Abstract

We employ the influence functional technique to trace out the pho-
tonic contribution from full quantum electrodynamics. The reduced
density matrix propagator then is constructed for the electron field.
We discuss the role of time-dependent renormalization in the prop-
agator and focus our attention to the possibility of obtaining a dy-
amically induced superselection rule. As a final application of our
formalism we derive the master equation for the case of the field be-
ing in an one-particle state in the non-relativistic regime and discuss
whether electromagnetic vacuum fluctuations are not sufficient to pro-
duce decoherence in a position basis.

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1 Introduction

Quantum field theory provides a broad framework within which problems in diverse branches of physics can be formulated and addressed. Its success is primarily based on its inherent flexibility: its roots lie within both Hamiltonian and statistical mechanics and this double aspect can be used invariably when dealing with particular physical phenomena.

Its large domain of applicability in conjunction with the increasing interest in non-equilibrium phenomena has led in recent years into the adoption of successful techniques from standard non-equilibrium statistical mechanics or the quantum theory of open systems and their upgrading to fit the field theoretic contexts. Hence the Feynman-Vernon influence functional technique, the Schwinger-Keldysh closed time-path formalism (for extensive bibliography see reference [1]) and Zwanzig’s projection method [2, 3] have been used to deal with diverse issues including early universe cosmology, black hole statistical mechanics condensed matter physics and quantum optics.

The earliest specimen among field theories is of course quantum electrodynamics (QED). Being a theory of photons and electrons, it can be used to describe most of the phenomena in “ordinary” low energy matter. Hence, within its range of applications many issues connected with non-equilibrium phenomena arise. In this context one of the older (dating back to Lorentz for the case of classical electromagnetism) interesting questions is what the effective dynamics of charged particles when its interaction with its own electromagnetic field is taken into account. Or stating it differently, how the vacuum fluctuations of the electromagnetic field affect the evolution of an electron.

More than that, the program of decoherence raises important questions relevant to the domain of QED. It has been proposed that the superselection rules for conserved charges could be another instant of the environment induced superselection rules [4]: that is, the electromagnetic field considered as an environment makes superpositions of states with different charge rapidly lose their coherence. Arguments in favor and against this proposal can be found in [5, 6].

Also of equal importance is whether single electron states exhibit decoherence in position by virtue of their interaction with the photon vacuum [7, 8]. The two-slit experiment suggests that this is not the case, but we would like to see in more detail the truth of this assertion.
These have been our physical motivations, but were not the only ones. We feel that the use of the influence functional technique in field theoretic situations has been rather less frequent than the strength of the method would require. One reason for that might be the fact that it gives a description of the system in a Schrödinger picture, which has been rather rare in most field theoretic applications. Issues like renormalization (which is explicitly time-dependent) seem to be more complicated in this picture, Lorentz covariance is not manifest and calculations are definitely messier.

Compensating for these problems are two important benefits:
1) Using the influence functional technique, we can more easily see the effects of the initial condition of the environment in the evolution equations. In particular, the effects of the unphysical factorizing initial condition (which is also used in the other techniques) are more easily understood and as we argue in section 3 we can find regimes where they can be removed by a renormalization process.
2) The main object in the influence functional formalism is the reduced density matrix propagator, the knowledge of which enables us to construct master equations, study the classical limit, decoherence phenomena and pass from a field description to a particle one.

From the above discussion, we believe that our aim in this paper has been made clear. We intend to use the influence functional technique to trace out the effects of the electromagnetic field in full QED, hence obtaining the reduced density matrix propagator. This is meant to be our tool for examining the possibility of a dynamical origin of the charge superselection rules. As a special application we shall construct from this the evolution equation for the regime where the initial field state lies in the one-particle sector, aiming to obtain a master equation for a single electron as it interacts with the vacuum fluctuations of the photon field.

In the next section we give a brief review on the influence functional formalism, fixing primarily the notation and introducing some important objects. The calculation of the propagator for the free spinor field with external sources (of great importance for perturbation theory) is also briefly sketched. Section 3 is the largest and from the technical side the most important one. Herein the reduced density matrix propagator for the electron field is calculated, using perturbation theory. The section includes a discussion of the factorizing initial condition, how one can remove the inevitable divergences through a renormalization procedure and the implications of our result for
dynamically induced superselection rules. In section 4 the special case of a
single non-relativistic particle is studied. A master equation is obtained and
the question of whether the photon vacuum can play the role of a decohering
environment is studied in detail.

Before proceeding any further we should mention that a master equation
in the context of quantum electrodynamics for the reduced dynamics of the
charges has been derived before [9]. It is, to the best of our knowledge, the
first attempt to derive such an equation in QED but it does not utilize the
functional Schrödinger picture and it focuses on different issues.

2 The model

Our starting point is the QED Lagrangian

$$\mathcal{L} = \mathcal{L}_e + \mathcal{L}_{ph} + \mathcal{L}_{int} =$$
$$\bar{\psi}(i\gamma^\mu \partial_m - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + e\bar{\psi}\gamma^\mu A_\mu \psi$$  \hspace{1cm} (2.1)$$

We are going to use the influence functional technique to trace out the photon
field and derive the master equation for the reduced dynamics of the electron
field. To do this we shall work in the Bargmann representation (considering
essentially coherent state path integrals) and under the Schrödinger pic-
ture. For more details in the various elements we employ in our formalism
the reader is referred to [10, 11, 12]. The relevant object is the evolution op-
erator $e^{-iHt}$, the matrix elements of which can be read from its path-integral
expression

$$\langle \bar{\psi}_f, A_f | e^{-iHt} | \psi_i, A_i \rangle = \int [d\bar{\psi}][d\psi][dA]e^{iS[\bar{\psi}, \psi, A]}$$   \hspace{1cm} (2.2)$$

where the sum is over fields such that $\psi(0) = \psi_i$, $\bar{\psi}(t) = \bar{\psi}_f$, $A^\mu(0) = A^\mu_i$
and $A^\mu(t) = A^\mu_f$. The knowledge of this object is sufficient to determine the
reduced density matrix at time $t$

