Theoretical Analysis of $\bar{B} \rightarrow D^{**} \pi$ Decays

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Abstract

The decays $\bar{B} \rightarrow D^{**} \pi$, where $D^{**}$ is a narrow p-wave charm resonance, are investigated in the context of a generalized factorization hypothesis, including the leading nonfactorizable corrections. The decay rates for these processes are related to the corresponding semileptonic rates at maximum recoil. It is pointed out that the current data on the branching ratio for the decay $B^- \rightarrow D_2^{*0} \pi^-$ may pose a problem for theory. We predict that future, more accurate measurements of this branching ratio will find a value $B(B^- \rightarrow D_2^{*0} \pi^-) \approx 4 \times 10^{-4}$, a factor 5 lower than the current central value.
1 Introduction

Recently, the CLEO Collaboration has reported the first measurement of the branching ratios for the decays $B^- \rightarrow D_1^0 \pi^- $ and $B^- \rightarrow D_2^0 \pi^-$, where $D_1$ and $D_2^*$ (generically denoted as $D^{**}$) are p-wave charm mesons. In the heavy-quark limit, these are the members of a spin-symmetry doublet in which the light degrees of freedom have total angular momentum $j = \frac{3}{2}$. The measured branching ratios are [1]

$$B(B^- \rightarrow D_1^0 \pi^-) \times B(D_1^0 \rightarrow D^{*+} \pi^-) = (7.8 \pm 1.9) \times 10^{-4} ,$$
$$B(B^- \rightarrow D_2^0 \pi^-) \times B(D_2^0 \rightarrow D^{*+} \pi^-) = (4.2 \pm 1.7) \times 10^{-4} .$$ (1)

The purpose of this letter is to interpret these observations together with the existing data on the semileptonic decays $B^- \rightarrow D_1^0 \ell^+ \nu$ and $B^- \rightarrow D_2^0 \ell^+ \nu$. For the corresponding branching ratios, the CLEO Collaboration quotes [2]

$$B(B^- \rightarrow D_1^0 \ell^+ \nu) \times B(D_1^0 \rightarrow D^{*+} \pi^-) = (0.37 \pm 0.10)\% ,$$
$$B(B^- \rightarrow D_2^0 \ell^+ \nu) \times B(D_2^0 \rightarrow D^{*+} \pi^-) = (0.059 \pm 0.067)\%$$
$$< 0.16\% (90\% CL) .$$ (2)

These numbers are consistent with results reported by the ALEPH Collaboration [3]:

$$B(\bar{B} \rightarrow D_1 \ell^- \bar{\nu}X) \times B(D_1 \rightarrow D^{*\pm} \pi^0) = (0.49 \pm 0.11)\% ,$$
$$B(\bar{B} \rightarrow D_2^0 \ell^- \bar{\nu}X) \times B(D_2^0 \rightarrow D^{*+} \pi^-) < 0.34\% (95\% CL) ,$$
$$B(\bar{B} \rightarrow D_2^0 \ell^- \bar{\nu}X) \times B(D_2^0 \rightarrow D^{*+} \pi^-) < 0.33\% (95\% CL) ,$$
$$B(\bar{B} \rightarrow D_2^{*+} \ell^- \bar{\nu}X) \times B(D_2^{*+} \rightarrow D^0 \pi^+) < 0.26\% (95\% CL) ,$$ (3)

if one assumes that in most cases no additional particles $X$ are produced. Semileptonic $B$ decays into p-wave charm mesons are important with regard to understanding how the total inclusive semileptonic branching ratio of $B$ mesons is composed out of exclusive modes. The ALEPH Collaboration finds a sizable branching ratio of $(2.26 \pm 0.44)\%$ for the sum of all (resonant or nonresonant) decays $\bar{B} \rightarrow D^{(*)} \pi \ell^- \bar{\nu}$ [3]. These channels make up for about 20% of the total semileptonic branching ratio.

