimum with \( \langle \bar{B}_D \rangle \neq 0 \), since there are no classical flat directions that can connect this vacuum to the other branch of the moduli space. The supersymmetry-breaking order parameter is

\[ F \approx \frac{\Lambda^{(2N-1)/(N+1)} \Lambda_{Sp} \Lambda_{SU} \left( \frac{\Lambda_{Sp}}{\Lambda_{SU}} \right)^{3/(N+1)}}{4\pi}. \] (2.11)

We see that this model has two descriptions: a “Higgs” description in which the gauge group \( SU(2N - 1) \) is broken, and a “confining” description in which it confines. This model therefore realizes the “complementarity” picture described in Refs. [17]. Neither of these descriptions is quantitatively under control near the vacuum of the theory, but both pictures should be a reliable guide to qualitative features of the low-energy physics. We are not able to determine whether or not \( \langle \bar{B}_U \rangle \) is nonzero. (This can be thought of as the question of whether the induced soft mass-squared for \( \bar{B}_U \) is positive or negative at \( \bar{B}_U = 0 \).) If \( \langle \bar{B}_U \rangle = 0 \), the global symmetry is broken down to \( SU(2N - 1) \times U(1) \), and there is a massless composite fermion

\[ \psi \sim (\Box; -6). \] (2.12)
If \langle \bar{B}_D \rangle \neq 0, the global symmetry is broken down to $SU(2N - 2) \times U(1)$ (where the unbroken $U(1)$ is a linear combination of the original $U(1)$ and a broken $SU(2N - 1)$ generator), and there are massless composite fermions

$$\psi \sim (\Box; -30), \quad \chi \sim (1; 0).$$

(2.13)

In the confined description, the composite fermions correspond to the fermion components of $\bar{B}_D$, and in the Higgs description they correspond to the fermion component of $D$.

It is amusing that the model above does not have gauge anomalies if we replace $Sp(2N)$ by either $SU(2N)$ or $SO(2N)$. The $SO$ model breaks supersymmetry by a mechanism very similar to the one described above, but the $SU$ model does not break supersymmetry! The reason is that the analog of the dynamical superpotential Eq. (2.7) in the $SU$ model is

$$W_{\text{eff}} \sim \bar{U}^{(2N-1)/(2N)},$$

(2.14)

which gives rise to a potential that runs away for large $\bar{U}$. We will not analyze the $SO$ version of the model in this paper.

We now briefly consider the analysis for small field values when $\Lambda_{Sp} \gg \Lambda_{SU}$. The analysis depends on the value of $N$.

For $N = 2$, the $Sp(4)$ group has a confined description with a deformed moduli space. The tree-level superpotential turns into a mass term that combines with the quantum constraint to force some of the composite fields in this description to run away. This shows that there is no supersymmetric vacuum for small fields in this model.

For $N = 3$, the $Sp(6)$ group s-confines, and the low-energy theory is an $SU(5)$ gauge theory with matter content $\Box \oplus \bar{\Box}$ plus singlets. This theory is known to break supersymmetry [18], so there is no supersymmetric vacuum in this model for small fields. This mechanism leads to a class of models that are discussed in the Appendix.

For $N \geq 4$, the $Sp(2N)$ group has a dual description in terms of a $Sp(2N - 6)$ gauge group. The $SU(2N - 1)$ matter content is $\bar{\Box} \oplus (2N - 5) \cdot \Box$ plus singlets. This theory has a dynamically-generated superpotential [19], and this combines with the tree-level superpotential to give a runaway behavior. This again shows that there is no supersymmetric vacuum for small fields.
2.2 The “Lepton” Branch

We now consider the branch where \( \langle L \rangle \neq 0 \). Along this branch, we have

\[
\langle L \rangle = \begin{pmatrix}
v_1 & 1_2 \\
& \ddots \\
& & v_{N-1} & 1_2 \\
& & & 0 \\
& & & & 0
\end{pmatrix}.
\] (2.15)

