Report of the working group on the measurement of triple gauge boson couplings


* convenors

Abstract: The working group discussed several aspects of triple gauge coupling analysis viewed in the light of experiences with the first high energy data recorded at energies above the W pair threshold. Some analysis methods were reviewed briefly, and consideration given to better ways of characterising the data. The measurement of CP violating parameters was discussed. Results were prepared to further quantify the precision attainable on anomalous couplings in the four-quark channel using jet-charge methods, and finally the trade off between maximum LEP energy-vs-luminosity was quantified.
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1 Introduction

The triple-gauge-coupling (TGC) working group was composed of members of the theoretical and experimental community who are actively involved with the measurement and interpretation of TGCs at LEP2. Many of the participants also contributed to the LEP2 workshop [1] in 1995.

A review of the current status of the first experimental results from LEP2 was given in the plenary review [R.Sekulin - these proceedings] which also gives an overview of some of the calculational tools presently available as well as methods of analysis which have been used.

At the LEP2 workshop many prophecies were made in advance of the first data taking above the W pair threshold. The group decided that the best use of this workshop would be to review how some of the ideas envisaged at that time had actually evolved in the light of experience with the first data, and to discuss some of the problems which have been brought to light. We also identified some quantitative studies which were considered useful to present for future reference.

In section two we consider briefly some of the limitations of present analysis methods. We then consider in some detail how experimental measurements might be better characterised in order to avoid, or minimise, some of these limitations in an unbinned maximum likelihood fit. Finally the possibilities for an improved fitting tool are described.

In the third section we describe a development of a method employing optimal observables, which makes use of fully simulated distributions to extract TGC parameters.

In the fourth section a study is presented in which the precision which can be obtained using the $q\bar{q}q\bar{q}$ channel is quantified in the light of current experience of jet pairing and charge assignment purity.

In section five we present a parametrization of the CP violating trilinear gauge couplings restricting ourselves to the gauge invariant operators with the lowest dimension. We also present some estimate of the sensitivity expected at LEP2.

Finally in section six we present a study to quantify the variation of TGC measurement precision which would be obtained for different scenarios of maximum LEP2 energy versus the luminosity increase which results from sacrificing a few GeV from the absolute maximum energy which LEP can achieve.

Throughout the following we make use of abbreviations for different W pair decay types. These are: (i) $q\bar{q}q\bar{q}$ events where both W bosons decay to a quark-
antiquark final state, (ii) $q\bar{q}\ell \nu_\ell$ events where one W decays to a quark-antiquark final state, and the other W decays to an electron, muon or tau, plus a neutrino and (iii) $\ell \nu_\ell \ell' \nu_\ell$ events where both W bosons decay leptonically. We also refer to two distinct ways in which the phase space variables are normally characterised for each event. The first is to use quantities which are very close to those measured by the detectors, such as the four-vectors of the observed “fermions” (leptons and jets), this is referred to as $4$-vector analysis. The second is to instead use the set of five angles given by the $W^-$ flight direction with respect to the incoming beam direction, and the polar and azimuthal angles of the decay products of each W measured in the rest frame of the respective W. A full description of these angles can be found in [1, 2, 3]. These angles are commonly written as \{\cos \theta_W, \cos \theta^*_1, \phi^*_1, \cos \theta^*_2, \phi^*_2\} and methods using these are referred to as $5$-angle analyses. We refer to both schemes generically as the set \{\Phi\}.

2 Data characterisation for maximum likelihood fitting.

Previous studies [1] have examined possible observable effects from anomalous TGCs. Anomalous TGCs can affect both the total production cross-section and the shape of the differential cross-section as a function of the $W^-$ production angle. Additionally, the relative contributions of each helicity state of the W bosons are changed, which in turn affects the distributions of their decay products.

The most straightforward approach which has been considered is to use an un-binned multi-dimensional maximum likelihood method (ML) where one identifies the most likely measurement of a TGC $\alpha$, by varying $\alpha$ to minimise the quantity

$$-\log L = - \sum_{\text{events } i} \log P(\Phi_i, \alpha)$$

where $\Phi_i$ represents the phase space measurements for event $i$. To do this is necessary to have some function, $P$, which returns the probability density function for a particular $\Phi_i$ to occur, and this can be conventionally derived as the normalised differential cross section. It is reasonably straightforward to code an analytic form for $P$ if one restricts to on-shell formula without initial state radiation (ISR). The relevant expressions can be found in [2] and an example code is given in [4]. Such approaches are simple, not very CPU intensive, but as well as neglecting ISR and $\Gamma_W$ also neglect detector acceptance and resolution. The neglect of such effects was shown [1] to lead to biases of roughly the same magnitude as the expected eventual experimental precision for $500\text{pb}^{-1}$. 
More sophisticated approaches have been implemented, which attempt to allow for all of the above effects. A typical solution is to compare an observed data distribution with that predicted using fully simulated Monte Carlo events passed through the same selection requirements as the data. By using Monte Carlo distributions corresponding to different generated values of \( \alpha \) one can make fits in order to measure the most likely true value of \( \alpha \). Examples of this are (i) a coarse binned maximum likelihood analysis [5] (ii) use of distributions of optimised observables [6], (iii) \( \chi^2 \) fits to a reduced number of variables [7] and (iii) nearest neighbour counting in Monte Carlo distributions [8]. These methods are straightforward in the case of one variable, but tend to require either a very large number of Monte Carlo events as the number of variables increases, or to necessitate very coarse binning (the possible exceptions to this are the optimised observables methods described below). There are various ways to try to make these methods more efficient, however we decided to turn our attention back to the more straightforward maximum likelihood approach and consider how its failings could be rectified.

