Do experiments suggest a hierarchy problem?

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The hierarchy problem of the scalar sector of the standard model is reformulated, emphasizing the role of experimental facts that may suggest the existence of a new physics large mass scale, for instance indications of the instability of the matter, or indications in favor of massive neutrinos. In the see-saw model for the neutrino masses a hierarchy problem arises if the mass of the right-handed neutrinos is larger than approximately \(10^7\) GeV: this problem, and its possible solutions, are discussed.

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1. We speak of a hierarchy problem when two largely different energy scales are present in the theory, but there is no symmetry that stabilizes the light scale from corrections coming from the large scale [1].

This problem is commonly invoked to argue against the simple structure of the Higgs potential of the standard model, since the massive parameter \(\mu^2\) appearing as \(-\mu^2|H|^2\) in the potential (the light scale) can in principle receive corrections from any larger scale. Which kind of mass the problem pertains? It can be formulated in terms of the renormalized mass, let us say in \(\overline{MS}\) scheme, noticing that at external momenta above a heavy threshold scale \(M_{\text{heavy}}\) the parameter \(\mu^2\) will acquire loop contributions of the order of \(M_{\text{heavy}}^2\), times the coupling to the heavy particle. In this case, the renormalization group flow in the standard model is unnatural, in the sense that the initial conditions at some large scale have to be extremely fine-tuned to reproduce a Higgs mass below the TeV scale, if the coupling of the Higgs particle with the particle of mass \(M_{\text{heavy}}\) is not very small. From another point of view, it was remarked that the bare scalar mass receives quadratic corrections, if the theory is regulated with a cutoff in the momenta [2]. This may be considered a less relevant aspect, since the standard model is a renormalizable theory, and there is no way to give sense to bare parameters in this context; the cutoff can be thought of as a technical device, and in last analysis, other regulators can be chosen.

Notice that to speak of a “problem” one is taking a theoretical point of view: one does not like to assume, without motivations, that a hypothetical fundamental theory that should explain the observed quantities and the various parameters of the standard model should be forced to have a fine-tuning like the one discussed above.

This principle can be used to select possible extensions of the standard model: What is needed is simply stating a quantitative criterion of naturalness (such a program was formulated in [3]).

Once this principle is accepted, the discussion about its actual relevance is reduced to two experimental terms. The first is if a fundamental Higgs particle exists. Assuming that it exists, we face the other aspect: before speaking of a hierarchy problem, one has to understand if there are signals of physics beyond the standard model, that, in turn, point to the existence of larger energy scales.

We will not rely on the Planck mass scale in the following discussion, since in our opinion the formulation of a quantum theory of the gravity is in a preliminary stage, and the experimental perspectives are unclear [4]. We want instead to discuss the relevance of signals of violations of the global symmetries of the standard model, the baryon and the lepton numbers B and L, paying attention to the experimental perspectives that we can foresee at present.

2. Let us start to discuss possible signals of matter instability. If discovered, they would strongly suggest the existence of a large mass scale, most probably related to a deeper layer of gauge unification (the alternative hypothesis of light mediators of matter instability, very weakly coupled with the matter, should be seriously considered if B+L conserving nucleon decay modes would be observed [5,6]). Suppose that proton decay signals would be within reach, say at Superkamiokande. To be concrete, let us imagine the case in which the decay channels involving strange mesons are the dominating ones, that may indicate us that the physics responsible of the proton decay and of the origin of the (family hierarchical) fermion masses is the same. Assuming that the couplings involved in the decay are of the order of a typical Yukawa couplings \(m_s/v \approx 10^{-3}\), a sufficient suppression of the nucleon lifetime can be obtained only if the mass of the mediator \(M_X\) is close to \(10^{12}\) GeV (we assumed: \(\Gamma_p \sim M_X^{-4}\). Therefore, \(\mu^2\) receives the contribution \(\delta\mu^2 \approx y^2 M_X^2/(4\pi)^2\) that is much larger than \(1\) TeV\(^2\), unless the effective coupling \(y\) of the light Higgs with the heavy particle is very small, approximately \(y < 10^{-8}\). It is easy to understand that for a typical theoretical scheme (in which \(y\) can appear at one-loop or even at tree level) the contributions to \(\delta\mu^2\) can be very large. In conclusion, this scenario would probably require us to wonder about the hierarchy problem and about its solution.

It is remarkable that the supersymmetric extensions
of the standard model, with masses of the supersymmetric particles around the electroweak scale, are able to offer a way out from the hierarchy problem due to the non-renormalization theorem [7] and at the same time are compatible with the hypothesis of a minimal SU(5) unification group structure at an energy scale around $2 \times 10^{16}$ GeV [8,9]. This may be regarded as the solution [10], but in the present stage of development it is not clear if a gauge hierarchy problem has to be addressed, since no signal of matter instability has been found yet. In this connection, it is important to remark that supersymmetric grand unified models that predict that nucleon decay signal may be within reach (in the close future) have been indeed proposed [11]. However, one should not forget that some supersymmetric grand unified model can be already excluded by present experimental information on matter stability [12], or, on the opposite extreme, that some model entails an essentially stable nucleon [13]. Even if somewhat disappointing, it may be fair to say that this is due to the fact that the “supersymmetric grand unification” is still not a completely defined program. Coming back to the main focus of the present work, we conclude that (despite the theoretical promises) the experimental studies of matter stability do not permit us at present to infer the existence of a hierarchy problem.

