Visualization of chiral condensate at finite temperature on the lattice *

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We perform an analysis of the topological and chiral vacuum structure of four-dimensional QCD on the lattice at finite temperature. From correlation functions we show the existence of local correlations between the topological charge density and the quark condensate on gauge average. We comment on sizes of clusters of nontrivial chiral condensate and of instantons in full QCD. By analysis of individual gauge configurations, we demonstrate that at the places in Euclidian space-time, where instantons are present, amplified production of quark condensate occurs.

It is believed that the instantons as carriers of the topological charge might play a crucial role in understanding the confinement mechanism of four-dimensional QCD, if one assumes that they form a so-called instanton liquid [1]. Instantons have topological charge $Q$ being related to the zero eigenvalues of the fermionic matrix of a gauge field configuration via the Atiyah-Singer index theorem [2]. Recently, it was demonstrated that monopole currents which constitute a different topological excitation of compact SU(3) gauge theory, appear preferably in the regions of non-vanishing topological charge density [3,4]. It has been conjectured that both instantons and monopoles are related to chiral symmetry breaking [1, 5, 6]. In this contribution we further support this idea by the following results of a direct investigation of the local topological correlation lengths exponential to the tails of the shorter correlation lengths. The corresponding screening masses are $\zeta = 0.59$ and $\zeta = 1.56$ in

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Figure 1. Correlation function of the quark-antiquark density and the topological charge density for 0 and 11 cooling steps. The correlations extend over two lattice spacings and indicate local coexistence of the quark condensate and topological objects.

In a series of papers we found that at the local regions of clusters of topological charge density, which are identified with instantons, there are monopole trajectories looping around in almost all cases for both pure SU(2) and SU(3) gauge theories [3]. In Fig. 2 three typical topologically nontrivial configurations from SU(3) theory with dynamical quarks on the $8^3 \times 4$ lattice in the confinement phase are shown for fixed time slices. We display the topological charge density (hypercube definition) by dark dots if the absolute value $|q(x)| > 0.003$. The quark-antiquark density is indicated by light dots whenever a threshold for $\bar{\psi}\psi(x) > 0.006$ is exceeded. Monopole currents are defined in the maximum Abelian projection and only one type is shown by lines. By analyzing dozens of quark and quark field configurations we found the following results. The topological charge is hidden in quantum fluctuations and becomes visible by cooling of the gauge fields. For 0 cooling steps no structure can be seen in $q(x)$, $\bar{\psi}\psi(x)$ or the monopole currents, which does not mean the absence of correlations between them. After about 5 cooling steps clusters of nonzero topological charge density and quark condensate are resolved. These particular configurations contain a single instanton, a pair of antiinstantons and an instanton-antiinstanton pair. For more than 10 cooling steps both topological charge and chiral condensate begin to die out and eventually vanish. Combining the above finding of Fig. 1 showing that the correlation functions between $\bar{\psi}\psi(x)$ and $q^2(y)$ are not very sensitive to cooling together with the cooling history of the 3D images in Fig. 2, we conclude that instantons go hand in hand with clusters of $\bar{\psi}\psi(x) \neq 0$ also in the uncooled QCD vacuum [8].

Figure 2 has demonstrated that the quark-antiquark density attains its maximum values at the same positions where the extreme values of the topological charge density are situated. This behavior is further substantiated in Fig. 3 where the $\bar{\psi}\psi(x)$-values are plotted against $q(x)$ for all points $x$. The nine selected configurations at 10 cooling steps have different topological content. At first sight a linear relationship between the absolute value of the topological charge density and the virtual quark density is suggested.

We comment on the behavior of the topological structure and the quark condensate when crossing the phase transition. The normalized correlation functions do not change qualitatively [3]. The strength of the correlations becomes, however, 2-3 orders of magnitude smaller. This means that the topological activity becomes much weaker in
Figure 2. Cooling history for time slices of three gauge field configurations of SU(3) theory with dynamical quarks. The columns represent a single instanton, a pair of anti-instantons and an instanton-anti-instanton pair as the configuration gets cooled. The dark dots represent the positive and negative topological charge density; the light dots the density of the quark condensate. It turns out that the quark condensate takes a non-vanishing value at the positions of instantons.
Figure 3. Scatter plots of the local chiral condensate $\bar{\psi}\psi(x)$ and the topological charge density $q(x)$ for nine configurations after 10 cooling steps. A linear relationship is suggested.
the deconfinement phase as expected. Most of the configurations have trivial net topological charge. This does not exclude the existence of instanton-antiinstanton pairs which become more difficult to be resolved at the smaller physical volume at $\beta = 5.4$ which we also considered. In less than one percent of the gauge field configurations instantons could be identified. The configurations we scanned did not show pronounced pictures of instantons and quark condensate. The quark-antiquark density is considerably lower and does not have the tendency to cluster anymore, $\bar{\psi}\psi(x)$ becomes uniformly distributed over the lattice sites. The maximum values of $\bar{\psi}\psi(x)$ in configurations in the deconfinement after 10 cooling steps are one order of magnitude smaller.

In summary, our calculations of correlation functions between topological charge and the quark condensate yield an extension of about two lattice spacings. The correlations suggest that the chiral condensate takes a non-vanishing value predominantly in the regions of instantons and monopole loops. It was well known before that the chiral condensate is related to the topological charge and topological susceptibility. The visualization exhibited that the distribution of the “chiral condensate” concentrates around areas with enhanced topological activity (instantons, monopoles). We demonstrated that exactly at those places in Euclidian space-time, where tunneling between the vacua occurs, amplified production of quark condensate takes place. It must be emphasized that this represents the situation on a finite lattice with finite quark mass without the extrapolation to the thermodynamic and chiral limit. We found for full SU(3) QCD with dynamical quarks that the clusters of non-vanishing quark condensate have a size of about 0.4 fm, which corresponds to the instanton sizes observed in the same configurations. Visualization of quark and gluon fields might be especially useful to decide if the instanton-liquid model is realized in nature and if instanton-antiinstanton pairs appear in the deconfined phase. It might also help to clarify the question of the existence of a disoriented chiral condensate with consequences for heavy-ion experiments.

REFERENCES


DISCUSSIONS

Heinz J. Rothe, Univ. Heidelberg

1. Your results suggest that the dynamical mechanism for the deconfinement phase transition and chiral symmetry restoration is the same. Is this also your opinion?
2. How does the cooling procedure affect the number of monopoles you see?

Harald Markum

1. It was shown by lattice simulations that the temperature and chiral phase transition coincide. My opinion is that there is a common driving force, maybe the gluon field itself.
2. Cooling smooths quantum fluctuations and uncovers topological charges. One of the caveats is that monopoles are gradually lost. This is a reason why one tries to develop alternative methods.