Unfortunately, the $D^{**} \rightarrow D^{(*)} \pi$ branching ratios entering the above measurements have not yet been determined experimentally. Under the assumption that all $D_1$ mesons decay into $D^{*+} \pi$ (decays into $D_d$ are forbidden by angular momentum conservation) and using isospin invariance, one obtains $B(D_1^0 \rightarrow D^{*+} \pi^-) = 2/3$. For $D_2^*$ mesons, the ratio $B(D_2^{*0} \rightarrow D^{*+} \pi^-)/B(D_2^{*0} \rightarrow D^{*+} \pi^-) = 2.3 \pm 0.8$ has been measured [4, 5]. Under the assumption that $D_2^* \rightarrow D \pi$ and $D_2^* \rightarrow D^{*+} \pi$ are the only decay modes and using isospin invariance, one then obtains $B(D_2^{*0} \rightarrow D^{*+} \pi^-) = (20 \pm 5)\%$ and $B(D_2^{*0} \rightarrow D^{+} \pi^-) = B(D_2^{*+} \rightarrow D^0 \pi^+) = (47 \pm 5)\%$.

In the case of neutral $B$ mesons, there is a simple relation between hadronic decays involving a negatively charged pion in the final state and the corresponding semileptonic
decays with the pion replaced by a lepton–neutrino pair [6]. In general, the amplitude for the hadronic decay \( \bar{B}^0 \to H_c^+ \pi^- \), where \( H_c^+ \) represents an arbitrary charm meson (or a sum of hadrons with total charm number one), can be written as [7]

\[
A_{\text{had}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1 \langle \pi^- | d \gamma_{\mu}(1 - \gamma_5) u | 0 \rangle \langle H_c^+ | \bar{c} \gamma^\mu(1 - \gamma_5) b | \bar{B}^0 \rangle ,
\]

where \( a_1 \) parameterizes the complicated hadronic matrix elements of four-quark operators, which enter the theoretical description of nonleptonic amplitudes. This will be discussed in more detail below. The squared amplitude takes the form

\[
|A_{\text{had}}|^2 = \frac{G_F^2}{2} |V_{cb} V_{ud}^*|^2 |a_1|^2 f_\pi^2 q_\mu q_\nu H^{\mu\nu} ,
\]

where \( q = p - p' \) is the pion momentum, \( f_\pi \) the pion decay constant, and

\[
H^{\mu\nu} = \langle H_c^+ | \bar{c} \gamma^\mu(1 - \gamma_5) b | \bar{B}^0 \rangle \langle \bar{B}^0 | \bar{b} \gamma^\nu(1 - \gamma_5) c | H_c^+ \rangle
\]

the “hadronic tensor”. The same tensor appears in the description of semileptonic decays. Indeed, the squared amplitude for the process \( \bar{B}^0 \to H_c^+ \ell^- \bar{\nu} \) is given by

\[
|A_{\text{sl}}|^2 = \frac{G_F^2}{2} |V_{cb}|^2 L_{\mu\nu} H^{\mu\nu} ,
\]

where \( L_{\mu\nu} \) is the leptonic tensor. In the special case where \( q^2 = (p_\ell + p_\nu)^2 = 0 \), the leptonic tensor takes the simple form

\[
L_{\mu\nu} \bigg|_{q^2=0} = 16 x (1 - x) q_\mu q_\nu ,
\]

where \( x = E_\ell / E_\ell^{\text{max}} \) is the scaled lepton energy in the rest frame of the \( B \) meson. Thus, in the limit where the pion mass is neglected, the nonleptonic decay rate is proportional to the semileptonic rate at \( q^2 = 0 \). Working out the trivial phase-space factors, we obtain Bjorken’s relation [6]

\[
\frac{\Gamma(\bar{B}^0 \to H_c^+ \pi^-)}{d\Gamma(\bar{B}^0 \to H_c^+ \ell^- \bar{\nu})/dq^2 \bigg|_{q^2=0}} = 6\pi^2 f_\pi^2 |V_{ud}|^2 |a_1|^2 + O \left( \frac{M_\pi^2}{M_B^2} \right)
\]

irrespective of the nature of the hadron state \( H_c \). The corrections of order \( M_\pi^2 / M_B^2 \) have been worked out for the cases \( H_c = D \) and \( D^* \) and are found to be negligible [8]. Relation (9) still contains an unknown hadronic parameter \( a_1 \). However, for energetic two-body decays such as \( \bar{B}^0 \to D^{(*)+} \pi^- \) one can argue that the value of this parameter is close to unity. The reason is that the pion has a large energy in the rest frame of the decaying \( B \) meson. Once its constituent quarks have grouped together in a colour-singlet state, they form a fast-moving colour dipole which decouples from long-wavelength gluons. Only hard gluons with virtualities of order \( M_B \) are effective in rearranging the quarks. This “colour transparency argument” [6] suggests that the nonfactorizable contributions
to the hadronic decay amplitude are switched off at low momentum scales. Hence, one expects that $a_1 \approx 1$ for energetic two-body decays. This assertion is further strengthened by the $1/N_c$ expansion, which shows that $a_1 = 1 + O(1/N_c^2)$ [8]. A determination of $a_1$ from $\bar{B}^0 \rightarrow D^{(*)+} \pi^-$ decays gives $|a_1| = 1.08 \pm 0.11$, supporting the theoretical argument just presented. A similar value is expected to apply for two-body decays into p-wave charm mesons.