$$\rho(t) = Tr_{ph}(e^{-iHt}\rho_0e^{iHt})$$ \hspace{1cm} (2.3)$$

where the trace is over the photon field degrees of freedom. The reduced
density matrix propagator $J$ is defined by

$$\rho_t(\bar{\psi}_f, \psi_f') = \int D\bar{\psi}_f D\psi_i J(\bar{\psi}_f, \psi_f'; t|\bar{\psi}_i, \psi_i; 0)\rho_0(\bar{\psi}_i, \psi_i)$$ \hspace{1cm} (2.4)$$
and can be written in a path integral form

\[
J(\bar{\psi}_f, \psi_f; t|\bar{\psi}_i, \psi_i; 0) = \int [d\bar{\psi}] [d\psi] \int [d\bar{\psi}'] [d\psi'] \exp \left\{ iS_e[\bar{\psi}, \psi] - iS_e[\bar{\psi}', \psi'] \right\} W[\bar{\psi}, \psi; \bar{\psi}', \psi']
\]

(2.5)

with integration over paths $\bar{\psi}(t) = \bar{\psi}_f$, $\psi(0) = \psi_i$, $\bar{\psi}'(0) = \bar{\psi}_i$ and $\psi'(t) = \psi'_f$. Here, $W$ is the influence functional containing the information of the dynamics and initial state of the photon field. In the case of factorisable initial conditions $\rho_0 = \rho_0 \otimes \rho_{ph}$ this has a path integral expression

\[
W[\bar{\psi}, \psi; \bar{\psi}', \psi'] = \int DA_i DA_i' \int [dA][dA'] \exp \left\{ iS_{ph}[A] - iS_{ph}[A'] \right\} \times \exp \left\{ iS_{int}[A, \bar{\psi}, \psi] - iS_{int}[A', \bar{\psi}', \psi'] \right\} \times \rho_{ph}(A_i, A_i')
\]

(2.6)

with path integration over vector fields such that $A(0) = A_i$, $A'(0) = A_i'$ and $A(t) = A'(t) = A_f$. Since the Lagrangian contains an interaction term of third order to the fields, we have to rely on perturbation theory. For this it is sufficient that we compute the spinor propagator with external sources.

### 2.1 The spinor propagator with external sources

Let us consider the Lagrangian density

\[
\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + \bar{\eta}\psi + \bar{\psi}\eta
\]

(2.7)

in terms of the Grassmann fields $\psi$, $\bar{\psi}$ and sources $\eta$, $\bar{\eta}$. Written in terms of the Fourier transformed fields and sources

\[
\psi = \sum_p y_p e^{-ipx} \quad \psi^\dagger = \sum_p y_p^\dagger e^{ipx}
\]

(2.8)

\[
\eta = \sum_p \eta_p e^{-ipx} \quad \eta^\dagger = \sum_p \eta_p^\dagger e^{ipx}
\]

(2.9)

\(^1\text{We denote the path-integration measure (over spacetime fields) by } [d.\ ]\text{ and the measure over fields at one Cauchy surface (single moment of time ) as } D; \text{ the former is a formal expression while the latter is the Gaussian measure with respect to which the field Hilbert space is defined.}\)
it reads
\[
L = \int d^3x \mathcal{L} = \sum_p y_p^\dagger (i\partial_0 - i\gamma^0 \gamma^i p_i - m\gamma^0) y_p + \eta_p^\dagger \gamma^0 y_p + y_p^\dagger \gamma^0 \eta_p
\] (2.10)

which corresponds to a Hamiltonian
\[
H = \sum_p y_p^\dagger (\gamma^0 \gamma^i p_i + m\gamma^0) y_p - \eta_p^\dagger \gamma^0 y_p - y_p^\dagger \gamma^0 \eta_p
\] (2.11)

Now, \(y_p\) is a four-component Grassmann variable. In order to write the Hamiltonian as a sum of single-component Grassmann variables one has to diagonalise the matrix
\[
\gamma^0 \gamma^i p_i + m\gamma^0
\] (2.12)

Writing in the standard way (suppress the index \(p\)) \(y = y_i u_i + z_i v_i\), \(\eta = \eta_i u_i + \eta_i' v_i\), \(i = 1, 2\), there is a particular choice of \(u, v\) such that the above matrix becomes diagonal. The Hamiltonian becomes therefore a sum of terms of the form
\[
H = \omega \bar{y} y - \bar{\eta} \eta - \bar{y} \eta
\] (2.13)

where \(\omega_p^2 = (\mathbf{p}^2 + m^2)\). This is just the Hamiltonian of a Grassmann harmonic oscillator with external sources, the propagator of which reads in a path-integral form
\[
\langle \bar{y}_f, t | y_i, 0 \rangle = \int D\bar{y} D\bar{\eta} e^{iS[y, \bar{y}]}
\] (2.14)

where summation is over paths with \(y(0) = y_i\), \(\bar{y}(t) = \bar{y}_f\) and the action is
\[
iS[y, \bar{y}] = \bar{y} y(t) + i \int_0^t ds (i\bar{y} \dot{y} - H(y, \bar{y}))
\] (2.15)