Ignoring global \( U(1) \) factors, this breaks the gauge and flavor symmetries down to

\[
G_{\text{eff}} = SU(2) \times SU(2N - 1) \times [SU(2)^{N-1}],
\] (2.16)

with light fields

\[
L' \sim (\square, 1) \times 1,
Q' \sim (\square, \square) \times 1,
\bar{U}' \sim (1, \bar{\square}) \times 1,
\bar{D}' \sim (1, \bar{\square}) \times 1,
L'' \sim (1, 1) \times \square,
L''' \sim (1, 1) \times \square,
\] (2.17)

and superpotential

\[
W = \lambda Q'L'\bar{U}'.
\] (2.18)

(Each \( SU(2)^{N-1} \) representation is denoted by a \( SU(2N - 2) \) representation that is understood to be decomposed under \( SU(2N - 2) \rightarrow SU(2)^{N-1} \).) The only flat directions are excitations of \( L \), which correspond to the fields \( L'' \) and \( L''' \) in Eq. (2.17). The remaining light fields have quartic potentials from the \( D \)-term potential.

We will assume that the \( SU(2N - 1) \) group in the effective theory above is stronger than the \( SU(2) \). This is always true for \( N \geq 6 \), where the \( SU(2) \) group is not asymptotically free. For \( N \leq 5 \), it is sufficient to assume that the in the fundamental theory \( \Lambda_{SU} \gg \Lambda_{Sp} \). In the effective theory, the \( SU(2N - 1) \) gauge group has 2 flavors, and if the fields \( Q', \bar{U}', \) and \( \bar{D}' \) were flat directions, the model would have a runaway supersymmetric vacuum where these fields are infinite. The \( D \)-term potential does not allow these fields to run away, and so there is no supersymmetric vacuum in
this region of moduli space. (This is the same mechanism that operates in the 3–2 model, but the present model has classical flat directions.) Since we have explored all regions of the classical moduli space, we conclude that supersymmetry is broken in this theory.

We would like to know whether there are local minima on the lepton branch of the moduli space, and if so, whether these have lower energy than the local minimum found on the baryon branch. For $\Lambda_{SU} \gg \Lambda_{Sp}$ we can show that the only minimum is the one found on the baryon branch above. The reason is simply that if we minimize the energy with $\langle L \rangle$ held fixed, the energy depends only on the scale $\Lambda_{SU,\text{eff}}$ where the $SU(2N - 1)$ gets strong. The scale at which the unbroken $SU(2)$ gauge group becomes strong is irrelevant, because we have seen that supersymmetry is broken in the limit where we ignore the non-perturbative effects of the $SU(2)$ gauge interactions. The scale $\langle L \rangle$ appears in the effective theory through the scale $\Lambda_{SU,\text{eff}}$, but otherwise it only controls the size of higher-dimension operators that give only small corrections to the vacuum energy. Therefore, we expect that the vacuum energy as a function of $\langle L \rangle$ is $V(\langle L \rangle) \sim |\Lambda_{SU,\text{eff}}(\langle L \rangle)|^4$. This grows with $\langle L \rangle$, and so we do not expect a vacuum for large $\langle L \rangle$. The analysis above breaks down for $\langle L \rangle \sim \Lambda_{SU}$. For $\langle L \rangle \ll \Lambda_{SU}$, we can use the confined description of the $SU(2N - 1)$ dynamics of the previous subsection, so the only remaining possibility is a vacuum with $\langle L \rangle \sim \Lambda_{SU}$. However, in this case, we expect the vacuum energy to be of order $|\Lambda_{SU}|^4$, which is larger than the vacuum energy Eq. (2.11) found on the baryon branch. We conclude that the global minimum of this theory is on the baryon branch.

The case where $\Lambda_{Sp} \gg \Lambda_{SU}$ appears to be more complicated, and we cannot rule out the possibility that the global minimum is on the lepton branch in that case.