Unfortunately, the situation is more complicated for the decays of charged $B$ mesons. Whereas the semileptonic decay rates of $B^-$ and $\bar{B}^0$ mesons are related to each other by isospin invariance, this is not so for the hadronic rates. We will now discuss how relation (9) must be modified in this case.

2 Hadronic decay amplitudes for $\bar{B} \rightarrow D^{**}\pi$

The part of the effective weak Hamiltonian relevant to $b \rightarrow c\bar{u}d$ transitions is given by

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left\{ c_1(\mu) \langle \bar{u}d \rangle \langle c\bar{b} \rangle + c_2(\mu) \langle \bar{u}u \rangle \langle c\bar{d} \rangle + \ldots \right\}, \quad (10)$$

where $(\bar{u}d) = \bar{d}\gamma^\mu(1 - \gamma_5)u$ etc. are left-handed, colour-singlet quark currents. The Wilson coefficients $c_i(\mu)$ are known to next-to-leading order. At the scale $\mu = m_b$, they have the values $c_1(m_b) \approx 1.1$ and $c_2(m_b) \approx -0.3$. These coefficients take into account the short-distance corrections arising from the exchange of hard gluons. The effects of soft gluons (with virtualities below the scale $\mu$) remain in the hadronic matrix elements of the local four-quark operators. A reliable field-theoretic calculation of these matrix elements is the obstacle to a quantitative theory of hadronic weak decays.

Using Fierz identities, the four-quark operators in the effective Hamiltonian may be rewritten in various forms. It is particularly convenient to rearrange them in such a way that the flavour quantum numbers of one of the quark currents match those of one of the hadrons in the final state of the considered decay process. In the case of $\bar{B} \rightarrow D^{**}\pi$ transitions, omitting common factors, the various decay amplitudes may be written as

$$A(\bar{B}^0 \rightarrow D^{**+}\pi^-) = \left( c_1 + \frac{c_2}{N_c} \right) \langle D^{**+}\pi^- | (\bar{u}d) \langle c\bar{b} | \bar{B}^0 \rangle + \frac{c_2}{2} \langle D^{**+}\pi^- | (\bar{d}t_a u) \langle c\bar{t}_a b | \bar{B}^0 \rangle, \right.$$

$$A(\bar{B}^0 \rightarrow D^{**0}\pi^0) = \left( c_2 + \frac{c_1}{N_c} \right) \langle D^{**0}\pi^0 | (\bar{u}u) \langle c\bar{d} | \bar{B}^0 \rangle + \frac{c_1}{2} \langle D^{**0}\pi^0 | (\bar{c}t_a u) \langle \bar{d}\bar{t}_a b | \bar{B}^0 \rangle, \right.$$

$$A(B^- \rightarrow D^{**0}\pi^-) = A(\bar{B}^0 \rightarrow D^{**+}\pi^-) - \sqrt{2} A(\bar{B}^0 \rightarrow D^{**0}\pi^0), \quad (11)$$

where $t_a$ are the SU(3) colour matrices. The last relation follows from isospin symmetry of the strong interactions. The three classes of decays shown above are referred to as class-1, class-2, and class-3, respectively [7].

3
The class-1 amplitude $A(\bar{B}^0 \to D^{*+} \pi^-)$ contains the “factorizable contribution”

$$A_{\text{fact}} = \langle \pi^- | (\bar{d}u) | 0 \rangle \langle D^{*+} | (\bar{c}b) | \bar{B}^0 \rangle,$$  \hspace{1cm} (12)

which can be calculated in terms of the pion decay constant $f_\pi$ and the $\bar{B}^0 \to D^{*+}$ transition form factors. It also contains other, nonfactorizable contributions, which can be accounted for by introducing hadronic parameters $\varepsilon_1$ and $\varepsilon_8$ such that

$$\langle D^{*+} \pi^- | (\bar{d}u)(\bar{c}b) | \bar{B}^0 \rangle = [1 + \varepsilon_1(\mu)] A_{\text{fact}},$$

$$\langle D^{*+} \pi^- | (\bar{d}t_a u)(\bar{c}t_a b) | \bar{B}^0 \rangle = 2\varepsilon_8(\mu) A_{\text{fact}}.$$  \hspace{1cm} (13)