The path integral integral is readily evaluated by the saddle-point method to yield
\[
\langle \bar{y}_f, t | y_i, 0 \rangle = e^{iS_{cl}}
\] (2.16)

with
\[
iS_{cl} = \bar{y}_f y_i e^{-i\omega t} + i \int_0^t ds [e^{-i\omega(t-s)} \bar{y}_f \eta(s) + e^{-i\omega s} \bar{\eta}(s) y_i] - \int_0^t ds \int_0^s ds' e^{-i\omega |s-s'|} \bar{\eta}(s) \eta(s)
\] (2.17)
One can thus use the inverse Fourier transform to construct the propagator for the full spinor field

\[ \langle \bar{\psi}_f, t | \psi_i, 0 \rangle_{\eta \bar{\eta}} = \exp \left\{ \int d^3x d^3x' \left[ \bar{\psi}_f(x) \Delta(x, x'; t) \psi_i(x') ight. 
\quad + i \int_0^t ds \left( \bar{\psi}_f(x) \Delta(x, x'; t - s) \eta(x', s) + \bar{\eta}(x, s) \Delta(x, x'; s) \psi_i(x') \right) 
\quad - \left. \int_0^t ds \int_0^s \bar{\eta}(x, s) \Delta(x, x'; |s - s'|) \eta(x', t) \right\} \]  

where \( \Delta \) is a “propagating kernel”

\[ \Delta(x, x'; t) = \int \frac{d^3p}{(2\pi)^3} e^{-ip(x-x')} \left( \frac{\omega_p \gamma^0 - \gamma^i p_i}{m} \cos \omega_p t - i \sin \omega_p t \right) \]  

The reduced density matrix propagator

Let us concentrate now on the photon field dynamics. The relevant part of the Lagrangian density is

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + J^\mu A^\mu \]  

where \( J^\mu = \bar{\psi} \gamma^\mu \psi \)

The main idea in the influence functional formalism is to proceed and use the propagator for the above Lagrangian, while treating \( J^\mu \) as an external source. Now this is clearly a linear action, hence the propagator in presence of sources can be computed explicitly.

It is more convenient to fix the gauge for the photon field \( (\partial_\mu A^\mu = 0, A^0 = 0) \) and expand the spatial components of the field in normal modes

\[ A^i(x) = \sum_k \sum_r \epsilon^i_r(k) \left( q_{1r}(k) \cos kx + q_{2r}(k) \sin kx \right) \]  

\( \Delta \) is essentially a unitary operator in the single-particle Hilbert space out of which the field Fock space is constructed.
where the polarization matrices satisfy
\[
\sum_r e^i(r)(k)e^j(r)(k) = T^{ij}(k) = \delta^{ij} - k^i k^j / k^2 \tag{3.3}
\]

Substituting this into (2.6) we get the following expression for the relevant part of the action
\[
S_{ph} + S_{int} = \frac{1}{2} \int_0^t ds \sum_k \sum_r \sum_l (\dot{q}_{rl}(k) + k^2 q_{rl}(k) + 2q_{rl}(k)J_{rl}(k) \tag{3.4}
\]

In this expression the index \( l \) takes values 1 and 2 according to the decomposition (3.2) and the external fields \( J \) read
\[
J_{1r}(k) = e \int d^3 x \cos kx J^i(x) \epsilon_{i(r)}(k) \tag{3.5}
\]
\[
J_{2r}(k) = e \int d^3 x \sin kx J^i(x) \epsilon_{i(r)}(k) \tag{3.6}
\]

The action corresponds to an infinite number of harmonic oscillators under external source and the propagator can be computed the standard way. Moreover if we assume the initial state factorisable and the photon part to correspond to the vacuum \( \rho_{ph} = |0\rangle \langle 0| \) we can use equation (2.19) to compute the influence functional. It is a standard calculation to show that
\[
W[J, J'] = C(t) \prod_\alpha \exp \left[ -i \int_0^t ds_1 \int_0^{s_1} ds_2 [J_\alpha - J'_\alpha](s_1) \right. \\
\times \left( \frac{e^2}{2\omega_k} \sin \omega_k (s_1 - s_2) \right) [J_\alpha + J'_\alpha](s_2) \\
- \int_0^t ds_1 \int_0^{s_1} ds_2 [J_\alpha - J'_\alpha](s_1) \left( \frac{e^2}{2\omega_k} \cos \omega_k (s_1 - s_2) \right) [J_\alpha - J'_\alpha](s_2) \tag{3.7}
\]

where \( \alpha \) is a collective index containing \( k, l \) and \( r \). Using the inverse Fourier transform we obtain the final result for the influence functional
\[
W[J, J'] = C(t) \exp \left[ \delta_{ij} \int d^3xd^3x' \int_0^t ds_1 \int_0^{s_1} ds_2 \right. \\
\left. \left( -i[J^i - J'^i](x, s_1) \eta_{ij}(s_1 - s_2, x, x)[J^j + J'^j](x', s_2) \\
- [J^i - J'_i](x, s_1) \nu_{ij}(s_1 - s_2, x, x)[J^j - J'_j](x', s_2) \right) \tag{3.8}
\]
Here, \( \eta \) and \( \nu \) are the dissipation and noise kernel respectively which read

\[
\eta_{ij}(s, x, x) = e^2 \int \frac{d^3k}{2\omega_k(2\pi)^3} e^{-i\mathbf{k}(x-x)} T^{ij}(\mathbf{k}) \sin \omega_k s \tag{3.9}
\]

\[
\nu_{ij}(s, x, x) = e^2 \int \frac{d^3k}{2\omega_k(2\pi)^3} e^{-i\mathbf{k}(x-x)} T^{ij}(\mathbf{k}) \cos \omega_k s \tag{3.10}
\]

Having the expression for the influence functional we can proceed to evaluate the reduced density matrix propagator. But first we should recall that its derivation depended upon the assumption of factorised initial state. We should first then discuss what is the physical meaning of such a condition.

### 3.1 The initial condition

Our choice of the initial condition (factorisable density matrix, vacuum for the photon field) has been such as to make the calculations easier, but on the other hand it introduces a number of problems. This condition has been extensively used in quantum Brownian motion models and is generally deemed unphysical. Such a separation between system and subsystem requires a large amount of energy (in the case of fields an infinite amount) and hence cannot be taken as realistic. The effects of this initial condition have not been fully understood yet. It was conjectured [13] that it is related to a jolt in the diffusion coefficients at early times, dissapearing in a timescale proportional to \( \Lambda^{-1} \), where \( \Lambda \) is the high frequency cut-off of the environment. Nevertheless this is not probably the case. Recent studies [14] have shown that this jolt is present even if one assumes correlated initial states. The typical behavior in those systems is that the preferred degrees of freedom at early times couple very fast to the high frequencies of the environment, until a dynamical equilibrium is established and then at later times the low frequency modes start playing an important effect (in essence the long time limit is governed solely by the latter). In this class of systems it is usually the case that \( \Lambda \gg \omega \) where \( \omega \) is the typical frequency of the preferred degrees of freedom. To conclude, it is the existence of the above dynamical equilibrium allowing us to trust the analysis with the given initial condition.