In order to perform a “complete” ML analysis one would like to include all of the effects of ISR, \( \Gamma_W \), detector resolution and detector acceptance into a likelihood function which we will call \( P_{\text{full}} \). The issues involved in doing this fall into three classes: (i) to include ISR requires the inclusion of some internal integration over a radiated photon spectrum (ii) to include the effect of \( \Gamma_W \) depends upon whether one attempts to supply \( W \) mass information with each data event or instead wishes to internally integrate over the \( W \) virtualities and (iii) the facilities which can be included in \( P_{\text{full}} \) depend upon the choice of quantities which one measures for each data event.

To completely specify an event requires 8 pieces of information. Each of the four decay fermions requires 3 quantities (masses are fixed). The total energy and momentum provide 4 constraints (assuming that the 4-momentum of any ISR photon is known). The total number of pieces of information required is therefore \( 4 \times 3 - 4 = 8 \) (although the overall azimuthal angle of the event carries no information). In the 5-angle formulation the missing information is equivalent to ignoring the two \( W \) masses. It is also necessary to know the total energy and momentum to calculate the decay angles, and it is normally assumed that there is no ISR. This makes it impossible to then carry out a consistent integration over ISR in \( P_{\text{full}} \). Conversely if the 4-vectors of all of the decay fermions are specified then these completely determine both the \( W \) masses and the momentum carried by ISR. There is therefore no flexibility left for these to be integrated over in \( P_{\text{full}} \).

Another important consideration is that if one chooses to characterise each event into 5-angles then all the quantities are complicated transforms of the basic de-
ector measurements such as lepton energy and direction etc. As a result the 5-angles are highly correlated and rarely have Gaussian resolutions. Both of these facts make it non-trivial to incorporate integrations over detector resolution or acceptance. If instead one chooses to characterise each event using the 4-vectors then some of these limitations are removed.

The group therefore considered the most desirable way to characterise each type of event from an experimental point of view. This was done separately for each W pair decay channel, and the results are summarised in table 1.

2.1 $q\bar{q}e\nu_e$ and $q\bar{q}\mu\nu_\mu$ events

In these events the lepton is normally well measured and can be characterised by its energy, $E_l$, and its polar and azimuthal angles, $\cos \theta_l$ and $\phi_l$. These three quantities are largely uncorrelated and often have resolutions which may be approximated as being Gaussian.

The hadronically decaying W results in two jets. It is often (although not invariably) the case that the jet angle measurements are better than energy resolutions. It is therefore highly likely that any apparent jet mass is mainly determined by energy resolution rather than any physical mass. It therefore also follows that the measured invariant mass of the two jet system is poorly related to the true W virtuality on an event by event basis. This last point is important, as several fitting tools which exist take as input the pairs of measured 4-vectors belonging to each W, and then interpret the invariant mass of each pair literally when calculating the matrix element for the decay. This means that the inclusion of the effect of $\Gamma_W$ is not via any integration internal to the fitting tool, but comes from the actual mass distribution of the data being fitted, and this is determined almost entirely by resolution errors. Although at this time no quantative study has been done, it seems unlikely that this is sensible. It is therefore desirable to have the option to allow $P_{full}$ some freedom to integrate internally over W virtuality. We therefore concluded that the two jets should be characterised by their directions, $\cos \theta_1$, $\phi_1$, $\cos \theta_2$ and $\phi_2$ and optionally the sum of both jet energies, $E_{tot}^{had}$.

This choice of these variables is designed to allow those that are well measured to be used, and those that may not be well measured to be integrated over easily. If $E_{had}^{tot}$ is used then 8 quantities are specified. Since it is not required to assume the total energy and momentum of the event to specify the 8 quantities, then the freedom is left for $P_{full}$ to either assume no ISR or to integrate over a photon

\footnote{The energy will either be measured in a calorimeter or by the magnetic tracking detector, in which case either the measurement or its inverse is approximately Gaussian.}
radiation spectrum. However there is no freedom left to integrate over the W virtualities. If, however, it is considered that the hadronic energy measurement is relatively poor then one can either perform an explicit integration over the assumed resolution (see section 2.5) or allow a complete integration over all possible values of $E_{\text{had}}^{\text{tot}}$ to be included in $P_{\text{full}}$.

2.2 $q\bar{q}\tau\nu\tau$ events

These events differ from the previous category because one observes only the tau decay products. It is likely that the direction of the decay products can serve as a useful approximation to the actual tau direction (with a larger resolution) but the energy will always be low as there is always at least one missing neutrino. Therefore the simplest way to specify the event is to use exactly the same quantities as for the $q\bar{q}e\nu_e$ and $q\bar{q}\mu\nu_\mu$ events, but in addition allow the integration over $E_l$ in $P_{\text{full}}$.