Then the class-1 decay amplitude takes the form shown in (4), i.e. $A(\bar{B}^0 \to D^{*+} \pi^-) = a_1 A_{\text{fact}}$, with [8]–[10]

$$a_1 = \left( c_1(\mu) + \frac{c_2(\mu)}{N_c} \right) [1 + \varepsilon_1(\mu)] + c_2(\mu) \varepsilon_8(\mu).$$  \hspace{1cm} (14)

The hadronic parameter $a_1$ takes into account all contributions to the matrix elements and is thus $\mu$ independent. The scale dependence of the Wilson coefficients is exactly balanced by that of the hadronic parameters $\varepsilon_i(\mu)$.

In the case of $\bar{B} \to D^{(*)} \pi$ transitions, a similar discussion can be done for the class-2 amplitude, which contains the factorizable contribution $\langle D^{(*)0} | (\bar{c}u) | 0 \rangle \langle \pi^0 | (\bar{d}b) | \bar{B}^0 \rangle$. Nonfactorizable contributions can be accounted for by introducing a parameter $a_2$, which has a similar structure as $a_1$ except for an interchange of $c_1$ and $c_2$. In the present case of $\bar{B} \to D^{**} \pi$ transitions, however, things are more subtle, since the factorizable contribution to the class-2 decay amplitude vanishes. The reason is that the p-wave charm mesons do not couple to the vector or axial vector currents, i.e. $\langle D^{*0} | (\bar{c}u) | 0 \rangle = 0$. Nevertheless, the matrix elements of the four-quark operators need not vanish. Taking into account that there is only a single helicity amplitude for the decays considered here, we choose to normalize these matrix elements to the factorized class-1 amplitude $A_{\text{fact}}$ introduced in (12) and define

$$\langle D^{*0} \pi^0 | (\bar{c}u)(\bar{d}b) | \bar{B}^0 \rangle = \delta_1(\mu) A_{\text{fact}},$$

$$\langle D^{*0} \pi^0 | (\bar{c}t_a u)(\bar{d}t_a b) | \bar{B}^0 \rangle = 2\delta_8(\mu) A_{\text{fact}}.$$  \hspace{1cm} (15)

Then the class-2 amplitude takes the form $A(\bar{B}^0 \to D^{*0} \pi^0) = \bar{a}_2 A_{\text{fact}}$, where

$$\bar{a}_2 = \left( c_2(\mu) + \frac{c_1(\mu)}{N_c} \right) \delta_1(\mu) + c_1(\mu) \delta_8(\mu).$$  \hspace{1cm} (16)

Additional insight can be gained by combining these results with the $1/N_c$ expansion [8]. At a scale $\mu = O(m_b)$, the large-$N_c$ counting rules of QCD imply\(^1\) $c_1 = 1 + O(1/N_c^2)$

\(^1\)For scales much lower than $m_b$, the counting rules for the Wilson coefficients $c_i(\mu)$ are spoiled by large logarithms.
and \( c_2 = O(1/N_c) \) for the Wilson coefficients, and \( \varepsilon_1, \delta_1 = O(1/N_c^2) \) and \( \varepsilon_8, \delta_8 = O(1/N_c) \) for the hadronic parameters. Using these results, we find

\[
a_1 = 1 + O \left( \frac{1}{N_c^2} \right), \quad \frac{\tilde{a}_2}{a_1} = \delta_8(m_b) + O \left( \frac{1}{N_c^3} \right). \tag{17}
\]

Hence, for the class-1 amplitude we recover the result \( a_1 \approx 1 \), which also follows from the colour-transparency argument. The class-2 amplitude, on the other hand, is governed by a nontrivial hadronic parameter \( \delta_8(m_b) \) of order \( 1/N_c \), which is process depend and will, in general, take different values for \( D_1 \) and \( D_2^* \). For the ratios of the various hadronic decay rates, we obtain

\[
\frac{\Gamma(\bar{B}^0 \to D^{*0}_\pi^0)}{\Gamma(B^0 \to D^{*+}_\pi^\pi^-)} \approx \frac{1}{2} |\delta_8(m_b)|^2, \\
\frac{\Gamma(B^- \to D^{*0}_\pi^-)}{\Gamma(\bar{B}^0 \to D^{*+}_\pi^\pi^-)} \approx |1 + \delta_8(m_b)|^2. \tag{18}
\]