Doing field theory just makes things more difficult as one would expect. In QBM models the separation between system and environment follows naturally by assuming the typical timescales of the problem and somehow
the environment appears to be “robust” to the relevant degrees of freedom. Backreaction effects need not be taken into account as far as the environment is concerned. One of the consequences is that a cut-off naturally appears only in the frequencies related to the environmental degrees of freedom ($\Lambda$). In QED this separation is not obvious. We could equally well formulate the inverse problem i.e. what happens to the photon field if we trace out the fermionic degrees of freedom? This, for example, is a question of relevance to solid state physics. Still we expect that some of the features of QBM models will be relevant but we have to be very careful in our analysis.

Our aim is to find the regime where our initial condition can be a good approximation, in the sense that a suitable renormalization of the parameters can be expected to give reliable results. What we assume is that at times $t < 0$ the electron - photon field combined system lies on its ground state and at time $t = 0$ an operation is performed on the spinor sector, which effects to a local excitation of spinor degrees of freedom. Our aim is to see how the reduced density matrix describing these degrees of freedom evolves in time. The vacuum of the full QED theory is a correlated one, so no operation carrying finite energy is sufficient to fully separate the spinor from the electromagnetic degrees of freedom. There will be correlations stemming from the ultra- violet and infra-red sector of the photon spectrum.

Now, in perturbative quantum field theory, one is starting from the Hilbert space of free fields (hence considering a factorised vacuum state) and then the renormalization procedure provides a mapping (though a mathematically ill-defined one $^3$) from the free fields Hilbert space to the interacting one [12, 15]. This observation suggests a rephrasing of our problem : is there any renormalization procedure enabling us to remove the effects of the initial condition?

The expected behavior is the following. At early times the high frequency modes of the photon field couple very fast to the slow ones of the fermion field and vice-versa. This latter coupling expresses the backreaction of the relevant degrees of freedom to the environment (We believe backreaction effects to be important as far as decoherence is concerned). Our experience tells as that it will only affect the initial moments of the dynamics of the photon field had we wished to study them. Hence in the times where the problems

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$^3$Since the free and interacting field Hilbert space carry unitarily inequivalent representations of the canonical (anti)commutation relations
of the unphysical initial condition are expected to arise the contribution of
correlations with soft photon modes is not important. This leads us to expect
that the standard renormalization procedure of the propagator [12] must be
sufficient to account for the effect of the soft modes. This will be verified
in our analysis, that no divergent contributions stemming from low energy
modes appear in the density matrix propagator apart from the standard ones
due to perturbative evaluation.

The case is more complicated for the highly energetic modes, since there
exists an infinite energy barrier between our factorised initial state and a
physical one (i.e. the Hilbert space of the interacting theory does not contain
a factorized state). This implies that the time it will take for these modes to
come to equilibrium will tend to become infinite. To get over this difficulty we
may observe that no such modes can be excited with energy larger than the
total energy of the system. Hence there exists of a natural cut-off \( \Lambda \) (state-
dependent) for both spinor and photon modes (This is due to the fact that
photon and fermion fields involve typical frequencies which are of comparable
order of magnitude. There is not a preferred time scale distinguishing one
from the other in a natural way). Introducing this cut-off can be taken as
considering factorised initial state for only the modes with energy less than \( \Lambda \),
while higher energetic ones (for both fields) can be taken to be in dynamical
equilibrium \( 4 \). This implies that the effect of the spurious initial condition
is to be contained in the parts of the propagator containing the cut-off.
Hence a renormalization procedure (additional to the standard dynamical
one) removing these terms would yield reliable results for sufficiently large
times.

More formally, one could write that the physical density matrix should
be

\[
\rho_0 = \rho_e \otimes \rho_{ph} + e^2 \rho_{cor}
\]

The first term contains a factorized state for all modes it describes while
the second one gives the contribution of the correlated vacuum of the higher
modes. In other words we are asking for a state which in the low energy
modes is essentially uncorrelated while in the high energy ones contains the
correlations of the real vacuum state of the interacting theory. The later
contains differences from the factorized vacuum of the order of \( e^2 \). Time

\footnote{More precisely we can assume a truncation of the field theory such that high energy
modes do not contribute in the dynamics and hence lie close to their vacuum state}
evolution to lowest order in perturbation theory takes the low energy modes
to low ones and similarly for the high ones. To have mixing of the regimes
we must consider orders $e^2$. This means that the contribution of the initially
correlated terms (already of order $e^2$) to the low energy ones at late times will
be of order $e^4$, hence we can safely ignore it in the lowest order of perturbation
expansion.

To summarize: there is a regime of energies $E$, where a (time-dependent)
renormalization on the propagator is expected to remove all effects due to
our unphysical initial condition and our expressions can be valid for times
$t >> E^{-1}$. In particular, the non-relativistic limit for a single particle where
$E \simeq m$ should definitely lie within our range of validity.

3.2 The propagator

The index notation It is more convenient to use an index notation for
the spinor fields so that we will not have to carry the explicit dependence on
$x$. We have essentially four types of spinor fields: $\psi(x)$, $\bar{\psi}(x)$, $\psi'(x)$, $\bar{\psi}'(x)$. The first two propagate forward in time and the other backwards (note the
similarity with the CTP formalism). Now, $\psi$ and $\bar{\psi}'$ in the path integration
are fixed by their initial condition at $t = 0$; we shall assume that they carry
an upper index (in place of their x dependence), while $\bar{\psi}$ and $\psi'$ are fixed
by their values at $t$ and we shall use a lower index for them. This is an
elaboration of the notation used in [3].