2.3 $\ell\nu_\ell\bar{\ell}\nu_\ell$ events

Studies have indicated that in the case of electron and muon events there is useful information on anomalous couplings. It is in principle possible to fully reconstruct [1] these events up to a two fold ambiguity using the two observed leptons, the beam energy and momentum, and the W mass. However there are several problems in doing this due to experimental resolution and that one has no knowledge of the actual W masses for each event. The working group therefore considered that it would be better to use only the observed quantities and integrate over everything else in $P_{\text{full}}$. Thus such events would be characterised by $E_{l+}, \cos \theta_{l+}, \phi_{l+}, E_{l-}, \cos \theta_{l-}$ and $\phi_{l-}$ (note again that only the relative $\phi$ carries information). If in addition an integration over $E_{l+}$ and $E_{l-}$ can be included then tau events could also be used.

2.4 $q\bar{q}q\bar{q}$ events

These are the most problematic. The experience of the working group from various Monte Carlo studies is that the W production direction can be well measured from these events, but as for the $q\bar{q}\ell\nu_\ell$ events there is probably no useful information about the individual W masses on an event by event basis. However,

\footnote{It was not considered sensible (at this stage at least) to consider incorporating an integration over all tau decay modes into $P_{\text{full}}$.}
following the previous reasoning, we might think of specifying all of the jet angles, 
\[ \cos \theta_i, \phi_i, \quad i = 1, 4. \] In this case we would have given all 8 pieces of information, hence completely specifying the W masses, once the total energy and momentum are fixed (in fact this is used by DELPHI in one of their mass measurement methods). At the present time no detailed studies have done to resolve this point from the point of view of TGC analysis, and further work is required.

2.5 Inclusion of detector resolution and acceptance

The quantities chosen to specify each type of event are largely uncorrelated and are often described by Gaussian resolutions. As such it is possible to imagine performing a simple Gaussian integration over each to allow for detector resolution. This can be done externally to \( P_{\text{full}} \) using commonly available integrating functions.

Exactly how many quantities to integrate over depends upon the event type. In the case of \( q\bar{q}\ell\bar{\nu}_\ell \) events it may be that the precision of the lepton measurements is more than adequate for no resolution on either energy or direction to be included. In this case one needs to perform an integration over only the jet angles. By the same arguments it may be that in the case of \( \ell\nu_\ell \ell\nu_\ell \) events resolution can be neglected entirely. Exactly how to proceed in each case is an experimental question, and also depends upon the statistical precision of the data.

Similar approximations may be used in order to account for detector acceptance. For instance detectors normally have a sharp cut off in \( \cos \theta \) for lepton identification, but otherwise are near 100% efficiency and uniform in \( \phi \). A simple approximation such as this can be included in the normalisation of \( P_{\text{full}} \). In the case of jets the situation is not always so simple, but it may nevertheless be appropriate to apply a sharp fiducial cut such that jets are only used in a region where they are well reconstructed with high efficiency.

The treatment of resolution and acceptance suggested here is an experimentally motivated approximation, and as such will never be “exact”. However since the biases incurred by completely ignoring these effects are only expected to be approximately the size of the final LEP2 statistical error, it may be that the level of approximation involved here is adequate for all practical purposes. Further studies are needed to quantify this issue.
2.6 Prospects for a new semi-analytic analysis tool

A feasibility study of a semi-analytic analysis tool has been made. For this purpose two different problems should be dealt with. In the first place an efficient program should be developed which for each set of variables from Table 1 calculates the probabilities. In that evaluation the effects of the W-width and ISR should be taken into account. Secondly, it should be studied whether this analyzer properly determines the TGC parameters from a Monte Carlo sample of unweighted events.

The first issue, developing a program which calculates cross sections differential in the appropriate variables has been accomplished. In fact, two independent programs have been made in order to have cross checks. One program uses the matrix elements of ERATO, the other of EXCALIBUR. All kinematical variables of table 1 have been implemented, except the case with the tau-lepton. Without ISR the chosen variables require two, one integration, sometimes none. With ISR two more integrations have to be carried out.

The second issue, whether maximum likelihood fits based on probabilities related to the above specific kinematic variables can really determine TGC parameters is at present under study. One generates unweighted events, uses of these events only the variables of table 1 and calculates the likelihood function for e.g. nine values of a specific TGC parameter. From this the TGC parameter is determined. In this way one could try to determine any parameter on which the matrix element depends.

In fact, so far the most extensive studies have been made for a parameter other than a TGC. The W-mass has been chosen as first parameter to be studied. One reason for this choice is that the question of experimentally motivated quantities can also be relevant for the mass determination. Another is, that one may eventually like to determine simultaneously both the mass and a TGC coupling.

So, as a first test of the method the mass determination from experimentally motivated quantities was chosen. To this end typically 1600 unweighted events were generated with ERATO or EXCALIBUR. The semi-leptonic case has been studied in great detail. The data samples are CC03, CC10 and CC20 based sets including ISR and have been analyzed with the same matrix elements or simpler ones (e.g. a data set with ISR analyzed without ISR).

