The Parton Structure of Real Photons*

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Abstract

The QCD treatment of the photon structure is recalled. Emphasis is given to the recently derived momentum sum rule, and to the proper choice of the factorization scheme and/or boundary conditions for the evolution equations beyond the leading order. Parametrizations of the photon’s parton content are examined and compared. The small-x behaviour of the photon structure is briefly discussed.

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1 Introduction

Deep-inelastic electron–photon scattering has been the classical process for investigating the hadronic structure of the photon. This process is kinematically analogous to the usual lepton–nucleon scattering. It has quite early received special interest, since the structure function $F_2^\gamma(x, Q^2)$ can be completely calculated in perturbation theory at large Bjorken-$x$ and large resolution $Q^2$. At scales accessible at present or in the near future, however, these results are unfortunately not applicable. Hence the photon structure functions have to be analyzed in terms of non-perturbative initial distributions for the QCD evolution equations, very much like the nucleon case.

Experimentally $F_2^\gamma$ has been determined, albeit with rather limited accuracy, via $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ at all electron–positron colliders since PEP and PETRA. The longitudinal structure function $F_1^\gamma$ has been unaccessible so far, and will presumably remain so in the foreseeable future. On the other hand, the past months have witnessed a substantial amount of new results on $F_2^\gamma$ from LEP, and many more can be expected from forthcoming LEP2 runs. If systematic problems in extractions of $F_2^\gamma$ from final-state modeling can be overcome, these results will be able to challenge seriously the present, model-driven theoretical understanding of the photon structure.

In this talk a brief survey is given of the present theoretical and phenomenological status of this subject. In Section 2 we recall the evolution equations for the photon’s parton distributions, including the recently derived momentum sum rule. The factorization scheme ambiguities are more relevant here as in the usual hadronic case, this issue is discussed in Section 3. Some of the most relevant parametrizations of the quark and gluon densities of the photon are discussed in Section 4 with respect to their assumptions and lim-
2 The evolution of the photon’s parton densities

The photon is a genuine elementary particle, unlike the hadrons. Hence it can directly take part in hard scattering processes, in addition to its quark and gluon distributions arising from quantum fluctuations, \( q^\gamma(x,Q^2) \) and \( g^\gamma(x,Q^2) \). Denoting the corresponding photon distribution in the photon by \( \Gamma^\gamma(x,Q^2) \), the evolution equations for these parton densities are generally given by

\[
\frac{dq^\gamma}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \mathcal{P}_{q\gamma} \otimes \Gamma^\gamma + \frac{\alpha_s}{2\pi} \left\{ \sum_{k=1}^f \mathcal{P}_{q_k\gamma} \otimes q_k^\gamma + \mathcal{P}_{g\gamma} \otimes g^\gamma \right\}
\]

\[
\frac{dg^\gamma}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \mathcal{P}_{g\gamma} \otimes \Gamma^\gamma + \frac{\alpha_s}{2\pi} \left\{ \sum_{k=1}^f \mathcal{P}_{g_k\gamma} \otimes q_k^\gamma + \mathcal{P}_{g\gamma} \otimes g^\gamma \right\}
\]

\[
\frac{d\Gamma^\gamma}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \mathcal{P}_{\gamma\gamma} \otimes \Gamma^\gamma + \frac{\alpha_s}{2\pi} \left\{ \sum_{k=1}^f \mathcal{P}_{\gamma_k\gamma} \otimes q_k^\gamma + \mathcal{P}_{\gamma\gamma} \otimes g^\gamma \right\}.
\]

Here \( \alpha \approx 1/137 \) is the electromagnetic coupling constant, and \( \alpha_s \equiv \alpha_s(Q^2) \) denotes the running QCD coupling. \( \otimes \) represents the Mellin convolution, and \( f \) stands for the number of active (massless) quark flavours. The antiquark distributions do not occur separately in Eq. (1), as \( q_i^\gamma(x,Q^2) = q_i^{-\gamma}(x,Q^2) \) due to charge conjugation invariance. The generalized splitting functions read

\[
\mathcal{P}_{ij}(x,\alpha,\alpha_s) = \sum_{l,m=0} \frac{\alpha' \alpha_s^m}{(2\pi)^{l+m}} \mathcal{P}_{ij}^{(l,m)}(x),
\]

with \( \mathcal{P}_{q_k\gamma} \) being the average of the quark–quark and antiquark–quark splitting functions. The parton densities are subject to the energy–momentum sum rule

\[
\int_0^1 dx \left[ \Sigma^\gamma(x,Q^2) + g^\gamma(x,Q^2) + \Gamma^\gamma(x,Q^2) \right] = 1,
\]

where \( \Sigma \) represents the singlet quark distribution, \( \Sigma^\gamma = 2 \sum_{i=1}^f q_i^\gamma \).