Contraction of indices amounts to integration with respect to $x$, while
kernels carry indices according to the fields they are contracted. Hermitian
conjugation for a kernel amounts to inversion of all indices. Hence for exam-
ple our propagator (2.19) reads in this notation

$$\langle \bar{\psi}_f, t|\psi_i, 0 \rangle = \exp \left[ \bar{\psi}_{fa} \Delta^{ab} \psi^b_i + i \int_0^t ds (\bar{\psi}_{fa} \Delta^a_b (t-s) \eta^b(s) +
\bar{\eta}_a(s) \Delta^a_b (s) \psi^b_i - \int_0^t ds \int_0^s ds' \bar{\eta}_a(s) \Delta^a_b (|s-s'|) \eta^b(s') \right]$$

Whenever an upper and lower index do not denote integration and are to be
thought as free a bar will appear in top of them.

Perturbation expansion To compute the propagator one needs to eval-
uate the path integral (3.8) using the expression (2.6) for the influence func-
tional. Since the influence functional contains term of fourth order to the fields, an exact calculation is infeasible, hence we will have to rely on perturbation theory. For this we shall employ the following identity

\[
J(\bar{\psi}_f, \psi_f'; t|\bar{\psi}_i, \psi_i; 0) = W\left[-i \frac{\delta}{\delta \eta}, i \frac{\delta}{\delta \eta'}; i \frac{\delta}{\delta \eta}, -i \frac{\delta}{\delta \eta'}\right]
\]

\[
\langle \bar{\psi}_f, t|\psi_i, 0 \rangle \eta_0 \langle \bar{\psi}'_f, t|\psi'_i, 0 \rangle \eta'_0 |\eta = \eta' = 0 \rangle \quad (3.13)
\]

Expanding \( W \) in powers of \( e^2 \) one can use this formula to derive the perturbative series for the density matrix propagator.

Now, in the expansion we shall encounter loop terms (corresponding to \( Tr\Delta(0) \) or \( \Delta^a_b(t-s)\Delta^b_a(s) \)). Their effect would be to produce a change on the kernel \( \Delta \) and to necessitate a field and coupling constant renormalization. We will prefer to work in the zeroth loop order, that is consider only tree diagrams.

Hence, when taking only the tree diagrams into consideration we see that in the propagator there appear exponentiated four interacting terms.

\[
- \int_0^t ds \int_0^s ds' \left[ \bar{\psi}_f a \Delta^a \bar{\varepsilon}(t-s)\gamma^i \Delta^b \varepsilon(s) \psi^b_i \mu_{ij,eg}(s-s') \right.
\]

\[
\left. \bar{\psi}_{f'c} \Delta^c \bar{\varepsilon}(s)\gamma^j \Delta^d \varepsilon(t-s') \psi^d_{i'} \right] + \int_0^t ds \int_0^s ds' \left[ \bar{\psi}_{f'c} \Delta^c \bar{\varepsilon}(t-s)\gamma^i \Delta^b \varepsilon(s) \psi^b_i \mu_{ij,eg}(s-s') \right.
\]

\[
\left. \bar{\psi}_{f'c} \Delta^c \bar{\varepsilon}(s)\gamma^j \Delta^d \varepsilon(t-s') \psi^d_{i'} \right]
\]

\[
+ \int_0^t ds \int_0^s ds' \left[ \bar{\psi}_{f'a} \Delta^a \bar{\varepsilon}(s)\gamma^i \Delta^b \varepsilon(t-s) \psi^b_i \mu_{ij,eg}(s-s') \right.
\]

\[
\left. \bar{\psi}_{f'a} \Delta^a \bar{\varepsilon}(t-s)\gamma^j \Delta^d \varepsilon(s-s') \psi^d_{i'} \right] - \int_0^t ds \int_0^s ds' \left[ \bar{\psi}_{f'a} \Delta^a \bar{\varepsilon}(s)\gamma^i \Delta^b \varepsilon(t-s) \psi^b_i \mu_{ij,eg}(s-s') \right.
\]

\[
\left. \bar{\psi}_{f'a} \Delta^a \bar{\varepsilon}(s)\gamma^j \Delta^d \varepsilon(t-s') \psi^d_{i'} \right] \quad (3.14)
\]

\[5\]We could obtain the same result had we performed a saddle point approximation in the path integral and used perturbation theory in order to construct the corresponding "classical" equations of motion.
In the above expression \( \mu(s - s') \) stands for \((\nu + i\eta)(s - s')\) and \(\mu^*\) denotes its complex conjugate. We observe in these terms combinations of the form \(\Delta \gamma \Delta\), which amount to products of three and two \(\gamma\) matrices. For physical interpretation it would be more convenient to write these in terms of a fixed basis of the \(4 \times 4\) vector spaces where the Grassmann variables. Actually in our case (parity preservation) only \(\gamma^\mu\), \(1\) and \(\sigma^{\mu\nu}\) are relevant. One can use the identities

\[
\gamma^\mu \gamma^\nu = \eta^{\mu\nu} 1 + i\sigma^{\mu\nu} \\
\gamma^\mu \gamma^\nu \gamma^\rho = g^{\mu\nu} \gamma^\rho + g^{\nu\rho} \gamma^\mu - g^{\mu\rho} \gamma^\nu
\]

(3.15)

(3.16)

to bring all interaction terms into sums of the form \((\bar{\psi} A \psi)(\bar{\psi} B \psi)\) where \(A\) and \(B\) are matrices: \(1, \gamma^\mu, \sigma^{\mu\nu}\).

Let us denote by capital indices \(I, J, \ldots\) whether the indices correspond to scalar \(S\), vector \(V\), tensor \(T\) current (in terms of Lorentz indices \(S\) stands for no index, \(V\) for a single index and \(T\) for two antisymmetric indices).