The following results can be reported. Within the errors, typically 35-50 MeV, the input mass could be reconstructed when the same matrix element is used in the analysis as in the generation of the sample. When simpler matrix elements in the analysis than in the generation are used, shifts can arise. Large shifts (300
MeV in some cases) may occur due to the neglect of ISR in the analysis. Smaller shifts (up to 50 MeV) are found when a CC20 sample is analyzed with CC03 probabilities.

The mass determination is also successful in the four quark case when one assumes to know which quark pair comes from a W. This reduces the folding. When the method is applied to the purely leptonic case the mass reconstruction is not efficient, as one would expect as there is less kinematical information.

The results on the mass determination will be published in more detail [9] elsewhere. It seems that the maximum likelihood method to determine $M_W$ from experimental motivated quantities is a useful addition to the existing direct reconstruction method.

As to the TGC determination, the first results look promising, but more results should be obtained before a detailed discussion of the merits of the method can be presented. However it already now is clear that the method takes ISR and full matrix elements into account in a very satisfactory way.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Well measured</th>
<th>“Poorly measured”</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q\bar{q}e\nu_e$</td>
<td>$E_l$, $\cos \theta_l$, $\phi_l$, $\cos \theta_1$, $\phi_1$, $\cos \theta_2$, $\phi_2$</td>
<td>$E^\text{tot}_{\text{had}}$</td>
</tr>
<tr>
<td>$q\bar{q}\mu\nu_\mu$</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>$q\bar{q}r\nu_r$</td>
<td>$\cos \theta_l$, $\phi_l$, $\cos \theta_1$, $\phi_1$, $\cos \theta_2$, $\phi_2$</td>
<td>$E_l$, $E^\text{tot}_{\text{had}}$</td>
</tr>
<tr>
<td>$\ell\bar{\nu}<em>\ell \nu</em>\ell$</td>
<td>$E_{l+}$, $\cos \theta_{l+}$, $\phi_{l+}$, $E_{l-}$, $\cos \theta_{l-}$, $\phi_{l-}$</td>
<td>&quot;</td>
</tr>
<tr>
<td>$q\bar{q}q\bar{q}$</td>
<td>$\cos \theta_i$, $\phi_i$, $i = 1, 4$</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

Table 1: Experimentally motivated quantities to specify for different W pair event categories.

3 Development of optimal observable analysis.

In section 2 several methods of TGC analysis were mentioned which employ fully simulated Monte Carlo events in order to analyse the data. These methods include all effects of ISR, $\Gamma_W$, detector acceptance and resolution. One problem which often arises is the number of bins necessary in a multidimensional analysis. However methods which use “optimal observables” have the potential to avoid this problem.

The use of optimal observables (OO) has been discussed widely in the literature [1, 10, 11, 12]. The method consists in constructing observables, namely functions of
the observed kinematic variables $\Phi$, which maximise the sensitivity to anomalous couplings. There is one observable per coupling, $O_{\alpha}(\Phi)$, given by:

$$O_{\alpha}(\Phi) = \frac{d\sigma_1}{d\Phi}$$  \hspace{1cm} (1)

where the functions $\frac{d\sigma_i}{d\Phi}$ are obtained from the quadratic form of the Born level cross section:

$$\frac{d\sigma}{d\Phi} = \frac{d\sigma_0}{d\Phi} + \alpha \frac{d\sigma_1}{d\Phi} + \alpha^2 \frac{d\sigma_2}{d\Phi}$$  \hspace{1cm} (2)

The observable is calculated for each event from the measured $\Phi$ variables. In principle the mean value of $O$ (averaged over the data set) contains all of the information which can be extracted from the $\Phi$ distribution of the data, provided that it is small enough that the neglect of the second order term has a negligible effect. The spectrum of the optimal observables might be regarded as a projection of the five-dimensional differential cross section in $\Phi$ onto a direction which optimises the sensitivity.

As originally envisaged and discussed in the literature, the mean value is used to determine $\alpha$ through the inversion of a matrix relation between the two. The effects of ISR and $\Gamma_W$ can only be included if the the resulting functional form of the differential cross section of equation 1 is known, which is not generally the case, and therefore biases are introduced. It is also in principle possible to include detector effects in the procedure, but this can lead to problems of instability.

Instead of following this procedure a recent OPAL analysis [5] adopts a different approach. A similar method has also been developed by DELPHI [13].

In the OPAL analysis each event is analysed using the 5-angles as the phase space variables. Each OO quantity is calculated using the the on-shell/no ISR differential cross section (it is straightforward to obtain an analytic function for this). The value of the OO measured per event is then treated just as any other experimental quantity whose distribution is sensitive to variations in the relevant anomalous coupling. As example the observed distribution of the optimal observable for the $\alpha_{W\phi}$ parameter is shown in figure 1 for data and Monte Carlo. The expected distribution for all values of $\alpha$ is obtained using fully simulated EXCALIBUR [14] events generated for a few specific values, and then using the quadratic dependence upon $\alpha$ in each bin of the spectrum of $O_{\alpha}$. The Monte Carlo distributions are generated with ISR and $\Gamma_W$ switched on. The contribution from background contamination is added to the spectrum.