Whereas the first ratio may be used to extract the magnitude (but not the phase) of \( \delta_8(m_b) \), the second one determines the corrections to Bjorken’s relation (9) for the case of class-3 decays:

\[
\frac{\Gamma(B^- \to D^{*0}_\pi^-)}{d\Gamma(B^- \to D^{*0}_\pi^-)/dq^2} \bigg|_{q^2=0} \approx 6\pi^2 f^2_\pi |V_{ud}|^2 |1 + \delta_8(m_b)|^2. \tag{19}
\]

Note that the final state \( D^{*0}_\pi^- \) is a pure isospin \( I = \frac{3}{2} \) state. Therefore, the class-3 decays are unaffected by (elastic) final-state interactions.

## 3 Semileptonic decay amplitudes for \( \bar{B} \to D^{**} \ell^- \bar{\nu} \)

The theoretical description of semileptonic decays involves the hadronic matrix elements of vector and axial vector currents between heavy meson states. These matrix elements can be parametrized as

\[
\langle D_1(v', e) | \bar{c} \gamma^\mu b | \bar{B}(v) \rangle = f_{V_1}(w) \epsilon^{*\mu} + \left[ f_{V_2}(w) v^\mu + f_{V_3}(w) v'^\mu \right] \epsilon^{*} \cdot v, \\
\langle D_1(v', e) | \bar{c} \gamma^\mu \gamma_5 b | \bar{B}(v) \rangle = if_{A}(w) \epsilon^{*\alpha\beta\gamma} \epsilon^{*}_\alpha v^\beta_v \gamma, \\
\langle D_2^*(v', e) | \bar{c} \gamma^\mu \gamma_5 b | \bar{B}(v) \rangle = k_{A_1}(w) \epsilon^{*\alpha} v_\alpha + \left[ k_{A_2}(w) v^\mu + k_{A_3}(w) v'^\mu \right] \epsilon^{*\alpha\beta} v_\alpha v_\beta, \\
\langle D_2^*(v', e) | \bar{c} \gamma^\mu b | \bar{B}(v) \rangle = ik_{V}(w) \epsilon^{*\alpha\beta\gamma} \epsilon^{*}_\alpha v_\beta v'_{\beta} v_\gamma, \tag{20}
\]

where \( w = v \cdot v' \), and a mass-independent normalization of meson states is implied. The polarisation vector of the spin-one state \( D_1 \) satisfies \( \epsilon^{*} \cdot v' = 0 \); the symmetric, traceless Rarita-Schwinger spinor of the spin-two state \( D_2^* \) obeys the constraint \( \epsilon^{*\mu\alpha} v'^{\mu}_{\alpha} = 0 \).
The heavy mesons can be described by spin wave-functions with well-defined transformation properties under the Lorentz group and heavy-quark symmetry. To leading order in $1/m_Q$, expressions for the $\bar{B} \to D^{**}$ transition matrix elements are readily obtained using the covariant trace formalism developed in Ref. [11]. All form factors are proportional to a universal function $\tau_3(w)$, with calculable coefficients including the short-distance corrections to the decay amplitudes [12]. Explicitly, we find

$$\sqrt{2} f_{V_1}(w) = (w^2 - 1) C_1 \tau_3(w),$$
$$\sqrt{2} f_{V_2}(w) = [3C_1 + 2(w + 1)C_2] \tau_3(w),$$
$$\sqrt{2} f_{V_3}(w) = [(2 - w)C_1 + 2(w + 1)C_3] \tau_3(w),$$
$$\sqrt{2} f_A(w) = (w + 1) C_5 \tau_3(w),$$
$$\frac{1}{\sqrt{3}} k_{A_1}(w) = (w + 1) C_5 \tau_3(w),$$
$$\frac{1}{\sqrt{3}} k_{A_2}(w) = -C_2 \tau_3(w),$$
$$\frac{1}{\sqrt{3}} k_{A_3}(w) = -(C_1 + C_3) \tau_3(w),$$
$$\frac{1}{\sqrt{3}} k_V(w) = C_1 \tau_3(w).$$

(21)

Here $C_i$ and $C_5_i$ are the Wilson coefficients appearing in the heavy-quark expansion of the weak currents (explicit expressions for these functions, which depend on $w$ and the heavy-quark masses, can be found in Ref. [12]).