For $N \leq 5$ and $\Lambda_{Sp} \gg \Lambda_{SU}$, we have not explicitly shown that there is no supersymmetric vacuum on the lepton branch. However, we have examined the entire moduli space for $\Lambda_{SU} \gg \Lambda_{Sp}$ and shown that there is no supersymmetric vacuum. If there were a supersymmetric vacuum in the limit $\Lambda_{Sp} \gg \Lambda_{SU}$, there would have to be a critical condition on the interaction scales $\Lambda_{Sp}$ and $\Lambda_{SU}$ that gave the critical values at which the supersymmetric vacua are lifted. However, the moduli space of supersymmetric vacua structure is a holomorphic function of $\Lambda_{Sp}$ and $\Lambda_{SU}$ [20] and so the critical conditions must be holomorphic functions of $\Lambda_{Sp}$ and $\Lambda_{SU}$. This means there can be no critical lines in the space of gauge couplings separating a phase where supersymmetry is broken from a phase where it is unbroken [21]. This means that supersymmetry is broken also in the limit $\Lambda_{Sp} \gg \Lambda_{SU}$.
2.3 The $Sp(4) \times SU(3)$ Model

We now consider the special case $N = 2$, where the superpotential Eq. (2.7) is

$$W_{\text{eff}} \sim (\bar{U}^3)^{1/3}.$$  (2.19)

The vacuum is forced away from the origin for small $U$, but the potential becomes constant for $\langle \bar{U} \rangle \gg \Lambda_{SU}$. The location of the true vacuum therefore depends on the form of the Kähler potential. Yukawa couplings give corrections to the Kähler potential that push the field to the origin of moduli space, while gauge corrections do the reverse. Since $Sp(4)$ is asymptotically free, the contribution from the Yukawa coupling will dominate for large $\bar{U}$, while the $Sp(4)$ gauge contributions will dominate for small $\bar{U}$. For a range of couplings, there is a supersymmetry-breaking vacuum at large field values where the theory is fully calculable. This is an instance of the inverted hierarchy mechanism [22] similar to the ones in Refs. [9, 10].

3 Composite Messenger Models

In this Section, we consider realistic models of gauge-mediated supersymmetry breaking based on the models analyzed in Section 2. The model we consider is based on the $N = 3$ model of the previous section. This has a global $SU(5)$ symmetry into which we embed the standard model gauge group $SU(3)_C \times SU(2)_W \times U(1)_Y$ in the usual way. (We refer to this embedding as $SU(5)_{\text{SM}}$ for brevity.) The gauge group is therefore

$$Sp(6) \times SU(5) \times SU(5)_{\text{SM}}$$  (3.1)

with matter content

- $Q \sim (\Box, \Box, 1)$
- $L \sim (\Box, 1, \Box)$
- $\bar{U} \sim (1, \Box, \Box)$
- $\bar{D} \sim (1, \bar{1}, 1)$
- $D \sim (1, 1, \Box)$

and a tree-level superpotential

$$W = \lambda Q L \bar{U} + \frac{1}{M^3} (\bar{U}^4 \bar{D}) D.$$  (3.3)
This model differs from the models analyzed above only in that it contains an additional field $D$ (which cancels the standard-model anomalies) and there is a higher-dimension term in the tree-level superpotential. These new features are important for the phenomenology of the model, but they do not affect the qualitative features of the $Sp(6) \times SU(5)$ gauge dynamics discussed above. This model therefore has a supersymmetry-breaking vacuum with

$$
\langle \bar{U} \rangle \simeq \frac{A_{SU}}{4\pi}, \quad F \simeq \langle F_{\bar{U}} \rangle \simeq \frac{\lambda^{5/4}A_{SU}^{1/4}A_{Sp}^{7/4}}{4\pi}.
$$

The gauge symmetry is broken in the pattern

$$
SU(5) \times SU(3)_C \times SU(2)_W \times U(1)_Y \rightarrow SU(3)_C \times SU(2)_W \times U(1)_Y.
$$

Supersymmetry breaking is communicated to the standard-model fields via the messenger pairs $(Q, L)$, $(D, \bar{D})$, and the heavy $SU(5)$ gauge bosons. (We are using a Higgs description of the dynamics.) The masses of the messengers $Q$ and $L$ can be written

$$
\delta \mathcal{L} = \int d^2\theta M_{QL}QL + h.c.
$$

$$
+ B_{QL}\phi_Q\phi_L + h.c.
$$

$$
+ m_Q^2\phi_Q^4 + m_L^2\phi_L^4.
$$

Here, $M_{QL}$ is a supersymmetric mass term, $B_{QL}$ is the “$B$-type” supersymmetry breaking mass familiar from traditional gauge-mediated models, and $m_Q^2$ and $m_L^2$ are soft (non-holomorphic) masses for the messengers. All of these terms are induced by supersymmetry breaking, and we must estimate their size. The supersymmetric and $B$ masses are

