Creation of Strings in D-particle Quantum Mechanics *

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D-particle quantum mechanics in a type I' background is reviewed. It is also discussed how a string is created when a D-particle is taken through a D8-brane. The process is found to be dual to the creation of a D3-brane when a NS5 and D5 brane are passed through each other.

1. Introduction

The M\textit{atrix} model is a surprising approach to the study of strongly coupled type IIA string theory and M-theory [1]. Less explored is the corresponding approach for theories with half the number of supersymmetries. The heterotic string M\textit{atrix} model has been developed and studied in e.g. [2–8] but the number of non-trivial checks that have been performed are much less than its successful predecessor.

We will briefly review the quantum mechanics of the type I' D-particle as presented in [2], keeping in mind that this is also relevant for the heterotic string. We will furthermore discuss an interesting effect of string creation, [9–11] and its duality with a phenomenon discovered by Hanany and Witten [12].

The talk is based on the work [2] and on the work [10] together with I. Klebanov.

2. The Heterotic Life of the D-particle

The type IIA D-particle quantum mechanics is obtained by dimensional reduction of $N=1$, $D=10$ supersymmetric $SU(N)$ Yang-Mills theory [13–15]. (This system was also studied for different reasons in [16,17].) The Hamiltonian is given by

\begin{equation}
H = \frac{\lambda_{\text{IIA}}}{2}E_i^2 - \frac{1}{2\lambda_{\text{IIA}}} f^{abc} A_i^a \psi^b \gamma^i \psi^c + \frac{1}{4\lambda_{\text{IIA}}} \left( f^{abc} A_i^b A_j^c \right)^2,
\end{equation}

where $E_i$ is the conjugate momentum of $A_i$ and $\psi$ is a 16 component real spinor. All fields are in the adjoint representation of $SU(N)$ and the number of supersymmetries is 16. This system has been extensively studied and used as a starting point for the conjectured M\textit{atrix} description of M-theory.

Another quantum mechanical system of great interest is obtained if we consider type I' string theory, i.e. the T-dual of type I. From the type IIA point of view such a system contains two 8-orientifolds ($\Omega 8$), and 16 D8-branes. For simplicity we will consider a situation where the distance between the two $\Omega 8$ is large and 8 of the D8-branes are in the vicinity of each orientifold. Let us consider a system of $N$ D-particles near one of the orientifolds.

The quantum mechanics of the system is given by the following Hamiltonian:

\begin{align}
H &= \text{Tr} \left( \lambda_{\text{I}} \left( \frac{1}{2} P_i^2 - \frac{1}{2} E_i^2 \right) \right) \\
&+ \frac{1}{\lambda_{\text{I}}} \left( \frac{1}{2} [A_9, X_i]^2 - \frac{1}{4} [X_i, X_j]^2 \right) \\
&+ \frac{1}{2} \left( - S_a [A_9, S_a] - S_a [A_9, S_a] + 2X_i \sigma_i^{a\dot{a}} \{S_a, S_{\dot{a}}\} \right) \\
&+ \frac{1}{2} \left( - S_a [A_9, S_a] - S_a [A_9, S_a] + 2X_i \sigma_i^{a\dot{a}} \{S_a, S_{\dot{a}}\} \right)
\end{align}

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where the gauge group is $SO(2N)$. Here $A_9$ and its conjugate momentum $E_9$ are in the adjoint representation, while $X_i$ and its conjugate momenta $P_i$ are in the traceless symmetric representation. $A_9$ gives the distance between the D-particles and their mirror-images, while $X_i$ gives the relative distances parallel to the orientifold. Turning to the fermions, $S_a$ is in the adjoint, $S_a$ is in the traceless symmetric and, finally, $\chi_i$ is in the fundamental representation.

Let us briefly recall why this is the case. The orientifold projection $\Omega R$ does two things: through $R$ it performs a reflection in space and through $\Omega$ it reverses the orientation of a string. The figure below shows what happens.

![Fig. 1. The effect of $\Omega R$.](image)

In case of the field $A_9$ (where '9' refers to the transverse direction) it also gives an additional minus sign. This will kill the contribution from strings connecting a D-particle with its own mirror-image, since these strings are mapped to minus themselves. A simple example is $N = 2$, i.e. two D-particles with their two mirror-images. Excluding the strings connecting a D-particle with its own mirror-image we find four different strings not counting orientation. The projection also instructs us to take only one particular linear combination with respect to the orientations. Adding the two strings corresponding to the two independent positions, i.e. Cartan elements coming from strings starting and ending on the same D-particle, we therefore find $2 + 4 = 6$ strings which is the dimension of the adjoint of $SO(4)$.

$X_i$ does not have the extra minus sign and the strings joining a D-particle with its own mirror-image are therefore kept. In the example that we are discussing, this results in four extra strings (counting orientation), in total 10. This is the dimension of the symmetric representation of $SO(4)$. Taking out the trace, which just corresponds to a translation along the orientifold, we find the dimension to be 9.

