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NUCLEAR CLUSTERS IN EXIT AND ENTRANCE CHANNELS

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Nuclear clusters in exit and entrance channels

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Abstract

The exit and entrance channels leading rapidly from one sphere to two spheres and vice versa assuming volume conservation and neck formation in keeping quasi-spherical shapes have been investigated. The potential energy has been determined within a liquid-drop model including an accurate radius, a proximity energy, the mass asymmetry and the temperature. The calculated fission and fusion barrier characteristics, half-lives of radioactive nuclei emitting heavy clusters, fragment kinetic energies, critical angular momenta of light nuclei and rotating super and hyperdeformed state properties are compatible with the available experimental data.

1 Introduction

New observed phenomena like cluster radioactivity, formation of nuclear molecules in ^{24}Mg , cold fission of ^{252}Cf , asymmetric fission of intermediate mass nuclei, quasi fission of heavy dinuclear systems have renewed interest in investigating the fusion-like fission valley which leads rapidly to two touching spherical fragments and quasi-molecular configurations. Furthermore, the rotating super and hyperdeformed states as well as the very heavy and possible superheavy elements are and will be formed in heavy-ion reactions for which the starting shape is two close spheres.

The earlier fission studies assumed that the balance between the Coulomb and surface tension forces governs the nuclear shapes and the radius development in Legendre polynomials led to elongated one-body configurations. Later on, the inefficiency of the pure Coulomb barrier to reproduce the fusion cross sections has led to the introduction of a proximity term [1] in the development of the liquid drop model energy in order to smoothly describe the transition from two almost spherical nuclei to one-body shapes. Cluster emission, cold fission and fragmentation are exit modes exploring distorted compact shapes where the necks are not shallow and the proximity energy might play also an essential role in fission.

In this work, the study of the compatibility with the available data of a decay through these compact and necked shapes where the proximity forces play an essential role is presented. The ability of the adopted generalized liquid drop model to reproduce the fusion data has been firstly checked [2]. The main characteristics of the exit channel via these quasi-molecular shapes have been compared with new results on cluster radioactivity [3], with fission data and particularly asymmetric fission of intermediate mass nuclei [4-6], with critical angular momenta of light nuclei [7-8] and rotating hyperdeformed states [9-11]. Taking into account the temperature, the rapid fragmentation process with emission in a plane or in the whole space has been compared with the formation of toroids and bubbles [12-14].

2 Shape sequences

A two parameter shape sequence has been defined to describe the transition from one sphere to two tangent spheres (or vice versa) [2].

$$R(\theta)^2 = \begin{cases} a^2 \sin^2 \theta + c_1^2 \cos^2 \theta & (0 \leq \theta \leq \pi/2) \\ a^2 \sin^2 \theta + c_2^2 \cos^2 \theta & (\pi/2 \leq \theta \leq \pi) \end{cases} \quad (1)$$

c_1 and c_2 are the two radial elongations and a the neck radius. The two parameters $s_1 = a/c_1$ and $s_2 = a/c_2$ completely define the shape. For a given ratio $\eta = R_2/R_1$ between the radii of the fragments (or the colliding nuclei) s_1 and s_2 are connected by : $s_2^2 = \frac{s_1^2}{s_1^2 + (1-s_1^2)\eta^2}$.

Analytical expressions have been obtained for the various shape-dependent functions : volume, surface, moment of inertia, distance between the mass centres and quadrupole moment. The shape has been generalized to describe the ternary prolate fission.

3 Generalized liquid-drop model

The macroscopic energy of a rotating deformed nucleus is defined as the sum of the volume, surface, Coulomb, proximity and rotational energies. The three first terms read :

$$E_V = -a_v (1 - 1.8I^2) A, \quad (2)$$

$$E_S = a_s (1 - 2.6I^2) A^{2/3} (S/4\pi R_0^2), \quad (3)$$

$$E_C = 0.6e^2 (Z^2/R_0) \times 0.5 \int (V(\theta)/V_0)(R(\theta)/R_0)^3 \sin \theta d\theta. \quad (4)$$

$V(\theta)$ is the electrostatic surface potential. The volume, surface coefficients a_v , a_s and the sharp equivalent radius R_0 are given by :

$$a_v(T) = 15.494(1 + 0.00337T^2) \text{ MeV}, \quad (5)$$

$$a_s(T) = 17.9439(1 + 1.5T/17)(1 - T/17)^{3/2} \text{ MeV}, \quad (6)$$

$$R_0(T) = (1.28A^{1/3} - 0.76 + 0.8A^{-1/3})(1 + 0.0007T^2) \text{ fm}. \quad (7)$$

This last formula allows to reproduce the small increase of the ratio $R_0/A^{1/3}$ with the mass.