$$
M_{QL} \simeq \lambda \langle \bar{U} \rangle \simeq \frac{\lambda A_{SU}}{4\pi},
$$

$$
B_{QL} \simeq \lambda F.
$$

Using naïve dimensional analysis, the soft masses can be estimated from the gauge exchange diagrams of Fig. 1 to be

$$
m_Q^2 \simeq m_L^2 \simeq \frac{g_{SU}^4}{16\pi^2} \frac{F^2}{A_{SU}^2} + \ldots \simeq \frac{16\pi^2 F^2}{A_{SU}^2}.
$$

The messenger scale that sets the scale for the contributions of these messengers to the standard model superpartner masses is

$$
M_{mess} = \frac{B_{QL}}{M_{QL}} \simeq \frac{4\pi F}{A_{SU}}.
$$
We see that $M^2_{\text{mess}} \simeq m^2_Q \simeq m^2_L$, so the soft mass contributions to the standard-model superpartner masses are comparable to the usual gauge-mediated contributions. The soft mass contribution to the standard-model masses is not log enhanced from renormalization group running, since the supersymmetric mass is close to the scale $\Lambda_{SU}$ where the contribution is generated (as long as $\lambda \sim 1$).

In the Higgs picture we are using, the gauge group $SU(5) \times SU(5)_{SM}$ is spontaneously broken down to the diagonal $SU(5)$, which we interpret as the low-energy $SU(5)_{SM}$. This theory therefore contains gauge messengers, but their contribution is not calculable because the $SU(5)$ gauge group is strongly coupled. (At one loop, the gauge messenger contribution to the scalar masses is negative [23], but there is no reason to believe that this sign of this result is correct for the strongly coupled case.) The size of the supersymmetry-breaking masses is the same order as the $Q$ and $L$ messengers discussed above.

The fields $D$ and $\bar{D}$ also act as messengers, and they have mass terms analogous to those discussed above for $Q$ and $L$. The supersymmetric and $B$ masses are

\begin{align}
M_{DD} &\simeq \frac{(\bar{U})^4}{M^3} \simeq \frac{1}{M^3} \left( \frac{\Lambda_{SU}}{4\pi} \right)^4, \\
B_{DD} &\simeq \frac{(\bar{U})^3 F}{M^3} \simeq \frac{F}{M^3} \left( \frac{\Lambda_{SU}}{4\pi} \right)^3.
\end{align}

The $\bar{D}$ soft masses can again be estimated from the diagrams of Fig. 1 to be

\begin{equation}
|m^2_{\bar{D}}| \simeq \frac{16\pi^2 F^2}{\Lambda^2_{SU}}.
\end{equation}

Because $D$ does not feel the strong $SU(5)$ gauge interactions, $m^2_{\bar{D}} \ll m^2_{\bar{D}}$. We therefore have $(B_{DD}/M_{DD})^2 \simeq m^2_{\bar{D}}$, so the $\bar{D}$ soft masses are important for communicating.
supersymmetry breaking. In fact, the soft mass contribution is enhanced by renormalization group evolution from the scale \( \Lambda_{\text{SU}} \) to the scale \( M_{\bar{D}D} \) where the messengers are integrated out. This gives a contribution to the squark masses [24]

\[
\delta m^2_{\tilde{q}} \sim -\left( \frac{g_3^2}{16\pi^2} \right)^2 m^2_{\bar{D}} \ln \frac{\Lambda_{\text{SU}}}{M_{\bar{D}D}}.
\]  

(3.14)

This contribution is negative if \( m^2_{\bar{D}} > 0 \). The logarithm cannot be small: even if \( \Lambda_{\text{SU}} = M \), the logarithm is of order 10. It therefore seems sensible to assume that this term dominates. We see that this model only works if we make the dynamical assumption \( m^2_{\bar{D}} < 0 \). In this case, we require that \( M^2_{\bar{D}D} \gtrsim |m^2_{\bar{D}}| \), so that the supersymmetric mass be large enough that \( \langle D \rangle = 0 \). This gives the constraint

\[
\Lambda_{\text{SU}} \gtrsim 4\pi (M_{\text{mess}}M^3)^{1/4}.
\]  