With this notation we can write the reduced density matrix propagator as

\[
J(\bar{\psi}_f, \psi'_f; t|\psi_i, \bar{\psi}'_i, 0) = \exp \left( \bar{\psi}_f a \Delta^a b \psi'_i + \bar{\psi}'_i a b \psi f_b + \epsilon_{ij}^{\alpha c} A_i^b A_J^c \right) \]

(3.17)

The coefficients \(\epsilon\) and \(\zeta\) can be explicitly computed. Their Fourier transforms are

\[
\epsilon_{IJ}(x, x', y, y') = \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} e^{ip(x-x')-ip'(y-y')} \epsilon_{IJ}(p, p', t) \]

(3.18)

\[
\zeta_{IJ}(x, x', y, y') = \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} e^{ip(x-x')-ip'(y-y')} \zeta_{IJ}(p, p', t) \]

(3.19)

**Renormalization** Divergences appear into the coefficients of the propagator. We have chosen a naive regularisation scheme of substituting \(\int d^3 k\) with \(\int d^3 k e^{-k}\). Divergences then take the form of either poles in \(\epsilon\) or logarithms of \(\epsilon\). The renormalization procedure we are going to follow consists in essentially dropping all terms containing the divergences. There is an ambiguity
as far as the logarithmic divergences go, in the sense that one can with a simple rescaling of $\epsilon$ terms with no dependence on the cut-off appear. Happily, in the case of such divergences the ambiguous terms are oscillatory in time and generically negligible at late times, when the approximation is thought to be valid.

3.3 The master equation

From the form (3.17) of the reduced density matrix propagator it is straightforward to obtain the master equation. Recall that the relevant procedure consists in writing all multiplicative action of initial variables operators on $J$ to differential action of final variables. It can be easily shown that the master equation is of the form

$$\frac{\partial}{\partial t} \hat{\rho} = \frac{1}{i} [\hat{H}, \hat{\rho}] + \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3} \left( i e^2 \alpha_{IJ}(p, p', k, t) [\hat{J}_I(p, k), \hat{J}_J(p', k), \hat{\rho}] - e^2 \beta_{IJ}(p, p', k, t) [\hat{J}_I(p, k), [\hat{J}_J(p', k), \hat{\rho}]] \right)$$

(3.20)

The kernels $\alpha$ and $\beta$ can be computed from the knowledge of $\epsilon$ and $\zeta$.

It is important to remark that the interaction of the spinor field with its environment comes through terms quadratic in the currents $J_I(p, k) = \bar{\psi} A_I \psi(p + k)$.

What are the consequences of the expression (3.20.) as far as decoherence phenomena for the electron field are concerned? The complicated expressions for our coefficients does not support the probability of giving a general argument or proof. What they hint is that if we are to deal with those issues we must restrict ourselves in particular regimes. Such a regime (of a single particle state in the non-relativistic limit) we shall study in the next section.

Still, based on the experience obtained by study of simpler open systems [16] one can make a number of conjectures with high plausibility. Both dissipation and diffusion terms in the master equation are expressed in terms of the three currents (scalar, vector, antisymmetric two-tensor). This means that they form the channels through which the spinor field interacts with its environment. In particular, the diffusion term for each current suggests that the reduced density matrix has the tendency to become approximately
diagonal in some basis close to eigenstates of the current operators. As it stands, we cannot determine which such a basis can be, let alone whether the suppression of the off-diagonal terms is sufficiently strong. But if such regimes exist, then superposition of states with large difference in the expectation values of the current operators would decohere fastly. We could then say that we have some sort of “local current” environment-induced superselection rules. This is a more general case than a single charge superselection rule and actually seems more physical: Charge superselection rule is a special case of this, but we could also have decoherence between states with the same global charge but different local charges, i.e., between two seventeen-electron states with expectation values of currents having support in spatial neighborhoods with large separation. The study of such regimes would be invaluable towards understanding the role of decoherence in quantum field theories and can in principle be performed starting from the propagator (3.17). In connection to the above it is also worth noticing that not all of the currents appearing in Eqn (3.20) are conserved. Only the vector current has got this property. This current is thus related to a conserved quantity, a charge, given as a volume integral over the corresponding local density. When this volume is taken to infinity it will be an exactly conserved quantity. Then in this case according to [6] we have exact decoherence which because of the conservation law is related to the existence of an exact superselection rule. Nothing like that is obvious from the form of the above master equation. It is not very clear to us how the results of [6] could be reproduced in the given formalism. We believe it to be an issue worth investigating. Another interesting point could be made in relation to the ideas of Gell-Mann and Hartle [17] that in decoherent histories hydrodynamic variables are the variables that habitually decohere, being the quasiclassical variables of genuinely closed systems. These variables are related to locally conserved currents. Since not all of our currents are conserved there is an open question of what decoherence in the basis of those currents (if the phenomenon occurs) means and whether the basis of the conserved current is in any sense preferred over the other ones as one would naively expect.
The master equation for a single particle

The reduced density matrix propagator for the spinor field (3.17) is quite general, and can act as a starting point for exploring particular problems. In this section, we will derive the master equation for a single non-relativistic particle, having as our starting point the field propagator.

The general technique for the derivation of master equations for particle dynamics, starting from field theoretic evolution equations has been developed in reference [3], the notation of which we shall employ. In our treatment we are going to make the following assumptions:

i) The initial state of the field lies within the one-particle sector.

ii) We work in frame of reference such that the momentum of the initial state lies in the non-relativistic regime: \(|p/m| << 1\). This means in particular that there is no pair creation and the master equation is expected to be trace-preserving.

iii) We work to the lowest non-trivial order in \(e^2\) and \(|p/m|\).

The assumption (i) guarantees that the propagator of the single particle is of the form \(\mathbf{PJP}\), where \(\mathbf{J}\) is the field propagator and \(\mathbf{P}\) projector on the space of field states that project to the one-particle sector. It reads explicitly

\[
\mathbf{P} = \mathbf{P}_1 \rho \mathbf{P}_1
\]

where the operator \(\mathbf{P}_1\) is defined by its symbol in the Bargmann representation

\[
\tilde{\mathbf{P}}_1(\tilde{\psi}, \psi) = \tilde{\psi} P \psi
\]

Here \(P\) is a 4 \(\times\) 4 matrix that projects into the two dimensional subspace of Dirac spinors that correspond to particles. In the Dirac-Fermi representation for the \(\gamma\) matrices it reads \(P = \text{diag}(1, 1, 0, 0)\).

