Anharmonic phonon excitations in subbarrier fusion reactions

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Abstract. Recently measured high precision data of fusion excitation function have enabled a detailed study on the effects of nuclear collective excitations on fusion reactions. Using such highly accurate data of the $^{16}\text{O} + ^{144,148}\text{Sm}$ reactions, we discuss the anharmonic properties of collective phonon excitations in $^{144,148}\text{Sm}$ nuclei. It is shown that subbarrier fusion reactions are strongly affected by the anharmonic effects and thus offer an alternative method to extract the static quadrupole moments of phonon states in a spherical nucleus.

INTRODUCTION

It has been well recognized that cross sections of heavy-ion fusion reactions at energies near and below the Coulomb barrier are strongly influenced by couplings of the relative motion of the colliding nuclei to several nuclear intrinsic motions [1]. In the eigen-channel approach, such couplings give rise to a distribution of potential barriers [2,3]. Based on this idea, a method was proposed to extract barrier distributions directly from fusion excitation functions using the second derivative of the product of the fusion cross section and the center of mass energy $E\sigma$ as a function of energy $E$ [4]. Based on coupled-channels calculations, it was shown that the fusion barrier distribution, i.e. $d^2(E\sigma)/dE^2$, is very sensitive to the details of the couplings. In order to deduce meaningful barrier distributions, excitation functions of fusion cross sections have to be measured with high precision at small energy intervals. Thanks to the recent developments in experimental techniques [5], such data are now available for several systems, and they have clearly demonstrated that the barrier distribution is indeed a sensitive quantity to channels couplings [6].

In this contribution, we analyse the recently measured accurate data on the $^{16}\text{O} + ^{144}\text{Sm}$ fusion reaction to discuss effects of nuclear surface vibrations on
heavy-ion fusion reactions [7]. The barrier distribution analysis of the recent high precision data on the $^{58}\text{Ni} + ^{60}\text{Ni}$ fusion reaction has shown clear evidence for coupling of multi-phonon states in $^{58}\text{Ni}$ and $^{60}\text{Ni}$ [8], while no evidence for double phonon couplings is seen in the $^{16}\text{O} + ^{144}\text{Sm}$ reaction [6,9]. We show that anharmonicities in nuclear vibrations play an important role in the latter reaction. We estimate the magnitude as well as the sign of the quadrupole moments of the quadrupole and octupole single-phonon states of $^{144}\text{Sm}$ from the experimental fusion barrier distribution. A similar analysis is performed also for the $^{16}\text{O} + ^{148}\text{Sm}$ reaction.

ANHARMONICITIES IN NUCLEAR VIBRATIONS

Collective phonon excitations are common phenomena in fermionic many-body systems. In nuclei, low-lying surface oscillations with various multipolarities are typical examples. The harmonic vibrator provides a zeroth order description for these surface oscillations, dictating simple relations among the level energies and the electromagnetic transitions between them. For example, all the levels in a phonon multiplet are degenerate and the energy spacing between neighboring multiplets is a constant. In realistic nuclei, however, there are residual interactions which cause deviations from the harmonic limit, e.g., they split levels within a multiplet, change the energy spacings, and also modify the ratios between various electromagnetic transition strengths. There are many examples of two-phonon triplets ($n02^+; n02^+; n04^+$) of quadrupole surface vibrations in even-even nuclei near closed shells. Though the center of mass of their excitation energies are approximately twice the energy of the first $2^+$ state, they usually exhibit appreciable splitting within the multiplet. A theoretical analysis of the anharmonicities for the quadrupole vibrations was first performed by Brink et al. [10], where they related the excitation energies of three-phonon states to those of double-phonon triplets. For a long time, however, the sparse experimental data on three-phonon states had caused debates on the existence of multi-phonon states. The experimental situation has improved rapidly in recent years, and data on multi-phonon states are now available for several nuclei. As a consequence, study of multi-phonon states, and especially their anharmonic properties, is attracting much interest [11]. It is worthwhile to mention that anharmonic effects are not restricted to low-lying vibrations but have also been observed in multi-phonon excitations of giant resonances in heavy-ion collisions [12].

In many even-even nuclei near closed shells, a low-lying $3^-$ excitation is observed at a relatively low excitation energy, which competes with the quadrupole mode of excitation [13]. These excitations have been frequently interpreted as collective octupole vibrations arising from a coherent sum of one-particle one-hole excitations between single particle orbitals differing by three units of orbital angular momentum. This picture is supported by large
E3 transition probabilities from the first $3^{-}$ state to the ground state, and suggests the possibility of multi-octupole-phonon excitations. In contrast to the quadrupole vibrations, however, so far there is little experimental evidence for double-octupole-phonon states. One reason for this is that E3 transitions from two-phonon states to a single-phonon state compete against E1 transitions. This makes it difficult to unambiguously identify the two-phonon quartet states ($0^+, 2^+, 4^+, 6^+$). Only in recent years, convincing evidences have been reported for double-octupole-phonon states in some nuclei, including $^{208}$Pb [14] and $^{144}$Sm [15].