The fermions $S_a$ and $S_a$ have an additional $\Gamma_9$ in the orientifold projection. Since

$$\Gamma_9 S_a = -S_a \quad \Gamma_9 S_a = S_a$$  (3)

we find that $S_a$ must be in the adjoint and $S_a$ in the traceless symmetric.

Finally, we note that the fermions $\chi$ correspond to strings going between the D-particles and the D8-branes. These, therefore, must be in the fundamental representation.

3. ... and the Born-Oppenheimer Approximation

A clear space-time interpretation is obtained when we use the Born-Oppenheimer approximation, where we integrate out the fast modes and only the slow ones (corresponding to Cartan elements of the algebra) remain. In order to do this we turn on $A_9$ to separate the D-particles from the orientifold and $X_i$ to separate them from each other. The various fields decompose into slow and fast as follows:

$$\dim(\text{adj } SO(2N)) + 8 \dim(\text{sym } SO(2N)) = N(2N - 1) + 8(N(2N + 1) - 1)$$

$$= 2N^2 + 7N - 8 + 16N^2$$  (4)

Modding out the gauge equivalent slow modes, whose number is

$$\dim(SO(2N)) - \text{rank}(SO(2N)) = 2N^2 - 2N$$  (5)

one sees that the dimension of the moduli space is $N + 8(N - 1)$. $N$ is the number of neutral elements in the adjoint while $N - 1$ is the number of neutral elements in the traceless symmetric. Let us now consider the special case $N = 1$. We
find one slow mode, i.e. the distance between the single D-particle and the orientifold, and 16 fast ones. There is also 8 heavy fermions, $S_a$, in the symmetric representation giving 16 real fermions.

It is important to note that this is half the number of fermions as compared to type IIA quantum mechanics.

Let us now quantize the theory using the Born-Oppenheimer approximation. If we consider the contribution from the orientifold $\Omega^8$, we find the following effective potential for the slow mode:

$$V_{\Omega^8}(r) = 2r \sum_{i=1}^{16} (N_{i}^R + 1/2) + 2r \sum_{i=1}^{8} (N_{i}^F - 1/2).$$

(6)

Here we have used the 16 real fermions to construct 8 raising and 8 lowering operators. We see that the ground state energy do not seem to cancel! However, we also need to take the D8-branes into account. We have $8 + 8 = 16$ fermions $\chi_i$ in the fundamental representation. 8 coming from strings connecting the D-particle and the D8-branes, and the same number from strings connecting the D-particle with the mirror-images of the D8-branes. In total we have 32 real fermions. Since we above found that we had half the required number of fermions, it now seems as if we are overshooting. However, the coupling between the two types of fermions and the field $A_9$ is different due to the different representations. This supplies the needed factor one half. This can also be understood from the fact that in one case we are dealing with the distance to the D-particle mirror-image, in the other case with the distance to a D8-brane at the orientifold. The contribution to the potential coming from the D8-branes is thus:

$$V_{D8}(r) = r \sum_{i=1}^{16} (N_{i}^F - 1/2),$$

(7)

and the total potential is hence given by:

$$V(r) = V_{\Omega^8}(r) + V_{D8}(r).$$

(8)

We now see that the ground state energy cancels, implying that the D-particle at rest does not feel any force. Furthermore we observe that a D8-brane seems to be repulsive, while a 8-orientifold seems to be attractive.

4. Taking a D-particle through a D8-brane

We can now generalize the above potential slightly by allowing the D8-branes to be positioned off the 8-orientifold. We then find that the potential (7) generalizes to

$$V_{D8}(r) = \sum_{i=1}^{8} \left( |r - m_i|(N_{i}^{FR} - 1/2) + |r + m_i|(N_{i}^{FL} - 1/2) \right).$$

(9)

We will now ask the following question: what happens if a D-particle is taken through one of the D8-branes? Looking at the potential we seem to conclude that there now is a net force on the D-particle, trying to push it towards the orientifold. This is due to the change of sign of the force from the D8-brane that the D-particle crossed. But if a string is created, i.e. $N_{i}^{FR}$ jumps to $N_{i}^{FR} = 1$, the force is canceled! Now, does this really happen?

5. A U-dual Situation

We will now show that a string is indeed created by using U-duality. Hanany and Witten have showed that a D3-brane is created when a D5-brane, spanning directions 1-2-6-7-8, is passed through a NS5-brane spanning directions 1-2-3-4-5. The resulting D3-brane will span the directions 1-2 along which the two five-branes intersect and the transverse direction 9.

This result can now be used in the situation that we are interested in. First we T-dualize along directions 1 and 2. The NS5-brane remains an NS5-brane, while the D5-brane becomes a D3-brane along 6-7-8 and the D3-brane becomes a D-string along 9. Then we use S-duality of type IIB string theory which maps the NS5-brane to a D5-brane. Similarly the the D-string becomes a fundamental string. The D3-brane remains a D3-brane. The final step involves T-dualities along the directions 6, 7 and 8. The D5-brane now becomes a D8-brane and the D3-brane becomes a D-particle. The fundamental string remains a fundamental string. This is precisely the setup that we are after. We conclude that a fundamental string must be created when a D-particle passes through a D8-brane.
Instead of using S-duality we might view everything from a M-theory perspective, as in the diagram below. We see how two different compactifications (on direction 10 and direction 1 respectively) followed up with T-dualities give on the one hand the Hanany-Witten result, and on the other hand the result that we are after.