The propagator \(J_{1\text{par}}\) for the single particle then reads

\[
J_{1\text{par}}(\bar{\chi}_f, \chi'_f; t|\bar{\chi}_i, \chi_i; 0) = \int D\bar{\psi}_f D\psi_f D\bar{\psi}_i D\psi_i D\bar{\psi}'_f D\psi'_f D\bar{\psi}'_i D\psi'_i \ e^{[-\bar{\psi}_f \psi_f - \bar{\psi}_i \psi_i - \bar{\psi}'_f \psi'_f - \bar{\psi}'_i \psi'_i]}
\times (\bar{\chi}_f P \psi_f)(\bar{\psi}'_f P \chi'_f) J(\bar{\psi}_f, \psi'_f; t|\bar{\psi}_i, \psi_i; 0)(\bar{\chi}'_i P \psi'_i)(\bar{\psi}'_i P \chi_i)
\]

It is easily shown that \(J_{1\text{par}}\) is of the form

\[
J_{1\text{par}}(\bar{\chi}_f, \chi'_f; t|\bar{\chi}_i, \chi_i; 0) = K^{a^b}_{c^d} \bar{\chi}_f a \chi'_f b \bar{\psi}'_i c \chi_i d
\]
The kernel \( K_{ab}^{cd} = K_{r,r,s,s}(x_f, y_f; t|x_i, y_i) \) is the single particle propagator (the indices \( r \) and \( s \) take values 1 and 2 and correspond to the two spin polarizations). To lowest order in \( e^2 \) the kernel reads

\[
K_{ab}^{cd} = \Delta_a^b \Delta_c^d + \int_0^t ds \int_0^s ds' \left[ \mathcal{P} \Delta_a^e(t-s) \gamma^i \Delta_b^e(s-s') P_{ij,eg}^*(s-s') \right] \\
+ \int_0^t ds \int_0^s ds' \left[ \mathcal{P} \Delta_a^e(s) \gamma^i \Delta_b^e(t-s) P_{ij,eg}(s-s') \right] \\
+ \int_0^t ds \int_0^s ds' \left[ \mathcal{P} \Delta_a^e(s) \gamma^i \Delta_b^e(t-s) P_{ij,eg}(s-s') \right] \\
+ \int_0^t ds \int_0^s ds' \left[ \mathcal{P} \Delta_a^e(t-s) \gamma^i \Delta_b^e(s-s') \right] \
\tag{4.5}
\]

The non-relativistic limit is more clearly obtained in the Dirac-Fermi representation for the \( \gamma \) matrices

\[
\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix} \tag{4.6}
\]

In this representation the kernel \( \Delta \) reads (its Fourier transform)

\[
\Delta(p; t) = \begin{pmatrix} \frac{\omega_p}{m} \cos \omega_p t - i \sin \omega_p t & -\frac{\sigma_p}{m} \sin \omega_p t \\ -\frac{\sigma_p}{m} \sin \omega_p t & -\frac{\omega_p}{m} \cos \omega_p t - i \sin \omega_p t \end{pmatrix} \tag{4.7}
\]

Substituting this into (4.5) we derive the following expression for the propagator

\[
K_{r,r,s,s}(x_f, y_f; t|x_i, y_i) = \int \frac{d^3p \, d^3p'}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} e^{i(p'(x_f-x_i)-p(y_f-y_i)-\frac{t}{m}(p^2-p'^2))} \\
\times [1 + e^2 \int \frac{d^3k}{(2\pi)^3} e^{ik(x_i-y_i)} T^{ij}(k) \int_0^t ds \int_0^s ds' \\
\times \Lambda^k(t,s)(\sigma^k \sigma^i)_{r,r} e^{-ik(s-s')} (\sigma^j \sigma^l)_{s,s} (\Lambda^l(t',s'))^* \\
\times (\Lambda^k(t,s))^*(\sigma^k \sigma^i)_{s,s} e^{ik(s-s')} (\sigma^j \sigma^l)_{s,s} \Lambda^l(t,s)] \
\tag{4.8}
\]

where the functions \( \Lambda \) denote,

\[
\Lambda^i(t, s) = ip^l f(t, s) + k^l g(t, s) \tag{4.9}
\]

\[
\Lambda^l(t, s') = ip^d f(t, s') + k^d g(t, s') \tag{4.10}
\]
with,
\[
f(t, s) = e^{-imt} + \cos m(t - 2s) \tag{4.11}
g(t, s) = e^{-im(t-s)} \sin ms \tag{4.12}
\]
and \((\cdot)^*\) means complex conjugation. Simplifications occur, when we ignore the spin degrees of freedom, that is we consider initial and final states of total ignorance for the spin. Hence we define the "spinless" propagator
\[
K(x_f, y_f; t|x_i, y_i) = \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} e^{i(p'(x_f - x_i) - p(y_f - y_i) - \frac{1}{2m}(p^2 - p'^2))} \times \left[ 1 + \frac{e^2}{m^2} \int \frac{d^3k}{(2\pi)^3} e^{ik(x_i - y_i)} T^{ij}(k) \int_0^t ds \int_0^s ds' \times \Lambda^k(t, s)e^{-ik(s-s')}((\sigma^i \sigma^j)_{sfsi}(\Lambda^k(t, s'))^*) \times ((\Lambda^k(t, s))^{*} e^{ik(s-s')}((\sigma^i \sigma^j)_{sfsi})_{sfsi}\Lambda^k(t, s')) \right] \tag{4.13}
\]
We now perform the \(k\) integration. Writing \(r = x_i - y_i\) we get
\[
\int \frac{d^3k}{(2\pi)^3} \frac{1}{2k} e^{-ikr -iks} = \frac{1}{4\pi^2} \frac{1}{s^2 - r^2} - \frac{i}{8\pi^2 r} \delta(s - r) = I(r, s) \tag{4.14}
\]
In this integral the principal value is assumed for the real part, and a delta function on advanced time \(s + r\) has been dropped out since we have positive \(s\) in our integration range.