3. There is, however, an independent way of arguing of a hierarchy problem in certain extensions of the standard model. This argumentation is based on the presence of non-zero neutrino masses, that could imply the solution of long standing problems with solar neutrino flux, and may be confirmed by the next round of experiments.

It is in principle possible that also the neutrino masses are related to a new gauge structure manifesting itself at higher scales; if this would be true, we should again face a gauge hierarchy problem [14]. However, we want to be conservative in the assumptions. So, instead of jumping to conclusions, we address the question: What can we learn, using the indications of non-zero neutrino masses, on the structure of the theory that should extend the standard model?

Let us consider the see-saw model for neutrino masses [15]. The heavy right-handed neutrinos, with mass $M_R$, couple with the Yukawa coupling $y_\nu$ to the left-handed neutrinos, and give them a mass $m_\nu = (y_\nu v)^2/M_R$ ($v = 174$ GeV). In non supersymmetric theories the renormalized mass $\mu^2$ will receive corrections order

$$\delta \mu^2 \approx \frac{y_\nu^2}{(2\pi)^2} M_R^2 \log(q/M_R),$$

for momenta $q$ larger than $M_R$ (see fig. 1). We can rewrite these corrections as:

$$\delta \mu^2 \approx m_\nu M_R^3 \frac{3}{(2\pi v^2) \log(q/M_R)}. \tag{2}$$

Equation (2) points to the hierarchy problem that is inherent to the see-saw models for the neutrino masses.

![Fig. 1. The Feynman diagram originating the corrections in eq. (1); $\nu_R$ denotes the right-handed neutrino of mass $M_R$, $\ell_L = (\nu_e, e_L)$ the leptonic- and $H$ the Higgs-doublet.](image)

We specify the previous formula in two concrete cases, considering neutrinos that may be relevant to the solution of the solar neutrino problem and may serve as the hot dark matter (HDM) candidate respectively. Assuming small mixing, the contribution to $\mu^2$ will not exceed $1$ TeV$^2$ if the following upper bounds hold true:

$$m_\nu(\text{solar}) = 3 \times 10^{-3} \text{ eV} \implies y_\nu \approx 3 \times 10^{-5}$$
$$m_\nu(\text{HDM}) = 6 \text{ eV} \implies y_\nu \approx 1.1 \times 10^{-3}. \tag{3}$$

In the previous estimation we assumed the logarithm of order unity (in other terms, we are using the criterion of naturalness: $d\mu^2/d\log q \approx 1\text{TeV}^2$). Let us stress that the figures in eq. (3) should be taken as indicative, since we assumed that the mixing angles and the phases in the lepton matrices are small; their presence can modify to a certain extent the relation between the masses of the light and the heavy neutrinos. However, for given values of the left-handed and right-handed neutrino masses, the radiative contribution to $\mu^2$ tend to increase in presence of mixing and phases.

Under the same assumptions, the conditions (3) on $M_R$ are equivalent to upper bounds on the Yukawa couplings:

$$m_\nu(\text{solar}) = 3 \times 10^{-3} \text{ eV} \implies y_\nu \approx 8.5 \times 10^{-5}$$
$$m_\nu(\text{HDM}) = 6 \text{ eV} \implies y_\nu \approx 1.1 \times 10^{-3}. \tag{4}$$

For comparison, we notice that if $M_R \approx 1$ TeV (of interest for search at accelerators) the Yukawa couplings are $y_\nu \approx 3.1 \times 10^{-7}, 1.4 \times 10^{-5}$ in the two cases considered.

Therefore, to be able to assess the presence of a hierarchy problem, we still lack the information on the scale of the Majorana neutrinos $M_R$, or on the size of the Yukawa couplings. A recent discussion [16] of the structure of the right-handed mass matrix in the see-saw model suggests masses larger than those in (3). Notice however that the underlying assumption is the unification of the Yukawa couplings of the neutrinos and of the up-type-quarks; for smaller neutrino Yukawa coupling, lighter $M_R$ are needed. For instance, this is what happens if neutrinos are Dirac particles, that is when $M_R \ll m_\nu$ (and there is no direct Majorana mass term); the neutrino mass reduces to $y_\nu v$, and the Yukawa couplings are very small ($y_\nu = 1.7 \times 10^{-14}$ for solar neutrinos and $y_\nu = 3.4 \times 10^{-11}$ for HDM component neutrinos).