To measure the TGC parameter a binned likelihood fit of the observed data to the expected distribution predicted by the Monte Carlo is performed. Since like
is compared with like the method includes all of the required effects and no bias should be introduced.

There are two drawbacks of the method. The first is that the OO is calculated using an on-shell/no ISR differential cross section. This can be considered as an approximation to the true OO. The approximation does not introduce any bias, nor should it be confused with the fact that the data is fitted to Monte Carlo which includes ISR and $\Gamma_W$, however it does in principle lead to loss of optimality which means that the measured statistical error will increase. It is for this reason that the full distribution of the $O_\alpha$ calculated for each event is used, making better use of the available information than the mean value in the case of this non-optimality. The second drawback is that the simple OO method is based upon an expansion about a single value $^3$ of a given TGC and is only optimal if the fitted values lie in a small range $^4$ around this value. At present the statistical precision of the data is rather poor and so the OO method will not necessarily give the true confidence levels for large values of the errors. However this limitation is expected to diminish as the statistical precision of the data increases in future years. However this method is in practice almost adequate to analyse the current small data set. The log $L$ distributions obtained from the method described, and from a more conventional binned maximum likelihood analysis of the same data give almost identical errors and central values. These log $L$ curves can be seen in reference [5].

4 The use of jet charge in $qq\bar{q}q$ events

In events where both W pairs decay hadronically the simple method of determining the W charge used in the semi-hadronic and leptonic events is no longer available. A method of assigning a charge to the Ws from the properties of the jet pairs into which they decay must be sought. Various possibilities suggest themselves, but none of them lead to identification of the W charges with 100% certainty. Previous studies have either assumed that the charge is unknown or completely determined; here we investigate the effect that varying the efficiency of the W charge determination has on the ability to measure $\alpha_{W\phi}$. We generated events with PYTHIA. The correct pairing of jets into Ws was taken and, for a fraction $(1 - f_g)$ of the events (chosen randomly), the correct charge assignment of the Ws was taken. For the remaining fraction $f_g$ of the events, the pair constituting the W$^-$ was called W$^+$ and vice-versa. This procedure defines the “assigned

$^3$Alternately, an extension of the method is possible [12] whereby the $\alpha^2$ term is included.

$^4$If the measured central value, $\alpha_0$, is different from the expansion point of $\alpha = 0$ then the method can be iterated to use OOs calculated from a functional form of equation 2 where $\alpha$ is replaced by $\alpha - \alpha_0$. 

Figure 1: The distribution of the optimal observable for $\alpha_{W\phi}$ obtained from the $q\bar{q}l\ell_\nu$ events. The hatched histogram shows the non WW background. These are compared with the distribution expected in the Standard Model using fully simulated Monte Carlo events. The predicted distributions for $\alpha_{W\phi} = +2(-2)$ are also shown as dotted (dashed) lines.

In the analysis of the events, the unnormalized probability for observing each event was taken as $(1 - f_a)p(1, 2) + f_a(2, 1)$, where $p(i, j)$ is a squared matrix element. For $(i, j) = (1, 2)$ the production and decay angles of $W^-$ and $W^+$ were calculated using the “assigned momenta” defined above, while for $(i, j) = (2, 1)$ the production and decay angles were taken with the “assigned” $W^-$ interpreted as the $W^+$ and vice-versa.

Results of extended maximum likelihood fits of $\alpha_{W\phi}$ to 1000 events at 190 GeV are shown in table 2. The decay angles from both Ws were included in the fits, but folded, thus assuming no quark flavour information in the W decay products. Thus each $p(i, j)$ is an average of four matrix element calculations.

There is a definite pattern in the results:

- If the correct misassignment probability is assumed in analysis (the diagonal entries in the table), the result is unbiased, but the precision decreases rapidly with increasing $f_g$. For $f_g = 0.5$, the result is identical with that obtained simply by folding production and both pairs of decay angles in the fit (an eightfold ambiguity), i.e. without applying any jet charge selection: $\alpha_{W\phi} = 0.024 \pm 0.064$. For $f_g = 0.2$ (a value which might be expected in
<table>
<thead>
<tr>
<th>( f_a ), Rate assumed in analysis</th>
<th>( f_g ), Misassignment rate applied to generated events</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0.0 )</td>
<td>0.004 ( \pm 0.039 )</td>
</tr>
<tr>
<td>( 0.05 )</td>
<td>-0.095 ( \pm 0.040 )</td>
</tr>
<tr>
<td>( 0.10 )</td>
<td>-0.011 ( \pm 0.048 )</td>
</tr>
<tr>
<td>( 0.15 )</td>
<td>0.057 ( \pm 0.055 )</td>
</tr>
<tr>
<td>( 0.20 )</td>
<td>0.103 ( \pm 0.058 )</td>
</tr>
<tr>
<td>( 0.25 )</td>
<td>0.056 ( \pm 0.051 )</td>
</tr>
<tr>
<td>( 0.30 )</td>
<td>0.084 ( \pm 0.061 )</td>
</tr>
<tr>
<td>( 0.40 )</td>
<td>0.060 ( \pm 0.063 )</td>
</tr>
<tr>
<td>( 0.50 )</td>
<td>0.024 ( \pm 0.064 ) ( \pm 0.064 )</td>
</tr>
</tbody>
</table>

Table 2: Results of fits to \( \alpha_{W\phi} \)
practice), the precision has already deteriorated to a value rather close to this.