Usually calculations involving the photon’s parton structure are restricted to first order in \( \alpha \ll 1 \). In this approximation all \( l \neq 0 \) terms in Eq. (2) can be neglected, since \( q_i^\gamma \) and \( g^\gamma \) are already of order \( \alpha \). This reduces the functions \( \mathcal{P}_{ij} \) to the usual QCD quantities \( P_{ij}(x,\alpha_s) \), with \( P_{q\gamma} \) and \( P_{g\gamma} \) dropping out.
completely. Moreover one has $P_{\gamma \gamma} \propto \delta(1 - x)$ to all orders in $\alpha_s$, as real photon radiations from photons starts at order $\alpha^2$ only. Thus the last line of Eq. (1) can be integrated immediately, at leading order (LO), $m = 0$, resulting in

$$\Gamma_{LO}^{\gamma}(x, Q^2) = \delta(1 - x) \left[ 1 - \frac{\alpha}{\pi} \left( \sum_q e_q^2 \ln \frac{Q^2}{Q_0^2} + c_1 \right) \right]. \tag{4}$$

Here $e_q$ stands for the quark charges, $Q_0^2$ is some reference scale for the evolution, and the constant $c_1$ will be discussed below. Only the $O(1)$ part of $\Gamma_{\gamma}$ affects the quark and gluon densities at order $\alpha$, as well as any observable involving hadronic final states like $F_2^{\gamma}$, leading to the evolution equations

$$\frac{d k_i^{\gamma}}{d \ln Q^2} = \frac{\alpha}{2\pi} P_{\gamma \gamma} + \frac{\alpha_s}{2\pi} \left\{ 2 \sum_{\lambda=1}^I P_{\gamma \lambda} \otimes q_k^{\gamma} + P_{\gamma g} \otimes g^{\gamma} \right\}, \tag{5}$$

$$\frac{d g^{\gamma}}{d \ln Q^2} = \frac{\alpha}{2\pi} P_{g^{\gamma}} + \frac{\alpha_s}{2\pi} \left\{ 2 \sum_{\lambda=1}^I P_{g \lambda} \otimes q_k^{\gamma} + P_{gg} \otimes g^{\gamma} \right\}.$$ 

The splitting functions $P_{ij}(x, \alpha_s)$ are presently known to next-to-leading order (NLO) in $\alpha_s$, $m = 1$, see refs. 8,9,10.

The momentum sum rule (3) holds order by order in $\alpha$, thus Eq. (4) implies

$$\int_0^1 dx x \left[ \sum_{LO}^{\gamma}(x, Q^2) + \sum_{LO}^{\gamma}(x, Q^2) \right] = \frac{\alpha}{\pi} \left( \sum_q e_q^2 \ln \frac{Q^2}{Q_0^2} + c_1 \right). \tag{6}$$

The photon's quark and gluon densities are therefore not related by a hadron-type sum rule. Instead their momentum fractions rise logarithmically with $Q^2$ as long as the lowest-order approximation in $\alpha$ is justified. Hence, on the level of Eq. (5) alone, an important constraint on the parton densities is missing. That deficit can be removed by inferring $c_1$ from elsewhere, as recently done in refs. 11,12 by connecting Eq. (4) to the cross section $\sigma(e^+ e^- \rightarrow hadrons)$ via a dispersion relation in the photon virtuality. This procedure yields

$$\left( \frac{c_1}{\pi} \right)_{LO} = \sum_{V=\rho,\omega,\phi} \frac{4\pi}{f_V^2} \simeq 0.55 \text{ at } Q_0^2 \simeq (0.6 \text{ GeV})^2. \tag{7}$$

An error of about 20% can be assigned to this value, arising from the uncertainties of $f_V^2$ (leptonic $\rho$ width vs. $\gamma p \rightarrow \rho^0 p$) and of the scale $Q_0^2$ where the connection of $c_1$ to the vector-meson decay constants holds. The numerical results of refs. 11,12 agree well within this margin.
The general solution of the inhomogeneous evolution equations (5) reads

\[
\bar{q}^\gamma = \begin{pmatrix} \Sigma^\gamma \\ g^\gamma \end{pmatrix} = \bar{q}^\gamma_{PL} + \bar{q}^\gamma_{had},
\]  