**EFFECTS OF PHONON EXCITATIONS ON FUSION**

Let us now discuss the effects of nuclear surface vibrations on heavy-ion fusion reactions. In this section we use the linear coupling approximation to describe the coupling between the relative motion of the colliding nuclei and the surface vibrations. This simple model enables us to understand easily the effects of anharmonicity. Extension of the model so as to include the couplings to all orders and comparisons with the experimental data is given in the next section.

**Harmonic limit**

The effects of nuclear surface vibrations on heavy-ion fusion reactions at energies below and near the Coulomb barrier has been investigated by many groups (see Ref. [1] for a recent review). These studies were later extended to include the effect of multi-phonon states within the harmonic oscillator approximation [16,17]. Using the no-Coriolis approximation [16] and the linear coupling approximation, the coupling Hamiltonian, which describes the coupling between the relative motion and the quadrupole surface oscillations, is assumed to be

$$V_{\text{coupl}}(r, \xi) = \frac{\beta}{\sqrt{4\pi}} f(r)(a_{20}^\dagger + a_{20}),$$

where $a_{20}^\dagger$ and $a_{20}$ are the creation and the annihilation operators for the quadrupole phonon, respectively, and $\beta$ is the quadrupole deformation parameter. The coupling form factor $f(r)$ consists of the nuclear and Coulomb parts and reads

$$f(r) = -R_T \frac{dV_N}{dr} + \frac{3}{5} Z_p Z_T e^2 \frac{R_T^2}{r^3}.$$  \hspace{1cm} (2)

Here $R_T$ is the radius of the target nucleus and $V_N$ is the nuclear potential.
For the quadrupole surface vibrations, the two phonon state has three levels \((0^+, 2^+, 4^+)\). In the harmonic limit, this two-phonon triplet is degenerate in the excitation energy. One can then introduce the two-phonon channel by taking particular linear combinations of the wave functions of the two-phonon triplet \([16,17]\). The wave function of the two-phonon channel then reads

\[
|2> = \sum_{I=0,2,4} <2|0|I0> |I0> = \frac{1}{\sqrt{2!}} (a_{20}^+)^2 |0> .
\]

In the same way, one can introduce the \(n\)-phonon channel as

\[
|n> = \frac{1}{\sqrt{n!}} (a_{20}^+)^n |0> .
\]

The dimension of the coupled-channels equations is reduced drastically with the introduction of the \(n\)-phonon channels. If we truncate to the two phonon states, the corresponding coupling matrix is given by

\[
V_{\text{coup}} = \begin{pmatrix}
0 & F(r) & 0 \\
F(r) & \hbar \omega & \sqrt{2} F(r) \\
0 & \sqrt{2} F(r) & 2 \hbar \omega 
\end{pmatrix} .
\]

Here, \(F(r)\) is defined as \(\frac{\beta}{\sqrt{4\pi N}} f(r)\).

**Anharmonic vibrator**

The \(sd\)-interacting boson model (IBM) in the vibrational limit provides a convenient calculational framework to discuss the effects of anharmonicity in the surface vibrations [18]. The vibrational limit of the IBM and the anharmonic vibrator (AHV) in the geometrical model are very similar, the only difference coming from the finite number of bosons in the former [19]. A model for subbarrier fusion reactions, which uses the IBM to describe effects of channel couplings, has been developed in Ref. [20]. Following Ref. [20], we assume that the coupling Hamiltonian is given as

\[
V_{\text{coup}}(r, \xi) = \frac{\beta}{\sqrt{4\pi N}} f(r) Q_{20} .
\]

Here, \(N\) is the boson number and we have introduced the scaling of the coupling strength with \(\sqrt{N}\) to ensure the equivalence of the IBM and the geometrical model results in the large \(N\) limit [20]. \(Q_{20}\) is the quadrupole operator in the IBM, which we take as

\[
Q_{20} = s^\dagger d_0 + s d_0^\dagger + \chi_2 (d_0^\dagger d_0)^{(2)} ,
\]
where tilde is defined as \( \tilde{b}_\mu = (-)^{l+\mu}b_{l-\mu} \).