It follows from these results that, in the heavy-quark limit, the matrix elements of the weak currents vanish at the zero-recoil point $w = 1$, reflecting the fact that the ground-state $B$ meson is orthogonal to the $p$-wave $D^{**}$ states. The important observation made in Ref. [13] was that the leading $1/m_Q$ corrections at zero recoil can be calculated in a model-independent way in terms of the masses of charm-meson states. In this reference, rather complicated formulae have been derived where the various power corrections to the form factors are parametrized in terms of unknown, subleading universal functions. However, all model-independent information can be incorporated if the result for the form factor $f_{V_1}(w)$ in (21) is modified according to

$$\sqrt{2} f_{V_1}(w) = (w + 1) (w - 1 + 2\delta) C_1 \tau_3(w),$$
$$\delta = \left(1 + \frac{C_3}{C_1}\right)_{w=1} \times \frac{M_{D_1} - M_D}{M_D} \approx 0.29,$$

(22)

so that $f_{V_1}(1)$ no longer vanishes. There are many other sources of $1/m_Q$ corrections; however, they do not yield contributions at zero recoil. Since the physical range of $w$
where \( \alpha \) is independent and free of large logarithms. For all practical purposes, it is sufficient to express these ratios at order \( \alpha \). The Wilson coefficient with one of the short-distance coefficients is scheme-invariant. What remains in the relations (21) and (22), it is convenient to introduce a new function \( \tau(w) \equiv C_1 \tau_2(w) \). The Wilson coefficient \( C_1 \) is included in this definition since only the product of \( \tau_2(w) \) with one of the short-distance coefficients is scheme-invariant. What remains in the expressions for the semileptonic rates are ratios of Wilson coefficients, which are scheme-independent and free of large logarithms. For all practical purposes, it is sufficient to evaluate these ratios at order \( \alpha_s \).

Of particular interest for the further discussion are the differential decay rates at maximum recoil, corresponding to \( q^2 = 0 \). We find

\[
\frac{d\Gamma(\bar{B} \to D^* \ell^- \bar{\nu})}{dw} \bigg|_{w_0} = \frac{G_F^2 |V_{cb}|^2 M_B^5}{768 \pi^3} \frac{(1 - r_i)^5 (1 + r_i)^7}{r_i^2} \tau^2(w_0) K^2(D^{**}),
\]

where

\[
K(D_1) = 1 + \frac{\delta}{1 - r_1} + \frac{1 + r_1}{2r_1} \left( r_1 \frac{C_2}{C_1} + \frac{C_3}{C_1} \right) \approx 0.95 + 1.85 \delta,
\]

\[
K(D_2^*) = \frac{C_1^5}{C_1} - \frac{1 - r_2}{2r_2} \left( r_2 \frac{C_2^5}{C_1} + \frac{C_3^5}{C_1} \right) \approx 0.90,
\]

and the short-distance coefficients are evaluated at \( w_0 \). Numerically, we obtain

\[
\frac{d\Gamma(\bar{B} \to D_1 \ell^- \bar{\nu})}{dw} \bigg|_{w_0} \approx 0.160 (1 + 1.944 \delta)^2 \times [\tau(1.32)]^2 \text{ps}^{-1}
\]

\[
\approx 0.394 \times [\tau(1.32)]^2 \text{ps}^{-1},
\]

\[
\frac{d\Gamma(\bar{B} \to D_2^* \ell^- \bar{\nu})}{dw} \bigg|_{w_0} \approx 0.136 \times [\tau(1.31)]^2 \text{ps}^{-1}.
\]

(26)
To calculate the total semileptonic rates requires an ansatz for the form factor $\tau(w)$. Since the accessible range of $w$ values is small, it is sufficient to adopt a linear approximation with a slope parameter $\rho^2$: $\tau(w) = \tau(1) [1 - \rho^2(w - 1) + \ldots]$. Higher-order terms can be partially taken into account by reexpressing the results for the decay rates obtained in linear approximation through values of the function $\tau(w)$ at intermediate points. In that way, we find

$$\Gamma(\bar{B} \to D_1 \ell^- \bar{\nu}) \approx (0.0158 + 0.0694\delta + 0.1169\delta^2) \times [\tau(1.23)]^2 \text{ps}^{-1}$$

$$\approx 0.0462 \times [\tau(1.23)]^2 \text{ps}^{-1},$$

$$\Gamma(\bar{B} \to D_2^* \ell^- \bar{\nu}) \approx 0.0207 \times [\tau(1.21)]^2 \text{ps}^{-1}.$$  (27)