(8)

where only the flavour singlet part has been indicated. The solution of the homogeneous ('hadronic') equation, \( \bar{q}^\gamma_{had}(x, Q^2) \), contains the perturbatively uncalculable boundary conditions \( \bar{q}^\gamma(x, Q^2_0) \). The inhomogeneous ('pointlike') part, on the other hand, is completely calculable once \( Q^2_0 \) has been specified.
At next-to-leading order these solutions can be written as\textsuperscript{10,13}

\[ q_{\text{had}}^\gamma = \left( \left[ \frac{\alpha_s}{\alpha_0} \right]^d + \frac{\alpha_s}{2\pi} \left( \hat{U} \otimes \left[ \frac{\alpha_s}{\alpha_0} \right]^d - \left[ \frac{\alpha_s}{\alpha_0} \right]^d \otimes \hat{U} \right) \right) \otimes q^\gamma (Q_0^2) \] (9)

and

\[ q_{\text{PL}}^\gamma = \left\{ \frac{2\pi}{\alpha_s} + \hat{U} \right\} \otimes \left\{ 1 - \frac{\left[ \frac{\alpha_s}{\alpha_0} \right]^{1+d}}{1+d} \right\} \otimes \frac{1}{1+d} \otimes \hat{a} + \left\{ 1 - \frac{\left[ \frac{\alpha_s}{\alpha_0} \right]^d}{d} \right\} \otimes \frac{1}{d} \otimes \hat{b} \] (10)

with \( \alpha_0 = \alpha_s(Q_0^2) \). \( \hat{a}, \hat{b}, \hat{d} \) and \( \hat{U} \) stand for known combinations of the splitting functions and the QCD \( \beta \)-function. The LO evolution is obtained from Eqs. (9) and (10) by putting \( \hat{U} = 0 \) and \( \hat{b} = 0 \). A convenient way to evaluate these expressions is by transformation to Mellin moments, which reduces the convolutions to simple products. The \( x \)-dependent distributions are then calculated by a numerical Mellin inversion of the final result (8).

3 Boundary conditions and factorization schemes

The structure function \( F_2^\gamma \) is, at first order in the electromagnetic coupling \( \alpha \),

\[ F_2^\gamma = \sum_{q=u,d,s} 2x e_q^2 \left\{ q^\gamma + \frac{\alpha_s}{2\pi} \left( C_{2,q} \otimes q^\gamma + C_{2,q} \otimes g^\gamma \right) + \frac{\alpha_s}{2\pi} e_q^2 C_{2,q} \right\} . \] (11)

Only the contribution of the light flavours has been written out here. The reader is referred to refs.\textsuperscript{14,15} for the heavy quark part \( F_{2,h}^\gamma \). At LO in \( \alpha_s \), just the first term in Eq. (11) is taken into account since \( q^\gamma \sim 1/\alpha_s \), see Eq. (10). At NLO the usual hadronic one-loop coefficient functions \( C_{2,q}(x) \) and \( C_{2,q}(x) \) enter, together with the direct-photon contribution \( C_{2,\gamma} \) given by\textsuperscript{3}

\[ C_{2,\gamma}^{\text{MS}}(x) = 3 \left[ x^2 + (1-x^2) \right] \ln \frac{1-x}{x} - 1 + 8x(1-x) . \] (12)

This term causes difficulties in this standard factorization scheme, as it leads to a large LO/NLO difference for the inhomogeneous part \( F_{2,\gamma,\text{PL}} \). In particular it is strongly negative at large \( x \), see Fig. 1. Thus \( F_{2,\gamma,\text{PL}} \) turns positive over the full \( x \)-range, for \( Q_0^2 = 1 \) GeV\(^2 \), only at \( Q^2 \approx 20 \) GeV\(^2 \). This unphysical behaviour has to be overcome in the complete \( F_2^\gamma \) by the \( \overline{\text{MS}} \) initial distributions, which are therefore forced to be very different from their LO counterparts.