An interesting information on the Yukawa couplings follows if we assume the Fukugita-Yanagida scenario for
baryogenesis [17] (see also [18–20]), in which the decay of the lightest right-handed neutrino, of mass $M_{R_1}$, originates a lepton asymmetry that, in a second stage, can be converted in the presently observed baryon asymmetry. In fact, this scenario can be realized if the Yukawa couplings provide sufficient mixing with a heavier neutrino of mass $M_{R_2}$:

$$M_{R_1} \approx \frac{\text{Im}(Y_{
u}^\dagger Y_{\nu})_{hl}}{(Y_{
u}^\dagger Y_{\nu})_{ll}} \approx 10^{-5},$$

(5)

in the case of hierarchical masses of right-handed neutrinos, as discussed in [19]. Considering the inequality: $|\text{Im}(Y_{\nu}^\dagger Y_{\nu})_{hl}| \leq (Y_{\nu}^\dagger Y_{\nu})_{hh} (Y_{\nu}^\dagger Y_{\nu})_{ll}$, that follows from the non-negativity of the matrix $Y_{\nu}^\dagger Y_{\nu}$, we obtain:

$$10^{-5} \lesssim (Y_{\nu}^\dagger Y_{\nu})_{hh}.$$  

(6)

Comparing with eq. (4), we come to the conclusion that the corrections to $\mu^2$ exceed the TeV$^2$; in other terms, eq. (6) suggests the vicinity of a hierarchy problem.

This conclusion is related to a conjectural mechanism for baryogenesis, that however is quite natural once the existence of right-handed neutrinos has been assumed. For this reason, it is of interest to search for a loophole in the above argument. Let us therefore abandon the hypothesis of hierarchical right-handed neutrinos, and contemplate the case in which these particles are nearly degenerate; it turns out that the estimation (5) is no longer correct. In fact, the lepton asymmetry produced in the decay is dominated by the “wavefunction” contribution [19,20], that increases for smaller mass splitting, and that eventually reaches its maximum when the splitting is comparable to the decay widths of the right-handed neutrinos [20]. This makes it possible to reproduce the observed baryon number with smaller Yukawa couplings than those implied by eq. (6), and gives a chance to avoid the hierarchy problem in the minimal framework we are considering. We will not address the question of the theoretical likelihood of this very constrained scenario for neutrino masses. However, it is important to stress again that even in this framework the right-handed neutrinos would be relatively light (eq. (3)).

4. Finally, we discuss possible solutions of the hierarchy problem that arises if the see-saw model is the true theory of the neutrino masses, and the right-handed masses are large in comparison with eq. (3) (as suggested by eq. (6), modulo the caveats above). In this case, one could advocate for supersymmetry at low energy on the basis of the criterion of naturalness. We recall the argument: The quadratic corrections to the massive parameters of the Higgs potential entail in supersymmetric theories $M_R^2 - \tilde{M}_R^2$, the mass splitting of the right-handed neutrinos and their scalar partners instead of $M_R^2$ (compare with eq. (1)); the natural expectation is that $M_R^2 - \tilde{M}_R^2 \lesssim 1$TeV$^2$, due, for instance, to a relation of this mass splitting and the splitting between the charged leptons and their scalar counterparts. As a conclusion, the presence of the large mass scale $M_R^2 \gg 1$TeV$^2$ does not imply any hierarchy problem.

In this supersymmetric context, we remark that the mass splitting $M_R^2 - \tilde{M}_R^2$ could affect via one-loop corrections the value of the lightest Higgs mass, in close similarity with what happens due to the stop-top corrections [21]. In fact, these loop corrections are of the same nature of the corrections to $\mu^2$ discussed in eq. (1).

Of course, the argument for supersymmetry is far-reaching, and does not apply only to the see-saw model. In fact, once the low energy supersymmetry hypothesis is accepted, the light scales are “protected” against the presence of the heavy scales, and the theoretical speculations involving very high energy scales do not meet these types of problem. The urgency of the remarks above stays in the consideration that the strongest indications in favor of physics beyond the standard model come from neutrino physics.

If the model of the neutrino masses is not the see-saw model we have other possibilities to elude the hierarchy problem: We can assume that the scale at which the neutrino masses are generated is not far from the electroweak one. This can happen in the models in which the smallness of the neutrino masses is related to loop effects [22]. Even in the context of minimal supersymmetric models (in particular without right-handed neutrinos) other mechanisms for the generation of the neutrino masses are possible. We are referring to the R-parity breaking models, in which $a$ priori large violations of the lepton number may be present [23,24]. Again, the crucial remark is that in these models no large scale (besides the scale of the supersymmetric particles) is present. Can we distinguish this possibility? If the neutrino masses originate in these kinds of models, the expectation is that other signals of R-parity breaking should show up [25].

5. To summarize, massive neutrinos point to a hierarchy problem in possible extensions of the standard model, independently from the assumption of grand unification. We discussed how this remark may result in an argument in favor of certain theoretical models.

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However, L. Gonzalez-Mestres in physics/9705031 suggested the possible relevance of Planck scale physics for the cosmic rays of highest energy.


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