- If an incorrect misassignment probability is assumed in analysis, the result is biased. For instance, if the true misassignment rate is \( f_g = 0.2 \), but a value \( f_a = 0.0 \) is taken in analysis, a large negative bias in \( \alpha_{W\phi} \) is induced. This arises from the fact that the jet charge misassignment has had the effect of flattening the production angular distribution, which is also predicted for negative values of \( \alpha_{W\phi} \). Unfortunately, in the region \( f_g = 0.2 \), the bias seen in the table varies rather rapidly as the assumed misassignment probability diverges from its true value, implying that a large systematic error might be incurred in the fitted TGC value.

These considerations will have to be taken into account in determining the usefulness of jet charge information in the analysis of the \( q\bar{q}q\bar{q} \) channel.

This above analysis assumes that the angle of the W is well measured. We then wished to answer the question “What effect will imperfections in our definition of the W direction have on our ability to measure \( \alpha_{W\phi} \)?”

The standard of way of estimating the W direction is to divide the tracks (and energy clusters) of the event into 4 “jets”, and to add the momenta of pairs of jets together to make the W momenta. The resulting directions are not aligned precisely with the directions of the Ws.

In events where the reconstructed W is badly misaligned, the discrepancy can sometimes be traced to a misallocation of the jets to the Ws. This leads naturally to the idea of a measurement degradation due to a jet misassignment. However, this is an oversimplification. In another class of badly measured events it is the “jets” themselves which fail to measure the direction of the fundamental decay fermions. To be more precise, in the context of PYTHIA, if the sets of tracks which make up the four jets is compared with the sets of particles which are the decay products of the four fermions, then for these events one or more of the reconstructed jets will not be made up of particles from the decay of a single fermion.

The details of the problems caused by these effects may depend rather sensitively on the apparatus used in the measurement and the algorithm used to extract the W directions. For this reason we made no attempt at the sort of generic measurement which is suitable for the effects of jet charge. Instead, in order to provide an idea of the sort of degradation which follows from these considerations, we have taken the example of the OPAL apparatus and processed EXCALIBUR events through a a full simulation of the detector. The resulting simulated data
is divided into 4 jets using a Durham type algorithm; a kinematic fit is performed on jets constraining the pairs to have the same mass, and the pairing with the highest mass is taken as the correct one. The charge of each jet is defined using a momentum weighted charge measure and the pair of jets which have the highest summed jet charge is taken as the positive W. For a dataset consisting of 7,000 fully simulated $q\bar{q}q\bar{q}$ events we fit the value of $\alpha_{W\phi}$ for all the events which are successfully reconstructed as four jet events. This defines a ‘realistic measure’ of our ability to measure $\alpha_{W\phi}$. Then the opening angles $\Omega_{j\phi}$ between each jet and fermion are calculated, and the subset of events is taken where the minimum value of $\Omega_{j\phi}$ allows a unique association between quark and jet. This looses about 10% of the data sample. For these events the W’s direction is calculated by taking the correct pairing as given by the generator information and from that a value of $\cos\theta_w$ is derived. The charge assignment is then made by the normal charge weighting methods. It turns out that for these events the alignment between jet and quark is good, but the jet energies are not nearly so tightly constrained. Thus the W direction is much less well measured than the individual quark directions. These are the “good jet matching events”. Finally we used the correct value of the W charge, and a value and error was calculated for these “charge correct” events.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Reconstructed Pairing correct</th>
<th>Charge correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{W\phi}$</td>
<td>0.03-0.077+0.082</td>
<td>-0.18-0.072+0.080</td>
</tr>
</tbody>
</table>

Table 3: Results of fits to $\alpha_{W\phi}$

The values of $\alpha_{W\phi}$ which were found can be seen in Table 3, where it can be seen that the dominant cause of measurement degradation is the mismeasurement of the W charge. Reanalysing the 90% sample without any generator information produces a value fully consistent with the original sample.

These values should not be extrapolated to other values of $\alpha_{W\phi}$. The accuracy of the measurement of $\alpha_{W\phi}$ depends on the value of $\alpha_{W\phi}$, and experimental effects add towards that total error in ways which may also vary with $\alpha_{W\phi}$.

5 CP violating TGC parameters

In the usual phenomenological parametrization of the TGC [1] the CP violating interactions are also included in the form:

$$\mathcal{L}_{TGC} = e g_{v_{WW}} \left[ g_4 \gamma^\nu W_\nu^+ W^-_\mu (\partial^\mu V^\nu + \partial^\nu V^\mu) \right]$$
\[ + i\tilde{\kappa}_V W_\nu^+ W_\mu^- \mathcal{V}^{\mu\nu} + \frac{i}{m_W^2} \tilde{\lambda}_V W_\rho^+ W^-_{\rho \mu} \mathcal{V}^{\mu\rho} \]  

(3)

where

\[ V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu, \quad W_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm, \]

and

\[ \mathcal{V}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} V_{\rho\sigma}. \]

In Eq.(3) \( W^\pm \) is the \( W \)-boson field, and the usual definitions \( g_{\gamma WW} = 1, g_{ZWW} = \text{ctg} \theta_w \) are used. Finally \( e = g \sin \theta_w = g' \cos \theta_w \).