When the two fragments (or colliding nuclei) are separated, E_v , E_s and E_c are :

$$E_V = -a_v [(1 - 1.8I_1^2)A_1 + (1 - 1.8I_2^2)A_2], \quad (8)$$

$$E_S = a_s [(1 - 2.6I_1^2)A_1^{2/3} + (1 - 2.6I_2^2)A_2^{2/3}], \quad (9)$$

$$E_c = \frac{3}{5}e^2 Z_1^2/R_1 + \frac{3}{5}e^2 Z_2^2/R_2 + e^2 Z_1 Z_2/r. \quad (10).$$

The surface tension energy E_s does not include the effects of the attractive nuclear forces in the neck or the gap between the nascent fragments or incoming nuclei. The nuclear proximity energy E_N takes into account this contribution [1] : $E_N = 2\gamma \int_{h_{min}}^{h_{max}} \phi(D/b) 2\pi h dh$. γ and b are the surface parameter and width. D and h are respectively the distance between the elements of surface in regard and the ring radius in the plane perpendicular to the fission axis. ϕ is the proximity function. In this LDM approach, the surface diffuseness is not taken into account and the proximity energy vanishes when there is no neck as for ellipsoids for example.

The rotational energy has been determined within the rigid body ansatz.

4 fusion process

For light and medium systems and moderate energies above the barrier, a good evaluation of the barrier heights and positions is sufficient to explain the data. It has been shown [2] that the generalized liquid drop model and the above-mentioned shape sequence are able to reproduce the heavy-ion fusion data, as the KNS potential [15].

Above the barrier the static approach is insufficient to reproduce the fusion of very heavy systems and fusion of medium systems at high energies. Indeed, the barrier top corresponds to close separated spheres and energy dissipation occurs before the descent towards the compound nucleus. The dynamical trajectories have been inspected within a simple dynamic model [2]. For light and medium systems, the mass inertia is small and the fusion cross sections are reproduced assuming the sticking limit for the angular momentum dissipation rule. For heavy systems, static double-humped fusion barriers appear as soon as $Z_1 Z_2 \geq 1800 \pm 100$. The inner barrier is the highest for $Z_1 Z_2 \geq 2300 \pm 100$. The well between the two barriers is strongly connected with quasi-fission phenomena. For the heaviest systems ($Z_1 Z_2 \geq 2100 \pm 100$) the dissipation around the first peak is so high that an additional energy (the so-called "extra-push") is necessary to pass this first barrier. For these very heavy systems, the inertia is very high and the sliding limit for the angular momentum dissipation rule is reached.

The determination of the best reaction leading to the possible superheavy elements is not easy since the excitation energy deposited in the compound nucleus decreases with the symmetry of the reaction but the additional "extra-push" needed to compensate for the friction forces increases as well as the probability of quasi-fission events.

5 Cluster radioactivity and asymmetric fission through quasi-molecular shapes

The half-lives of the radio-active nuclei emitting heavy clusters like C, O, Ne, Mg and Si have been calculated in this very asymmetric fission valley leading rapidly to two spherical touching nuclei at the early stage of the tunneling process [3]. In such an unified fission model, the decay constant of the parent nucleus is simply $\lambda = \nu_0 P$. There is no adjustable preformation factor. The assault frequency ν_0 can be evaluated from the zero point vibration energy $E_v = 1/2 h \nu_0$ and, in the harmonic oscillator approximation, $\nu_0 = 2.5 \times 10^{20} s^{-1}$. The barrier penetrability P is calculated within the general form of the action integral

$$P = \exp \left\{ -\frac{2}{\hbar} \int_{R_a}^{R_b} [2B(r)(E(r) - E(R_a))]^{1/2} dr \right\}, \quad (11)$$

with $E(R_a) = E(R_b) = Q_{EXP}$.

The expression proposed in Ref [16] for the inertia $B(r)$ in this new fission valley has been used

$$B(r) = \mu \left\{ 1 + f(r) \frac{272}{15} \exp \left[-\frac{128}{51} ((r - R_a)/R_0) \right] \right\}, \quad (12)$$

where

$$f(r) = \begin{cases} \left(\frac{R_{cont} - r}{R_{cont} - R_a} \right)^2; & r \leq R_{cont} \\ 0; & r \geq R_{cont} \end{cases} \quad \text{and} \quad R_{cont} = R_1 + R_2. \quad (13)$$

The partial half-life time is related to the decay constant λ by $T_{1/2} = \frac{\ln 2}{\lambda}$. No parameter used to determine the potential energy has been changed and the above-mentioned formulas do not introduce any artificial parameter.