As in the harmonic limit, one can introduce the multi-phonon channel if one assumes that the multi-phonon multiplets are degenerate in the excitation energy. The wave function of the \( n \)-phonon channel in the framework of the IBM then reads

\[
|n> = \frac{1}{\sqrt{n!(N-n)!}} (s^+)^{N-n} (d_0^l)^n |0>.
\]

The corresponding coupling matrix, truncated to the two-phonon states, is given by

\[
V_{coup} = \begin{pmatrix}
0 & F(r) & 0 \\
F(r) & \tilde{\hbar}\omega - \frac{2}{\sqrt{4N}} \chi F(r) & \sqrt{2(1-1/N)}F(r) \\
0 & \sqrt{2(1-1/N)}F(r) & 2\tilde{\hbar}\omega + \delta - \frac{4}{\sqrt{4N}} \chi F(r)
\end{pmatrix}.
\]

The parameter \( \delta \) is introduced to represent deviation of the energy spectrum from the harmonic limit. When the \( \chi \) parameter in the quadrupole operator is zero, quadrupole moments of all states vanish, and one obtains the harmonic limit in the large \( N \) limit. Non-zero values of \( \chi \) generate quadrupole moments and, together with finite boson number, they are responsible for the anharmonicities in electric transitions.

It has been shown that anharmonicities in level energies have only a marginal effect on the fusion excitation function and the barrier distribution [7]. In fact, our studies show that the fusion barrier distribution does not depend so much on the excitation energies of the multi-phonon states once the energy of the single-phonon state is fixed. We therefore set \( \delta \) to be zero in the following discussion. As we will see later, the main effects of anharmonicity on fusion barrier distributions come from the deviation of the transition probabilities from the harmonic limit as well as the reorientation effects.

**COMPARISON WITH EXPERIMENTAL DATA**

**The \( {^{16}}O + {^{144}}Sm \) reaction**

We now discuss the effects of anharmonicities on the \( {^{16}}O + {^{144}}Sm \) fusion reaction, whose excitation function has recently been measured with high accuracy [6]. It has been reported that inclusion of the double-phonon excitations of \( {^{144}}Sm \) in coupled-channels calculations in the harmonic limit destroys the good agreement between the experimental fusion barrier distribution and the theoretical predictions obtained when only the single-phonon excitations are taken into account [21]. On the other hand, there are experimental [15,22] as well as theoretical [23] support for the existence of the double-octupole-phonon
states in $^{144}$Sm. Reconciliation of these apparently contradictory facts may be possible if one includes the anharmonic effects, which are inherent in most multi-phonon spectra.

In order to address these questions, it is necessary to extend the models which were discussed in the previous section so that they include the octupole mode as well as the couplings to all orders. The full order treatment is crucial in order to quantitatively, as well as qualitatively, describe heavy-ion sub-barrier fusion reactions [20,24]. We therefore assume the following coupling Hamiltonian based on the $sd_{f}$-IBM.

$$V_{\text{coupl}}(r, \xi) = V_{C}(r, \xi) + V_{N}(r, \xi),$$

$$V_{C}(r, \xi) = \frac{Z_{p}Z_{Te}^{2}}{r} \left(1 + \frac{3}{5} \frac{R_{T}^{2}}{r^{2}} \sqrt{4\pi N} + \frac{3}{7} \frac{R_{T}^{3} \beta_{3} \dot{Q}_{30}}{r^{3} \sqrt{4\pi N}}\right),$$

$$V_{N}(r, \xi) = -V_{0} \left[1 + \exp \left(\frac{1}{a} \left(r - R_{0} - R_{T}(\beta_{2} \dot{Q}_{20} + \beta_{3} \dot{Q}_{30})/\sqrt{4\pi N}\right)\right)\right]^{-1}. \quad (11)$$

The quadrupole and the octupole operators are defined as

$$\dot{Q}_{2} = s^{\dagger}d + sd^{\dagger} + \chi_{2}(d^{\dagger}d)^{(2)} + \chi_{2j}(f^{\dagger}f)^{(2)};$$

$$\dot{Q}_{3} = sf^{\dagger} + \chi_{3}(d^{\dagger}f)^{(3)} + h.c., \quad (12)$$

respectively.

The results of the coupled-channels calculations are compared with the experimental data in Fig. 1. The upper and the lower panels in Fig. 1 show the excitation function of the fusion cross section and the fusion barrier distributions, respectively. The experimental data are taken from Ref. [6]. The dotted line is the result in the harmonic limit, where couplings to the quadrupole and octupole vibrations in $^{144}$Sm are truncated at the single-phonon levels and all the $\chi$ parameters in Eq. (12) are set to zero. The deformation parameters in Eq. (11) are estimated to be $\beta_{2}=0.11$ and $\beta_{3}=0.21$ from the electric transition probabilities. The dotted line reproduces the experimental data of both the fusion cross section and the fusion barrier distribution reasonably well, though the peak position of the fusion barrier distribution around $E_{\text{cm}}=65$ MeV is slightly shifted. As was shown in Ref. [21], the shape of the fusion barrier distribution becomes inconsistent with the experimental data when the double-phonon channels are included in the harmonic limit (the dashed line).