![Diagram](image)

Fig. 2. Creation through duality.

6. Putting it All Together

The previous results were all written in a gauge where \( A_0 = 0 \). In general it is found that not only an induced potential, but also an induced Chern-Simons term is generated. For instance, a D8-brane gives rise to, \[ -\frac{1}{2} \text{sign}(A_9 - m)(A_9 - m + A_0), \] generalizing the zero-point energy part of (9) if \( A_9 = r \). But this is not all, in addition to the induced terms we might also have bare terms. We will now discuss how to interpret these bare and induced terms from a stringy point of view.

We begin by discussing the force between two parallel \( p \) and \( q \) branes at rest, [18]. If \( p - q = 0 \) the contributions are schematically

\[ NS + R + NS(-1)^F = 0 \]  

from the open string point of view. From the closed string point of view the first two terms correspond to an NSNS-exchange of gravitons and dilatons giving an attraction, the last term is due to RR-exchange and gives a repulsion in such a way that the total force cancels.

When \( p - q = 4 \) we have

\[ NS + R = 0. \]  

The first two terms, due to graviton and dilaton exchange, gives a repulsion that is canceled by the third term.

The question now arises how to interpret the third term from the closed string perspective. For an isolated D-particle–D8-brane system in type IIA string theory there is an argument that interprets the \( R(-1)^F \) term as due to the creation of “half” a string between the branes. It would be interesting to put this on more rigorous grounds; however, for the case of real interest to us, that is the type I’ system, the situation is much better established: when the D-particle is in the “symmetric” position (with 8 D8-branes and one 8-orientifold on each side) all the forces cancel out and as it moves across a D8-brane a string is created to cancel the unbalanced force. Thus, due to string creation, the D-particle feels no force everywhere. We shall review these arguments below.

In the type IIA scenario, one can heuristically argue in the following way; the D8-brane is the source of a 10-form field strength with dual \( F_{10} = * F_2 \) with the following coupling to the D-particle:

\[ \mu_0 \int d\tau F A_0. \]  

This implies that it will appear like an electric charge on the D-particle world line. Since this is also the way that the endpoint of a fundamental string would appear, we conclude that a fundamental string must end on the D-particle. This is what gives rise to the extra \( R(-1)^F \) force. One should note, however, that the term corresponds to a tension equivalent to “half” a string.
As discussed in e.g. [20], $F$ is piece-wise constant so that when the D-particle goes through the D8-brane $F$ jumps by $\mu_8$, and therefore $\mu_0 F$ jumps by $\mu_0 \mu_8 = \frac{1}{2\alpha'}$. We are working in units where $\frac{1}{2\alpha'} = 1$. This can be interpreted as the creation of a full string that cancels the original half a string with orientation, say, towards the D-particle and gives half a string with orientation away from the D-particle.

The type I' setting luckily allows for an interpretation without half-strings. In that case, by T-dualizing a type I configuration without Wilson lines, one obtains the symmetric configuration described above with $F = 0$ in the region where the D-particle has 8 D8-branes on each side. The jump in $F$ felt by the D-particle as it moves through some of the D8-branes corresponds to the creation of a string that we have discussed. We believe that the creation of the string, with consequent cancellation of the net force is also necessary if one has to make sense of the construction of [3] where one obtains the extra components of the $E_8$ vector multiplets as bound states at threshold; a non zero potential would in fact lower the energy of such states below the continuum.

To summarize, we conclude that the NSNS repulsion corresponds to the induced term in the D-particle quantum mechanics. Recall that we found a D8-brane to be repulsive. The term $R(-1)^F F$ that we have identified as coming from $F$ must therefore correspond to the bare term. Looking at the potential we see that the induced terms correspond to the zero-point energy while the bare terms should be associated with excited states, i.e. real strings.

The string theoretical annulus calculation seems to give a result that do not include the string-creation effect. Without this effect, the potential is asymmetric with a force on one side and no force on the other, [11]. It should be noted that in the type I case, the contribution from $R(-1)^F$ actually cancels everywhere (excluding string creation effects). The reason why the force seems to jump when a D8-brane is passed is hence simply related to the change of direction of the well understood graviton/dilaton force. The string creation is an effect that have to be added and, indeed, cancels the jump.

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**REFERENCES**

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3In [21] these effects are explained from a M(atrix)-model point of view using 5-branes and membranes. Through T-duality the authors conclude that in the case of one D8-brane, the potential of [11] is reproduced. However, the authors also show how the adiabatic passage of a D-particle through the D8-brane creates a string. This cancels the force.
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