The results for the differential and total semileptonic rates in (26) and (27) do not include $1/m_Q$ corrections except those proportional to the quantity $\delta$, which are kinematically enhanced and specific for $\bar{B} \to D_1$ transitions. From the analysis of power corrections for $\bar{B} \to D^{(*)} \ell^- \bar{\nu}$ decays it is known that the remaining $1/m_Q$ corrections tend to be spin independent and thus cancel in ratios of decay rates [12]. Therefore, we expect that ratios of the decay rates estimated above are accurate to about 20%.

4 Implications and conclusions

We are now in a position to compare the theoretical predictions derived above with the available experimental data. Consider first the branching ratio for the decay $B^- \to D_1^0 \ell^- \bar{\nu}$. Multiplying the theoretical prediction for the total semileptonic rate in (27) by the lifetime $\tau_D = 1.65$ ps, we find $B(B^- \to D_1^0 \ell^- \bar{\nu}) = 7.9\% \times [\tau(1.23)]^2$. This value is about 3 times larger than the result obtained in the strict heavy-quark limit, where the kinematically enhanced terms involving $\delta$ are neglected. Under the assumption that $B(D^0_1 \to D^{*+} \pi^-) = 2/3$, the experimental value in (2) implies $B(B^- \to D_1^0 \ell^- \bar{\nu}) = (0.56 \pm 0.15)\%$. Comparing this with the theoretical result, we find that $\tau(1.23) = 0.27 \pm 0.04$ is required in order to fit the data. This value is in good agreement with theoretical predictions obtained using relativistic quark models ($\tau(1.23) \approx 0.3$ [14]) or QCD sum rules ($\tau(1.23) \approx 0.22$ [15]). We take this agreement as an indication that the terms involving the quantity $\delta$ in (26) and (27) do indeed capture the dominant corrections to the heavy-quark limit.

Our next goal is to understand the ratio of the hadronic and semileptonic widths for $\bar{B} \to D_1$ transitions. The experimental data in (1) and (2) imply

$$R_1 = \frac{\Gamma(B^- \to D_1^0 \pi^-)}{\Gamma(B^- \to D_1^0 \ell^- \bar{\nu})} = 0.21 \pm 0.08,$$  (28)

independently of the $D_1 \to D^{*} \pi$ branching ratios. Combining (19), (26) and (27), we obtain the theoretical prediction (using $dq^2 = 2M_B M_{D_1} dw$)

$$R_1 \approx \frac{3\pi^2 f^2 |V_{ud}|^2}{M_B M_{D_1}} [1 + \delta_5(m_b)]^2 \left. \frac{d\Gamma(B^- \to D_1^0 \ell^- \bar{\nu})/dw}{\Gamma(B^- \to D_1^0 \ell^- \bar{\nu})} \right|_{w_0}$$

8
\[ \approx 0.32 \left( |1 + \delta_8(m_b)| \frac{\tau(1.32)}{\tau(1.23)} \right)^2, \]  

(29)

which is rather insensitive to the value of \( \delta \). Obviously, there is no problem to account for the data. Making the conservative assumption \( \rho^2 = 1.5 \pm 0.5 \) for the slope parameter of the function \( \tau(w) \), we find \( R_1 \approx (0.20 \pm 0.06) |1 + \delta_8(m_b)|^2 \), which is in good agreement with experiment provided the nonfactorizable corrections parametrized by \( \delta_8 \) are moderate in size. Solving for these corrections we obtain \( |1 + \delta_8(m_b)| = 1.02 \pm 0.25 \), in accordance with the fact that \( \delta_8 = O(1/N_c) \).