These problems are circumvented by adopting the DIS\textsubscript{S} scheme introduced in refs.\textsuperscript{10,16} Here \( C_{2,\gamma} \) is absorbed into the quark distributions according to

\[ q_{\text{DIS}}^\gamma = q_{\text{MS}}^\gamma + \frac{\alpha_s}{2\pi} e_\gamma^2 C_{2,\gamma}^{\text{MS}} , \quad C_{2,\gamma}^{\text{DIS}} = 0 . \] (13)
The coefficient functions $C_{2,q}$ and $C_{2,g}$ in Eq. (11), as well as the definition of the gluon density remain unchanged, in contrast to the hadronic DIS scheme. Therefore $F_{2,\gamma}^\gamma$ assumes the usual hadronic $\overline{\text{MS}}$ form without a direct term in DIS$_\gamma$, resulting in a good LO/NLO stability of $F_{2,\gamma}^\gamma$ as illustrated in Fig. 1. Consequently physically motivated boundary conditions for the quark and gluon densities can be employed in this scheme also beyond leading order.

An additional advantage of the DIS$_\gamma$ scheme is that the leading $\overline{\text{MS}}$ terms for $x \to 0$ cancel in the transformed NLO photon–parton splitting functions

$$P_{\gamma q}^{(1)} \sim \left\{ \begin{array}{ll} \ln^2 x + \ldots & \text{MS} \\ 2 \ln x + \ldots & \text{DIS}_\gamma \end{array} \right.$$  

$$P_{\gamma g}^{(1)} \sim \left\{ \begin{array}{ll} 1/x + \ldots & \text{MS} \\ -3 \ln x + \ldots & \text{DIS}_\gamma \end{array} \right.$$  

(14)

In fact, this cancellation of the leading small-$x$ term of $P_{\gamma g}^{\overline{\text{MS}}}$ does not only take place at NLO, but persists to all orders in $\alpha_s$.

An equivalent $\overline{\text{MS}}$ formulation of the above solution to the $C_{2,\gamma}$ problem has been pursued in refs 20,21. It can be written as a modification of the pointlike part (PL) in Eq. (8) by an additional ‘technical’ NLO input density,

$$q_{\gamma q}^{\text{PL}}(x, Q_0^2) = -\frac{\alpha_s}{2\pi} C_{2,\gamma}$$(x), \quad q_{\gamma q}^{\text{PL}}(x, Q_0^2) = 0.$$  

This leads to $F_{2,\gamma}^{\gamma}(x, Q_0^2) = 0$ and thus allows for similar ‘physical’ initial distributions on top of Eq. (15). The resulting quark distributions, however,
Figure 2: The pointlike up-quark density \( u^\gamma_{PL} \) in LO and in NLO for the DIS\( _\gamma \) scheme, compared to the physically equivalent \( \overline{\text{MS}} \) distribution (\( \overline{\text{PL}} \)) with the input \([15]\). The parameters \( Q^2_0, f \) and \( \Lambda \) are as in Fig. 1. Also shown is the result for the PL\' boundary condition \([16]\).

Exhibit a rather unphysical shape. As displayed in Fig. 2, they are suppressed (strongly enhanced) at medium (large) values of \( x \), respectively, with respect to the pointlike LO and DIS\( _\gamma \) results. In a fully consistent NLO calculation, this \( \overline{\text{MS}} \) procedure is nevertheless strictly equivalent to the DIS\( _\gamma \) treatment. On the other hand, as soon as not all terms beyond NLO are carefully omitted, the \( \overline{\text{MS}} \) treatment turns out to be unstable at large \( x \), see refs. \([22,23]\).

Due to the non-universality of the coefficient function \( C_{2,\gamma} \), a special role is assigned to \( F^\gamma_2 \) in the redefinitions \((13,15)\) of the quark densities, similar to the hadronic DIS scheme. An alternative process-independent approach was worked out in ref. \([24]\). A universal technical \( \overline{\text{MS}} \) input has been inferred from a detailed analysis of the Feynman diagrams for \( \gamma^* \gamma \to \gamma^* \gamma \), which leads to

\[
q^\gamma_{PL}(x,Q^2_0) = -\frac{\alpha}{2\pi} e^2 \, C'_1(x), \quad g^\gamma_{PL}(x,Q^2_0) = 0
\]  

with

\[
C'_1(x) = 3 \left( x^2 + (1 - x^2) \right) \ln(1 - x) + 2x(1 - x) \]  