The predictions within this macroscopic LDM barrier tunneling and the experimental partial half-lives are compared in the table (first and third columns). The macroscopic part of the total energy leads to a reasonable agreement for the heaviest clusters but fails to reproduce the ^{20}O and ^{14}C emission.

Emitter and cluster	Theoretical $T_{1/2}$: macroscopic LDM barrier tunneling	Theoretical $T_{1/2}$: macroscopic and microscopic barrier tunneling	Experimental $T_{1/2}$ (s)
$^{222}\text{Ra} \rightarrow ^{14}\text{C} + ^{208}\text{Pb}$	2.7×10^{33}	2.0×10^{11}	1.2×10^{11}
$^{223}\text{Ra} \rightarrow ^{14}\text{C} + ^{209}\text{Pb}$	1.6×10^{34}	1.2×10^{14}	2.0×10^{15}
$^{224}\text{Ra} \rightarrow ^{14}\text{C} + ^{210}\text{Pb}$	1.1×10^{35}	1.9×10^{17}	7.4×10^{15}
$^{226}\text{Ra} \rightarrow ^{14}\text{C} + ^{212}\text{Pb}$	4.3×10^{35}	6.8×10^{22}	1.8×10^{21}
$^{228}\text{Th} \rightarrow ^{20}\text{O} + ^{208}\text{Pb}$	1.3×10^{26}	4.3×10^{22}	7.5×10^{20}
$^{230}\text{Th} \rightarrow ^{24}\text{Ne} + ^{206}\text{Hg}$	1.1×10^{26}	3.7×10^{26}	4.4×10^{24}
$^{231}\text{Pa} \rightarrow ^{24}\text{Ne} + ^{207}\text{Tl}$	2.9×10^{24}	1.2×10^{23}	1.7×10^{23}
$^{232}\text{U} \rightarrow ^{24}\text{Ne} + ^{208}\text{Pb}$	9.6×10^{22}	1.3×10^{21}	2.5×10^{20}
$^{233}\text{U} \rightarrow ^{24}\text{Ne} + ^{209}\text{Pb}$	3.3×10^{23}	4.7×10^{24}	6.8×10^{24}
$^{234}\text{U} \rightarrow ^{24}\text{Ne} + ^{210}\text{Pb}$	1.2×10^{24}	9.4×10^{27}	1.6×10^{25}
$^{234}\text{U} \rightarrow ^{28}\text{Mg} + ^{206}\text{Hg}$	7.6×10^{24}	1.4×10^{27}	3.5×10^{25}
$^{235}\text{U} \rightarrow ^{28}\text{Mg} + ^{207}\text{Hg}$	5.1×10^{25}	4.6×10^{30}	2.8×10^{28}
$^{236}\text{Pu} \rightarrow ^{28}\text{Mg} + ^{208}\text{Pb}$	2.0×10^{21}	1.7×10^{21}	4.7×10^{21}
$^{238}\text{Pu} \rightarrow ^{28}\text{Mg} + ^{210}\text{Pb}$	6.2×10^{22}	8.0×10^{27}	5.0×10^{25}
$^{238}\text{Pu} \rightarrow ^{32}\text{Si} + ^{206}\text{Hg}$	1.3×10^{25}	8.4×10^{27}	1.9×10^{25}

In order to simulate the microscopic corrections such as shell effects and pairing, the difference between the experimental Q-value and the theoretical one deduced from the present LDM has been added at the macroscopic potential energy of the initial spherical nucleus with a linear attenuation factor vanishing at the contact point of the nascent fragments. When the microscopic contributions are included (second column), our theoretical estimates agree very well with the data for all the C, O, Ne, Mg and Si clusters.

Therefore, it seems that the emission of clusters by heavy nuclei may be viewed as the limiting case of very asymmetric fission via compact and creviced shapes. It is important to emphasize that the main explanation of the good reproduction of the experimental data is the ability of the present model to reproduce the height and width of the potential barriers with the help of the experimental Q-value. The physical process which leads to two tangent spherical fragments may be an adiabatic fission process or, alternatively, the emission of a preformed cluster. Whatever the physical process is, the role of the proximity energy and the microscopic corrections are emphasized since their introduction allows to reproduce the potential barrier characteristics which govern the half-lives.