To see whether this discrepancy is due to neglecting of anharmonic effects, we have repeated the calculations including the $\chi$ parameters in the fits and using $N=2$ in the IBM. The $\chi^{2}$ fit to the fusion cross sections resulted in the set of parameters, $\chi_{2} = -3.30 \pm 2.30$, $\chi_{2j} = -2.48 \pm 0.07$, and $\chi_{3} = 2.87 \pm 0.16$, regardless of the starting values. The resulting fusion cross sections and barrier distributions are shown in Fig. 1 by the solid line. They agree with the experimental data much better than those obtained in the harmonic limit. Thus, inclusion of the anharmonic effects in vibrational motion appear to be
essential for a proper description of barrier distributions in the reaction $^{16}\text{O} + ^{144}\text{Sm}$.

One of the pronounced features of an anharmonic vibrator is that the excited states have non-zero quadrupole moments [13]. Using the $\chi$ parameters extracted from the analysis of fusion data in the $E2$ operator, $T(E2) = e_B Q_2$, we can estimate the static quadrupole moments of various states in $^{144}\text{Sm}$. Here, $e_B$ is the effective charge, which is determined from the experimental $B(E2; 0 \to 2^+)$ value as $e_B = 0.16$ $eb$. For the quadrupole moment of the first $2^+$ and $3^-$ states, we obtain $-0.89 \pm 0.63$ $b$ and $-0.70 \pm 0.02$ $b$, respectively. Fig. 2 shows the influence of the sign of the quadrupole moment of the excited states on the fusion cross section and the fusion barrier distribution. The solid line is the same as in Fig. 1 and corresponds to the optimal choice for the signs of the quadrupole moments of the first $2^+$ and $3^-$ states. The dotted and dashed lines are obtained by changing the sign of the $\chi_2$ and $\chi_{2f}$ parameters in Eq. (12), respectively, while the dot-dashed line is the result where the sign of both $\chi_2$ and $\chi_{2f}$ parameters are inverted. The change of sign of $\chi_2$ and $\chi_{2f}$ is equivalent to taking the opposite sign for the quadrupole moment of the excited states. Fig. 2 demonstrates that subbarrier fusion reactions are sensitive to the sign of the quadrupole moment of excited states. The experimental data are reproduced only when the correct sign of the quadrupole moment are used in the coupled-channels calculations.

The $^{16}\text{O} + ^{148}\text{Sm}$ reaction

A similar analysis was performed also for the $^{16}\text{O} + ^{148}\text{Sm}$ reaction [25]. The best fit to the experimental fusion cross section [6] was obtained with the quadrupole moments of $-1.00 \pm 0.25b$ and $+1.52 \pm 0.14b$ for the first $2^+$ and $3^-$ states, respectively. The total boson number $N$ was assumed to be 4 and the deformation parameters were estimated from the electric transition probabilities. Note that the value of the quadrupole moment of the first $2^+$ state which we obtained from the fusion analysis is very close to that measured from the Coulomb excitation technique, i.e. $-0.97 \pm 0.27b$ [26]. Fig. 3 shows the sensitivity of the fusion cross section and the fusion barrier distribution to the sign of the quadrupole moment of the first $3^-$ state. The experimental data are taken from Ref. [6]. The solid line corresponds to the optimal choice for the sign of the first $3^-$ state, while the dotted line was obtained by inverting it. We again observe that the use of the incorrect sign of the quadrupole moment destroys the good fit to the experimental data. This strongly suggests that subbarrier fusion can provide an alternative method to determine the sign as well as the magnitude of the quadrupole moments in spherical nuclei.
SUMMARY

We discussed the effects of multi-phonon excitations on subbarrier fusion reactions. We especially focused on the anharmonic properties of the phonon excitations. The experimental fusion excitation functions for the $^{16}$O + $^{144,148}$Sm reactions were analyzed with a model which explicitly takes into account the effects of anharmonicity of the vibrational modes of excitation in $^{144,148}$Sm. We found that the best fit to the experimental data requires negative quadrupole moments for the first $2^+$ and the first $3^-$ states of $^{144}$Sm. For the $^{148}$Sm nucleus, we obtained a negative quadrupole moment for the first $2^+$ state and a positive one for the first $3^-$ state. As a general conclusion, we find that heavy-ion subbarrier fusion reactions, and in particular, barrier distributions extracted from the fusion data, are very sensitive to the sign of the quadrupole moments of phonon states in the target nucleus and thus offer an alternative method to determine them.

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