Consider next the ratio of the two semileptonic rates for \( B \) decays into \( D_1 \) and \( D_2^* \) mesons. The available experimental data for this ratio depend on some \( D^{**} \rightarrow D^* \pi \) branching ratios that have not yet been measured directly. We define

\[
    h = \frac{B(D_1^0 \rightarrow D^{**+} \pi^-)}{B(D_2^{*0} \rightarrow D^{**+} \pi^-)} = 3.3 \pm 0.8, \tag{30}
\]

where the quoted numerical value is obtained under the assumptions stated in the introduction. The CLEO data in (2) imply

\[
    R_2 = \frac{\Gamma(B^+ \rightarrow D_2^{*0} \ell^- \bar{\nu})}{\Gamma(B^+ \rightarrow D_1^{0} \ell^- \bar{\nu})} = (0.16 \pm 0.19) h = 0.53 \pm 0.63, \tag{31}
\]

or \( R_2 < 1.48 \) (90% CL). From (27), we obtain the theoretical prediction

\[
    R_2 \approx \frac{1.31}{1 + 4.39\delta + 7.40\delta^2} \left( \frac{\tau(1.21)}{\tau(1.23)} \right)^2 \approx 0.48, \tag{32}
\]

which is in good agreement with the data. We have again assumed \( \rho^2 = 1.5 \pm 0.5 \) to estimate the ratio of form factors, which is a small effect in the present case. Although the experimental errors in the value of \( R_2 \) are large, to reproduce the data requires the presence of the terms involving the quantity \( \delta \) [13].

Consider finally the ratio of the two hadronic rates for \( B \) decays into \( D_1 \) and \( D_2^* \) mesons. The CLEO data in (1) imply

\[
    R_3 = \frac{\Gamma(B^- \rightarrow D_2^{*0} \pi^-)}{\Gamma(B^- \rightarrow D_1^{0} \pi^-)} = (0.54 \pm 0.26) h = 1.8 \pm 1.0. \tag{33}
\]

Combining (19) with the expressions for the semileptonic rates at maximum recoil given in (26), and assuming again \( \rho^2 \approx 1.5 \) to estimate the tiny form-factor difference between the two cases, we obtain the theoretical prediction

\[
    R_3 \approx \frac{0.86}{(1 + 1.94\delta)^2} \left| \frac{1 + \delta_8^{(D_2^*)}(m_b)}{1 + \delta_8^{(D_1)}(m_b)} \right|^2 \approx 0.35 \left| \frac{1 + \delta_8^{(D_2^*)}(m_b)}{1 + \delta_8^{(D_1)}(m_b)} \right|^2, \tag{34}
\]

(34)

which is significantly lower than the data. It must be stressed, however, that the experimental errors are large, and the discrepancy between theory and experiment is only
about 1.5 standard deviations. Indeed, it would be very difficult to reconcile the central experimental value in (33) with the theoretical expectation. The least understood aspect of the theoretical calculation is the question about the size of the nonfactorizable contributions to the hadronic decay widths. To reproduce the data would require that

$$\left| \frac{1 + \delta_8^{(D_2^*)}(m_b)}{1 + \delta_8^{(D_1^0)}(m_b)} \right| \approx 2.27 \pm 0.63 .$$

(35)

Since the deviation of this ratio from unity is of order $1/N_c$ in the large-$N_c$ limit, such a large value seems very unlikely. This is even more so as we have shown above that the nonfactorizable contributions to the $B^- \to D_1^0 \pi^-$ decay rate are very small. Another possibility would be to blame the discrepancy between (33) and (34) on the $D^{**} \to D^* \pi$ branching ratios, which have not yet been measured directly. To reproduce the data would require that $h \approx 0.65 \pm 0.31$, which is much smaller than the commonly assumed value quoted in (30). Besides the fact that such a small value would imply exotic (or at least not well understood) physics in the strong decays of p-wave charm mesons, this possibility is essentially ruled out by the good agreement of the theoretical prediction for the ratio $R_2$ with the experimental value for that ratio derived assuming the standard value for $h$. Finally, one may ask whether the theoretical prediction (34) could be spoiled by $1/m_Q$ corrections not included in our estimate of the semileptonic rates in (26). However, as we have argued above, those corrections tend to cancel in ratios of decay rates and are expected not to exceed the level of 20%.

To summarize, whereas there is in general good agreement between theoretical expectations and experimental data for semileptonic and hadronic $B$ decays into final states containing a p-wave charm meson, the experimental central value of the branching ratio for the decay $B^- \to D_2^{*0} \pi^-$ cannot be accommodated by theory. We predict that future, more accurate measurements of this branching ratio will find a value

$$\mathcal{B}(B^- \to D_2^{*0} \pi^-) \approx 0.35 \mathcal{B}(B^- \to D_1^0 \pi^-) \approx 4 \times 10^{-4} ,$$

(36)

which is about a factor 5 lower than the current central value reported by the CLEO Collaboration [1]. If, on the other hand, the current central value were confirmed, this would pose a serious problem for the theory of $B$ decays.

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References