(3.15)

If we take \( M_{\text{mess}} \simeq 10 \text{ TeV} \), and identify \( M \) with the reduced Planck mass \( M_* \simeq 2 \times 10^{18} \text{ GeV} \), we have \( \Lambda_{\text{SU}} \gtrsim 7 \times 10^{15} \text{ GeV} \). In order to solve the flavor problem, we want the supergravity-mediated contribution to the sparticle mass-squared \( m_{3/2} \sim F/M_* \) to be \( \lesssim 1\% \) of the gauge-mediated contribution. This is satisfied for

\[
\Lambda_{\text{SU}} \lesssim 2 \times 10^{16} \text{ GeV}.
\]  

(3.16)

We see that if \( m^2_{\bar{D}} < 0 \) there is a window where these models can be realistic even if the scale of the higher dimension operator is the Planck scale. For this choice of parameters,

\[
\sqrt{F} \sim 3 \times 10^9 \text{ GeV}.
\]  

(3.17)

In this model, the next-lightest supersymmetric particle (NLSP) will be very long-lived, and may decay late enough in the history of the universe that its hadronic final states can induce additional contributions to nucleosynthesis, spoiling the agreement between the standard theory and experiment. The authors of Ref. [10] obtained a bound of \( \sqrt{F} \lesssim 10^8 \text{ GeV} \) from these considerations. However, this bound is rather model-dependent: it assumes \( R \)-parity conservation, and is invalid in inflationary models with a reheat temperature below the NLSP mass.

Alternatively, if we identify the scale \( M \) of the higher-dimension operator with the grand unification scale, we obtain \( \Lambda_{\text{SU}} \gtrsim 10^{14} \text{ GeV} \) and \( \sqrt{F} \gtrsim (3 \times 10^8 \text{ GeV}) \), which may be safe given the uncertainties involved in these estimates. In any case, the models will work for sufficiently small \( M \).
In these models, the dominant contribution to the standard-model scalar masses is given by the log-enhanced contribution of the \((D, \bar{D})\) messengers

\[
m^2_\tilde{q} \sim \left( \frac{g^2}{16\pi^2} \right)^2 \left( \frac{4\pi F}{\Lambda_{SU}} \right)^2 \ln \frac{\Lambda_{SU}}{M_{D\bar{D}}},
\]

where \(M_{D\bar{D}}\) is given by Eq. (3.11). The gaugino masses are given by

\[
m_\lambda \sim \frac{g^2}{16\pi^2} \frac{4\pi F}{\Lambda_{SU}}.
\]

Therefore, in these models the scalar masses are heavier than the corresponding gaugino masses compared to minimal gauge-mediated models. However, the minimal gauge-mediation relations between squark and slepton masses (say) are still satisfied.

4 Conclusions

We have discussed a new class of supersymmetry-breaking models based on direct product groups with a tree-level superpotential. These models have a large space of flat directions at tree level, but nonetheless break supersymmetry via the mechanism of \(s\)-confinement. These models have a number of attractive features: they contain no dimensionful parameters, and large global symmetries are possible. By embedding the standard model gauge group in the global symmetry of a particular model, we have found that a realistic superpartner spectrum is possible provided that a soft mass term generated by the strong dynamics is negative. An interesting direction to explore is to consider a variation of this model in which the light composite fermions are identified with standard-model fermions [25].

5 Acknowledgments

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Appendix: More Supersymmetry Breaking by S-confinement

In this Appendix, we analyze some additional models related to those in the main text. The models have gauge and flavor symmetry group

$$G = Sp(2N) \times SU(5) \times [SU(2N - 1) \times U(1) \times U(1)_R],$$

(A.1)

where the global symmetries are written in brackets. The matter content is

\[ Q \sim (\square, \square) \times (1; 1, 1), \]
\[ L \sim (\square, 1) \times (\square; -\frac{5}{2N-1}, -\frac{3}{2N-1}), \]
\[ \bar{U} \sim (1, \bar{\square}) \times (\bar{\square}; -\frac{2N-6}{2N-1}, \frac{2N+2}{2N-1}), \]
\[ \bar{D} \sim (1, \bar{\square}) \times (1; -6, -12), \]

(A.2)

and there is a tree-level superpotential

$$W = \lambda QL\bar{U}.$$  \hspace{1cm} (A.3)

For \( N = 3 \), this is one of the models discussed in the main body of the text.