The resulting modified pointlike structure function \( F^\gamma_{2,PL'} \), also shown in Fig. 1, remains negative at large \( x \) due to the uncompensated \(-1\) in Eq. \((12)\) only close to the reference scale \( Q^2_0 \). At medium to small \( x \), \( F^\gamma_{2,PL'} \) is similar to the pointlike \( \overline{\text{MS}} \) results, i.e., larger than its LO and DIS\( _\gamma \) = PL counterparts. The corresponding up-quark distributions are also illustrated in Fig. 2.
4 Parametrizations of photonic parton distributions

In order to specify the photon's parton densities, the perturbatively uncalculable initial distributions, \( q_i^\gamma(x, Q_0^2) \) and \( g^\gamma(x, Q_0^2) \), have to be fixed at some scale \( Q_0^2 \). Only one combination of quark densities (dominated by \( u^\gamma \)) is presently well constrained, however, by \( F_2^\gamma \) data at \( 0.01 \lesssim x \lesssim 0.8 \) from PETRA \(^{25,26,27}\), PEP \(^{28}\), TRISTAN \(^{29,30}\), and LEP \(^{31,32,33}\). The complete present data, including new results presented at this conference, is shown in Fig. 3 together with the NLO parametrizations of refs. \(^{16,24}\). The gluon distribution is not tightly constrained either: there is sound evidence for \( g^\gamma = 0 \), and a very large and hard \( g^\gamma \) has been ruled out by jet production results \(^{34,35,36}\).

Due to these limitations, current parametrizations invoke theoretical estimates and model assumptions, in particular from vector meson dominance (VMD). For safely high reference scales, \( Q_0^2 \gtrsim 2 \text{ GeV}^2 \), however, purely hadron-like initial distributions are known to be insufficient. An additional hard quark component has to be supplemented there in order to meet the \( F_2^\gamma \) data at larger \( Q^2 \). In view of this situation two approaches have been used. First one can keep \( Q_0 \gtrsim 1 \text{ GeV} \), fit the quark densities to \( F_2^\gamma \) data, and estimate the gluon input. This method has been adopted in refs. \(^{37,38}\) and, more recently, in refs. \(^{11,21,39,40}\). The second option is to retain a pure VMD ansatz,

\[
(q_i^\gamma, g^\gamma)(x, Q_0^2) = \frac{4\pi\alpha}{f_\rho^2} (q_i^\rho, g^\rho)(x, Q_0^2) + \ldots ,
\]

(18)

together with assumptions on the experimentally unknown \( \rho \) distributions, and to start the evolution at a very low scale \( Q_0 \simeq 0.5 \ldots 0.7 \text{ GeV} \). Note that this boundary condition complies with the momentum sum rule (7) if the \( \omega \) and \( \phi \) contributions are appropriately added.

In the following, the three available NLO parametrizations \(^{16,21,24}\) are briefly compared, together with the recent LO sets of ref. \(^{11}\). For all these distributions \( \Lambda_{\text{LO}, \overline{\text{MS}}} = 200 \text{ MeV} \) have been employed at \( f = 4 \).

The resulting \( u^\gamma \)-quark densities are displayed in Fig. 4. Considering the LO results first, one notices that the parametrizations form two groups in the well-measured intermediate \( x \)-range, \( 0.2 \lesssim x \lesssim 0.7 \). The lower one consists of the two low-\( Q_0 \) sets, GRV \(^{16}\) and SAS1D \(^{11}\), which start the evolution at \( Q_0^2 = 0.25 \text{ GeV}^2 \) and \( 0.36 \text{ GeV}^2 \), respectively. The reference scales for the higher SAS2D \(^{21}\) and GS (96) \(^{21}\) distributions read \( Q_0^2 = 4 \text{ GeV}^2 \) and \( 3 \text{ GeV}^2 \). This difference has been driven at least partly by first LEP data \(^{32,42}\), see ref. \(^{41}\), which were considerably higher than previous results around \( x = 0.2 \).

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The additional SaS1M and SaS2M sets in ref. \(^{11}\) are theoretically inconsistent, as the leading-order evolution is combined with the scheme-dependent coefficient function \( C_{2\gamma} \).

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Figure 3: The presently available $F_2^\gamma$ data compared to NLO parametrizations of refs. 16, 24. The hadron-like VMD components of the latter are separately displayed at $Q^2 = 2.4$ GeV$^2$. 
A more consistent picture is now emerging from the new LEP data in this range.