What is the meaning of the above Lagrangian? One way to answer this question is to look at it as the low energy limit of a manifestly \( SU(2)_L \times U(1)_Y \) gauge invariant theory. This can be done [15, 16, 17] in an effective Lagrangian approach by considering gauge-invariant operators involving higher-dimensional interactions among the gauge bosons and the Higgs field. These operators are scaled by an unknown parameter \( \Lambda_{NP} \) describing the characteristic scale of some high energy New Physics, generating at low energies the effective interaction \( L_{TGC} \) as a residual effect. In order to generate all kinds of TGC appearing in Eq.(3), we need operators with dimension up to twelve. On the other hand, restricting ourselves to \( SU(2)_L \times U(1)_Y \)-invariant operators with dimension six, which are the lowest order ones in a \( 1/\Lambda_{NP} \) expansion, we end up with the following list of operators capable of inducing CP-violating TGC couplings [18]

\[ \tilde{O}_{BW} = \Phi^\dagger \frac{\tau}{2} \cdot \tilde{W}^{\mu\nu} \Phi B_{\mu\nu} \]

\[ \tilde{O}_W = \frac{1}{3!} (W^\mu_\rho \times W^\rho_\nu) \cdot \tilde{W}^{\nu}_\mu, \]

(4)

where

\[ \tilde{B}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} B_{\rho\sigma}, \quad \tilde{W}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} W_{\rho\sigma} \]

(5)

and \( \Phi \) is the Higgs doublet.

The New Physics contribution from the above operators is described by the effective Lagrangian

\[ L_{TGC} = \frac{gg'}{2} \tilde{\alpha}_{BW} \mathcal{O}_{BW} + \frac{g}{m_W^2} \tilde{\alpha}_W \mathcal{O}_W \]

(6)

where the relations between \( \tilde{\alpha}_{BW}, \tilde{\alpha}_W, \) and the parameters appearing in the Eq.(3) are given by

\[ \tilde{\kappa}_\gamma = \tilde{\alpha}_{BW} \quad \tilde{\lambda}_\gamma = \tilde{\alpha}_W \]

\[ \tilde{\kappa}_Z = - \tan^2 \theta_w \tilde{\kappa}_\gamma \quad \tilde{\lambda}_Z = \tilde{\lambda}_\gamma. \]

(7)
In order to get an estimate of the expected sensitivity at LEP2, we performed an analysis based on the event generator ERATO, using the method of Optimal Observables, but without taking into account any experimental effect such as resolution and related background. The results are given in Table 4 corresponding to an integrated luminosity of 500 pb$^{-1}$.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>161</th>
<th>172</th>
<th>192</th>
<th>205</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\alpha}_{BW}$</td>
<td>1.81</td>
<td>0.79</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>$\tilde{\alpha}_{W}$</td>
<td>0.43</td>
<td>0.14</td>
<td>0.02</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Table 4: One standard deviation errors on TGC parameters.

It should be emphasised that the limits on the CP violating couplings are expected to be of the same order as those on the CP conserving ones.

6 High energy -vs- high luminosity

The maximum centre-of-mass energy attainable at LEP is expected to be limited by the radio frequency accelerating gradient available. As a result, a trade-off will be inevitable between the maximum centre-of-mass energy achievable and the reliability of machine operation, by keeping a fraction of the RF in reserve. Other machine performance issues emphasise this point further. There are therefore a range of strategies for running LEP during the LEP2 programme. The two extreme cases may be characterized as:

1. run at the highest energy attainable, with reduced safety factors, and accept the loss in integrated luminosity;

2. run at a lower energy and attempt to collect as much integrated luminosity as possible.

In order to seek a balance between these extremes, one should take into account the extra reach for searches and the expected precision of measurements. One element in this accounting is the variation of the expected precision for TGC measurements as a function of the centre-of-mass energy and the integrated luminosity.
6.1 Sensitivity studies with simulated events

The prospective sensitivity to the anomalous couplings has been studied for the case of the $\alpha$ parameters in the $W\phi$, $B\phi$ and $W$ models. Two independent event generation and analysis procedures were applied and consistent results were obtained.

In the first analysis, events were generated with the EXCALIBUR [14] program at various $\alpha$ values, and analysed with a fitting program using the ERATO [19] matrix element routines to calculate the differential cross-section. The second analysis employed the WOPPER [20] event generator, and analysed events using the on-shell cross-section formulae from Bilenky et al. [2]. These two analysis strategies differ in sophistication, and include slightly different information about the anomalous coupling sensitivity [1], but for the purpose of this study, i.e. the determination of the sensitivity to the anomalous couplings, they give similar results.