The models have been constructed so that the \( Sp(2N) \) factor has s-confining dynamics. This can be used to analyze the model for \( \Lambda_{Sp} \gg \Lambda_5 \), in a region of moduli space where all vacuum expectation values are small compared to \( \Lambda_{Sp} \). In this regime, the theory has a confined description in terms of composite chiral superfields. The effective symmetry group is

$$G_{\text{eff}} = SU(5) \times [SU(2N - 1) \times U(1) \times U(1)_R],$$

(A.4)

with matter content (after integrating out massive fields)

\[ M_{QQ} = QQ \sim \square \times (1; 2, 2), \]
\[ M_{LL} = LL \sim 1 \times (\square; -\frac{10}{2N-1}, -\frac{6}{2N-1}), \]
\[ \bar{D} \sim \square \times (1; -6, -12), \]

(A.5)

with vanishing effective superpotential. (The \( \bar{U} \) equation of motion sets the dynamically generated superpotential to zero.) The low-energy theory consists of some singlets, together with a \( SU(5) \) gauge theory that is believed to break supersymmetry through non-calculable strong dynamics [18]. However, we cannot conclude from this that supersymmetry is broken. The point is that supersymmetry breaking
will induce a non-calculable potential for the classical flat directions $M_{LL}$, and this potential may make $M_{LL}$ run away from the origin to a regime where the confined description is no longer valid. (In fact, we will show that for $N \geq 3$ the theory has a runaway supersymmetric vacuum.) We must analyze the full moduli space of the theory before we can conclude that supersymmetry is broken. The analysis differs for various values of $N$, and we proceed on a case-by-case basis.

A.2 $N = 1$: Minimal Deconfinement

This theory has no classical flat directions when the superpotential is taken into account. In fact this is the minimal “deconfined” description of the model with gauge group $SU(5)$ and matter content $\mathbb{Z} \oplus \mathbb{Z}$ [18].

It is interesting that this theory has a calculable limit. If we turn off the $SU(2)$ gauge coupling, the theory has a classical moduli space that can be parameterized by the $SU(2)$ doublets $M_{QD} = Q \bar{D}$ and $L$, subject to the constraint

$$\epsilon_{\alpha \beta} (L)^\alpha M_{QD}^\beta = 0.$$

(A.6)

Far out along these flat directions, $SU(2)$ is completely broken, $SU(5)$ is broken down to $SU(4)$, and all fields charged under $SU(4)$ are massive. Gaugino condensation in the $SU(4)$ Yang–Mills theory generates a dynamical superpotential for the flat directions

$$W_{\text{dyn}} \simeq \frac{1}{16\pi^2} (\Lambda_5)^{13/4} \left( \frac{M_{LL}}{M_{QD}^2} \right)^{1/8}.$$

(A.7)

This superpotential forces $M_{QD}$ to run away to infinity.

If we now turn on an $SU(2)$ gauge coupling, all flat directions are lifted at the classical level. The potential due to $SU(2)$ gauge couplings is small near the origin and grows for large fields. Therefore, for small values of the $SU(2)$ gauge coupling, the minimum of the potential will be at large values of $\langle M_{QD} \rangle$ and $\langle L \rangle$, and supersymmetry is broken. This mechanism for supersymmetry breaking is the same as in the “3–2 model” [4]. We will not analyze this model further.

This analysis proves that there is no supersymmetric vacuum in the parameter region $\Lambda_5 \gg \Lambda_2$. However, as discussed in the main text, there can be no phase transitions as a function of $\Lambda_5/\Lambda_2$, and so supersymmetry is broken also in the limit $\Lambda_2 \gg \Lambda_5$, i.e. in the original $SU(5)$ model.