6 Fission process through quasi-molecular shapes

In the fusion-like fission valley surprising results emerge [4,6]. The potential barrier heights are similar to the fission barrier heights in the whole mass range whatever the asymmetry is. The Businaro-Gallone point is nicely reproduced [17-18]. The Coulomb energy where the proximity forces vanish corresponds roughly to the experimental kinetic energy of the fragments. The critical angular momentum that light nuclei are able to sustain is very close to the experimental data. If the shell effects are introduced as in Ref [19], then double-humped barriers appear for actinide nuclei. The first peak is due to the microscopic corrections but the outer peak is mainly governed by the balance between the Coulomb and proximity forces. It disappears for the heaviest actinides due to the Coulomb repulsion. The peak heights and the well depths follow the trends of the data.

So, it seems that there is no experimental indication allowing to exclude that fissioning nuclei take the path through compact quasi-molecular shapes. Then, the fission barriers would be scission barriers, the rupture of the neck between the fragments occurring before crossing the barrier.

7 Rotating quasi-molecular hyperdeformed states

Hyperdeformations in a high spin range of about $75 - 98 \hbar$ have been apparently observed in Dy at the Chalk River Laboratories [9]. The super and hyperdeformations are often studied within the spheroidal configurations even at very large deformations and the ratios between the semi-axes put forward. The geometrical characteristics of these prolate ellipsoidal shapes have been compared with the ones of the compact and creviced shapes [11]. Up to $\beta \approx 0.8 - 0.9$ the differences between the values of the moment of inertia and quadrupole moment in the two deformation valleys are very small. Therefore, the values deduced from experimental data lead to a poor understanding of the nucleus elongation and neck development.

In the deformation channel through quasi-molecular shapes it is possible to get low values of the fission barrier heights and high stability of rotating nuclei against fission [4, 10]. For Dy, a scission barrier exists up to spin of $115 \hbar$ and hyperdeformed states appear in the high spin range of $70 - 110 \hbar$. The barriers are also compatible with superdeformation data in Dy : spin from 22 to $60 \hbar$, excitation energy of 30 MeV and bands close to yrast at spins greater than $55 \hbar$. For the Ce, Eu, Dy, Hg nuclei, the moment of inertia and quadrupole moment of the second macroscopic well at intermediate spins correspond roughly to the experimental characteristics of the superdeformed states.

The agreement with the first data on rotating hyperdeformed states seems to confirm that the valley through compact quasi-molecular shapes might be, really, an important deformation channel. Furthermore, the initial configuration in heavy-ion reactions lies in this valley and it is perhaps possible that some memory of the entrance channel still plays a role.

8 Fragmentation and quasi-molecular shapes

In heavy-ion collisions at 10-100 MeV/nucleon, experimental signatures [20] indicate that strongly deformed shapes intermediate between non homogeneous toroids and bubbles develop before the fragmentation in several intermediate mass fragments. Quasi-molecular configurations have been selected to investigate the barriers of fragmentation with emission in a plane or in the whole space : equilateral triangle, square and tetrahedron, hexagon and spheres at equal distance along three axes and octogon and cubic arrangement. One parameter shape sequences have been defined for toroids and bubbles.

Calculations [12-14] show that for heavy systems formed in Pb+Au, Gd+U, U+U,...reactions, minima lying below the barriers of fragmentation in a plane exist in the toroidal deformation valley allowing some stability and relaxation before the decay of the excited system in several fragments due to the surface tension forces.

9 Conclusion

The deformation path leading a nucleus through compact and creviced shapes and to two spherical tangent nuclei or vice versa has been studied within a generalized liquid drop model taking into account both the proximity energy, an accurate sharp radius, the mass asymmetry and the temperature effects. The fusion barrier heights and positions are in very good agreement with the experimental data. A simple dynamic model allows to correctly reproduce the fusion cross sections and the additional energy needed to overcome the friction forces between the heaviest systems.

This decay path through quasi-molecular shapes seems compatible with the experimental fission data : symmetric and asymmetric fission barrier heights, half-lives of the radioactive nuclei emitting heavy clusters, double-humped barriers of actinides, fragment kinetic energies and critical momenta for light nuclei. The rotational hyperdeformed states recently observed might also come up and survive in this deformation valley.

The plane and three-dimensional fragmentation barriers as well the deformation energy of toroids and bubbles have been compared. For very heavy systems, minima lying below the barriers of fragmentation exist in the toroidal deformation valley allowing some stability and relaxation before the decay of the excited system in several fragments.

Analytical formulas given the fusion and fission barrier characteristics in these deformation path will be provided in a close future [18].

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