Following [1], only the semileptonic channels $q\bar{q}\nu_\epsilon$ and $q\bar{q}\mu\nu_\mu$ were considered, and no detector simulation was applied. These “ideal detector” studies do not, of course, correspond directly to real analyses of events observed in the LEP detectors, because various additional effects enter: limited acceptances, backgrounds, and detector resolution degrade the performance. On the other hand, including information from the $q\bar{q}\tau\nu_\tau$, $l^+\nu_l\bar{\nu}$ and $q\bar{q}q\bar{q}$ channels improves the precision relative to the two-channel ideal detector case.

It is possible to estimate the statistical precision that might be obtained in real analyses compared with these idealized studies by comparing the precision achieved and expected from the small 172 GeV data sample already taken. The calibration factors were found to be within 20% of unity within most cases, supporting the usefulness of the idealized studies. Only statistical errors were considered in these studies. While systematic effects may eventually become significant for these measurements, the dominant systematic contributions are expected to come from detector effects, which can presumably be understood with sufficient precision with sufficiently large data samples.

Figure 2 shows the predicted error on the $\alpha$ parameters for centre-of-mass energies ranging from 170 GeV to 210 GeV, for 500 pb$^{-1}$ collected by a single experiment. The errors are derived for the case where the central value of the $\alpha$ parameter is close to zero — it was found that the expected error decreases by $O(30\%)$ if the true $\alpha$ parameter is 1, for example, but the main effect is simply the substantially increased $W$-pair production cross-section for such non-standard couplings. The sensitivities at four canonical centre-of-mass energies are shown in table 5. It is evident that the gain in precision with energy is very marked below around
180 GeV, but that the gain then slows, although the size of this effect differs for the different couplings. It is interesting in particular to compare the 190 and 200 GeV points: a unit of luminosity at 200 GeV is approximately as sensitive as 1.4 to 1.8 units at 190 GeV.

6.2 Conclusion

The best measurement of anomalous coupling parameters at LEP2 requires that a large fraction of the luminosity should be taken above 180 GeV centre-of-mass energy, as is planned. It is important that the luminosity available should be high: the 500 pb$^{-1}$ considered in previous studies remains an apposite goal. The sensitivity per unit luminosity rises above 180 GeV, so it is useful to increase the centre-of-mass energy if only a modest cut in luminosity is taken. The sensitivity rise is, however, slow, so this analysis disfavour operating strategies where a substantial luminosity penalty is incurred for the sake of only a few GeV in centre-of-mass energy.

7 Summary

The first data which has been recorded at LEP2 above the W pair threshold has been followed by a rapid analysis by all of the LEP experiments to search for possible signals for anomalous triple gauge boson couplings. That this could be done so quickly is a tribute the the large amount of preparatory work performed by theoreticians and experimentalists in the provision of tools, and the development of analysis methods. In this working group we reviewed our experience in the light of this first data. We primarily discussed the limitations which arise when trying to incorporate all effects of W width, ISR, detector acceptance and resolution into an analysis. These effects had been previously shown to lead to biases, and several techniques have been developed to try to handle this. Most methods do however have some limitations, and so we have considered alternative ways to characterise the data which may be more amenable to the use of analytic fitting tools. One of the most important outcomes of the work was to nucleate the provision of a new fitting tool. Work on this is still underway and will be published separately at a later date. Another important topic is the extent to which use can be made of W pair events decaying into the four quark channel. Analysis of this channel is difficult due to the problems in identifying the correct jet pairing and the correct W charge assignment, and therefore a study was made to quantify the degradation in precision due to these effects. We also briefly discussed the formulation of CP violating TGC parameters in a gauge invariant
<table>
<thead>
<tr>
<th>Centre-of-mass energy</th>
<th>$\Delta \alpha_{W\phi}$</th>
<th>$\Delta \alpha_{B\phi}$</th>
<th>$\Delta \alpha_{W}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>172 GeV</td>
<td>0.042</td>
<td>0.25</td>
<td>0.067</td>
</tr>
<tr>
<td>183 GeV</td>
<td>0.027</td>
<td>0.12</td>
<td>0.042</td>
</tr>
<tr>
<td>190 GeV</td>
<td>0.023</td>
<td>0.092</td>
<td>0.035</td>
</tr>
<tr>
<td>200 GeV</td>
<td>0.020</td>
<td>0.068</td>
<td>0.029</td>
</tr>
</tbody>
</table>

Table 5: Expectations for triple-gauge coupling precisions from a single experiment, approximated by the “ideal” precision for the $q\bar{q}e\nu_e$ and $q\bar{q}\mu\nu_\mu$ channels. In each case the precision which would be obtained with 500 pb$^{-1}$ at that centre-of-mass energy is given.

Figure 2: Evolution of the expected statistical precision on the measurements of the anomalous coupling parameters $\alpha$ for different models, as a function of centre-of-mass energy. The precisions given correspond to a single ideal detector analysis of the $q\bar{q}e\nu_e$ and $q\bar{q}\mu\nu_\mu$ channels, employing the production and decay angles of both W’s, but without resolving the two-fold decay ambiguity from the hadronically decaying W.
framework. Finally we have presented a study of the trade off between achieving the highest possible LEP2 energy at the expense of integrated luminosity.

References