The supersymmetric $SU(5)$ model has also been related to a calculable model in Ref. [26] by adding additional vector-like matter and tree-level superpotential terms, and our conclusions are in agreement.
A.3  $N = 2$: Supersymmetry Breaking via Supersymmetry Breaking

The classical flat directions can be parameterized by the gauge-invariant operator

$$M_{LL} = LL \sim (\mathbb{R}; -\frac{10}{3}, -2).$$  \hspace{1cm} (A.8)

Now consider the effective theory far out along this flat direction. Na"ively, it appears that the $M_{LL}$ flat direction cannot be lifted, since the symmetries do not allow a dynamical superpotential for this field. However, a careful analysis of the effective theory in this region of moduli space shows that this argument is not correct because supersymmetry is broken!

To understand this, note that in terms of the elementary fields, we are considering vacua with

$$\langle L \rangle = \begin{pmatrix} v \\ v \\ 0 \\ 0 \end{pmatrix},$$  \hspace{1cm} (A.9)

and all other vacuum expectation values vanishing. This breaks $Sp(4) \rightarrow SU(2)$, and gives a tree-level mass $\lambda v$ to two components of $Q$ and $\bar{U}$. Working out the effective $SU(2) \times SU(5)$ gauge theory, one finds that it has precisely the matter content of the theory considered in the previous subsection (with three additional singlets). As shown above, this theory breaks supersymmetry dynamically, and this supersymmetry breaking is communicated to the flat fields $M_{LL}$ by higher-dimension terms in the effective Kähler potential.

This model serves as a reminder that an analysis of the flat directions using the standard arguments based on holomorphy, symmetry, and classical limits is correct only if the strong sector of the theory does not itself break supersymmetry. This subtlety is not present in models with no tree-level superpotential, since in those theories the effective theory at a generic point in moduli space is either trivial (the gauge group is completely broken) or is a pure Yang–Mills theory; in either case, the low-energy theory does not break supersymmetry. However, in models with a tree-level superpotential, the classical equations of motion can force the theory to a singular vacuum where the unbroken gauge group has charged matter fields. As illustrated here, such a low-energy theory can break supersymmetry and invalidate a naïve application of Seiberg’s arguments.

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A.4 \( N \geq 3 \): Supersymmetry Restoring “Phase Transition”

The theory has a classical flat directions that can be parameterized by the gauge-invariants

\[
M_{LL} = LL \sim (\mathbb{E} - 2, -\frac{6}{2N-1}),
\]

\[
\bar{B}_U = \bar{U}^4 \bar{D} \sim (\mathbb{O}, -\frac{20N - 30}{2N-1}, -\frac{16N + 20}{2N-1}),
\]

\[
\bar{B}_D = \bar{U}^5 \sim (1; -\frac{10N - 30}{2N-1}, \frac{10N + 10}{2N-1}).
\]

We consider the classical vacuum

\[
\langle \bar{U} \rangle = (u_{15}, 0), \quad \langle L \rangle = \begin{pmatrix}
0 & v_{12} \\
\ddots & \ddots \\
0 & v_{N-3} & 0 \\
0 & 0 & 0
\end{pmatrix}.
\]

(The fact that \( \langle L \rangle \) has rank \( 2(N-3) \) is enforced by \( \partial W/\partial Q = 0 \).) This breaks \( SU(5) \) completely and breaks \( Sp(2N) \to Sp(6) \times U(1) \). All light matter fields are uncharged under \( Sp(6) \), and \( Sp(6) \) gaugino condensation gives rise to a dynamical superpotential

\[
W_{\text{eff}} \sim \left( \frac{\bar{B}_D}{M_{LL}^{N-3}} \right)^{1/4}.
\]

For \( N \geq 4 \), this forces \( M_{LL} \) to run away, and there is a supersymmetric vacuum at infinity. It may be that there is a local supersymmetry-breaking minimum near the origin, but we cannot determine this from the present analysis. Another possibility is that the composite singlet in the s-confined description has a potential that slopes away from the origin, and the true vacuum is outside the range of validity of the s-confined description.

For \( N = 3 \), both the \( Sp(6) \) and \( SU(5) \) groups confine. The superpotential Eq. (A.12) is the same as the one discussed in the main text for the regime \( \Lambda_5 \gg \Lambda_6 \). For \( \Lambda_6 \gg \Lambda_5 \), the analysis in the first part of this Appendix shows that there is no supersymmetric vacuum for small values of the \( SU(5) \) singlet fields \( M_{LL} \), so we understand how supersymmetry is broken in this case as well.
References