The second striking feature in Fig. 4 is the large-\( x \) behaviour of the GS parametrization. Unlike SaS 2D, where a simple hard term \( \propto x \) is employed, GS choose the massive Born expressions for \( \gamma^* \gamma \rightarrow q \bar{q} \) at \( Q_0^2 \) on top of the hadronic VMD input. All power-law contributions \( O((m_q^2/Q^2)^n) \), \( n \geq 1 \), are retained, resulting in a threshold at \( x \simeq 0.9 \) for typical constituent quark masses. Such a procedure, however, may be considered as inadequate for the construction of leading-twist parton densities.

Let us now turn to the NLO distributions. The results of GRV and AFG are both based on the VMD ansatz (18), imposed at \( Q_0^2 = 0.3 \) GeV\(^2\) and 0.5 GeV\(^2\), respectively. The differences between these two parametrizations at \( x \gtrsim 0.1 \) can be understood in terms of the non-hadronic NLO boundary conditions discussed in Sec. 3, cf. Fig. 1. At lower \( x \), the deviations are dominated by the differing assumptions \(^{34,45}\) on the experimentally virtually unconstrained pion sea – both groups estimate the unknown \( \rho \) distributions by their pionic counterparts. The third NLO set, GS (96), is technically flawed: it should at \( Q_0^2 \), by construction, lead to the same \( F_2^\gamma \) results as the LO fit. However, \( u^\gamma \) turns out to be sizeably too small over the full \( x \)-range. Hence
Before considering the gluon density, it is appropriate to comment on the quark-flavour decomposition and the momentum sum rule. Both SaS 1D and AFG perform a coherent addition of the three light vector mesons at their respective input scales, with slightly differing assumptions on SU(3) breaking and the value of $f_{\rho}$. That leads to a suppression of the $d$-valence density by a factor of four with respect to the $\rho$-meson's $u$-valence component. This approach is able to describe the $F_2^\gamma$ data without any further adjustment, hence the momentum sum rule (7) is met in both cases. On the other hand, GRV use just a $\rho$ distribution, with a prefactor adjusted to the data. Although a factor of 1.6 perfectly mimics the $F_2$ of the (SU(3) symmetric) coherent superposition, too much momentum is spent due to the $u_\rho = d_\rho$ symmetry. Thus Eq. (6) is violated, e.g., by about $+40\%$ in LO at $Q^2 = 4$ GeV$^2$. Finally the high-$Q_0$ fits, SaS 2D and GS, do not impose the momentum sum rule at all.

The gluon distributions of these parametrizations are finally presented in Fig. 5. The pion distributions of AFG and GRV both describe the direct-photon production data in $\pi p$ collisions, that is why these photonic gluons are so similar except at very $x$. For the GS parametrization, the gluon densities have been constrained by a LO comparison to TRISTAN jet production.

Figure 5: Parametrizations of the photon's gluon density at LO $^{11,16,21}$ and NLO $^{16,21,24}$. This parametrization is unfortunately not usable in its present form $^{6}$.
data \cite{34,35}, which seem to prefer a relatively large gluon distributions. The shapes of the SaS gluon densities are fixed by theoretical estimates, no direct or indirect experimental constraint has been imposed here.

5 Photon structure at small $x$

The region of very small parton momenta, $10^{-5} \leq x \leq 10^{-2}$, has attracted considerable interest in the proton case since the advent of HERA. The quark and gluon distributions show a marked rise at small $x$ \cite{47,48}, in good agreement with perturbative predictions for a low input scale $Q_0 \simeq 600$ MeV \cite{49}. The corresponding NLO evolution of the photon structure is shown in Fig. 6.

![Figure 6: The NLO small-$x$ evolution of the photon's quark and gluon distributions as predicted in ref \cite{16}. The hadronic (VMD) and pointlike contributions are shown separately.](image)

The parton distributions of the photon behave very differently in the limits of large and small $x$. In the former case, the perturbative part (10) dominates, especially for the quark distributions. On the other hand, this calculable contribution amounts at most to about 20\% at very small-$x$, at scales accessible in the foreseeable future, in LO as well as in NLO. One may therefore expect, by VMD arguments for the hadronic component (9), a very similar rise as observed in the proton case. It will be very interesting to see whether this expectation is borne out by future $F_2^\gamma$ measurements.
Let me finally mention that there is an even more intriguing, if exotic, possibility here: the calculable pointlike contribution could be drastically enhanced by large logarithmic small-\(x\) terms in the perturbation series. If this component could be projected out, for example, by final-state observables, it would provide a rather unique small-\(x\) QCD laboratory.

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