Similarly we can obtain
\[
\int \frac{d^3k}{(2\pi)^3} \frac{1}{2k} e^{-ikr -iks} k^i = \frac{i}{\partial_{r_i}} I(r, s) \tag{4.15}
\]
\[
\int \frac{d^3k}{(2\pi)^3} \frac{1}{2k} e^{-ikr -iks} k^2 = -\frac{\partial^2}{\partial r^2} I(r, s) \tag{4.16}
\]
Using these expressions it is straightforward to perform the \(p\) and \(p\) integration to obtain our final expression for the propagator
\[
K(x_f, y_f; t|x_i, y_i) = C(t) \exp \left[ -\frac{im}{2t} [(x_f - x_i)^2 - (y_f - y_i)^2] \right. \\
\left. + e^2[\alpha(t, r)(x_f - x_i)(y_f - y_i)] \right]
\]
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The expressions for the coefficients $\alpha, \beta, \gamma$ are quite complicated. What we are more interested is their dependence on $r$. Recalling that we are working in the non-relativistic regime and for the dimensions of our system $r$ can be taken much smaller than $t$. Hence to a first approximation one can expand the real part of $I(r,s)$ in a power series with respect to $r$, which essentially corresponds to a power series with respect to $1/c$ (not so with the imaginary part due to the presence of the delta function). This gives for the coefficients following $r$ dependence

$$\alpha(t,r) = \alpha_1(t,r)/r + A(t)/r^2 + O(r)$$

$$\beta(t,r) = \partial_r b_1(t,r) + O(r)$$

$$\gamma(t,r) = \partial_r^2 \gamma_1(t,r) + O(r)$$

where $\alpha_1, \beta_1, \gamma_1$ denotes the integrals containing the delta function which are calculable in terms of elementary trigonometric functions (see appendix).

**4.1 The Gaussian (Coulomb ) approximation**

Still, the $r$ dependence of the propagator is very complicated. Things simplify if one considers the ”Coulomb” limit $c \to \infty$ or $r \to 0$. This essentially corresponds to ignoring the retarded propagation effects for the virtual photons and is expected to be a good approximation (at least for early times) when the spread of the wave packet is very small. Ignoring for the moment the domain of validity of this result, we can easily see that the functions $\beta$ and $\gamma$ vanish and $\alpha(t)$ contains only a time dependent contribution, hence the propagator becomes Gaussian in this limit

$$K(x_f, y_f; t|x_i, y_i) = C(t) \exp \left[ -\frac{i m}{2t} [(x_f - x_i)^2 - (y_f - y_i)^2] + e^2(\alpha(t)(x_f - x_i)(y_f - y_i) \right]$$
From this with the standard procedure one can construct the master equation for the reduced density matrix of the "spinless" particle:

\[
\frac{\partial}{\partial t}\rho(x, y) = -\frac{1}{2mi} \left( \frac{\partial^2}{\partial x^2} - \frac{\partial}{\partial y^2} \right) \rho(x, y) + \frac{2e^2}{m^2} \frac{\dot{\alpha} t - \alpha}{t^2} \frac{\partial^2}{\partial x \partial y} \rho(x, y)
\] (4.22)

or in operator form

\[
\frac{\partial}{\partial t} \hat{\rho} = \frac{1}{i} \left[ \hat{H}, \hat{\rho} \right] - \frac{2e^2}{m^2} f(t) [\hat{p}, [\hat{p}, \hat{\rho}]]
\] (4.23)

where

\[
f(t) = \frac{\dot{\beta} t - \beta}{t^2}
\] (4.24)

and its plot with time is given at the figure 1. We can see that it is oscillatory with a basic frequency of the order of \( m^{-1} \). It is rather straightforward having this simple expression for the propagator, to examine whether in this regime there is any decoherence in position for the electron. For this we consider the evolution of the initial state

\[
\psi(x) = \left( \frac{a}{2\pi} \right)^{1/4} \left( e^{-ax^2} + e^{-a(x-L)^2} \right)
\] (4.25)

which is a superposition of two wavepackets, with their centers separated by \( L \). Time evolution with the propagator (4.12) yields a mixed state with two diagonal terms propagating according to the classical equations of motion plus two off-diagonal terms. To examine the appearance of decoherence one is interested in computing the prefactor of the latter. This is found to be

\[
\exp \left( -\frac{aL^2 (m^2 \frac{\dot{\beta}}{i t^2} + e^4 \beta(t)^2)}{a^2 + m^2 \frac{\dot{\beta}}{i t^2} + \frac{2e^2}{t^2} \beta(t)} \right)
\] (4.26)

It is easy to check that the off-diagonal terms are not suppressed. A necessary condition for that would be a growth of \( \beta(t) \) of at least \( t^2 \) with time, while we can see from figure 2, that \( \beta(t)/t^2 \) is falling asymptotically as \( 1/t \). So within this approximation, no decoherence in position seems to be possible
Figure 1: The function $f(t)$, containing the effects of diffusion at times $t >> m^{-1}$. 

![Graph of function $f(t)$ showing oscillatory behavior and decaying amplitude over time. The x-axis represents time values from 0 to 100, and the y-axis represents values ranging from -0.075 to 0.075. The function oscillates around the x-axis with decreasing amplitude as time increases.]
Figure 2: The function $\beta(t)/t^2$ for $t \gg m^{-1}$. 
for the electrons. This is in accordance with the facts of the classic two-slit experiment.

But how reliable is the approximation of switching off the $r$ dependence of the propagator, or rather in which regime should it be trusted. There are two remarks we should make. First, the term containing $\gamma(t, r)$ is increasing with time, hence at later times (depending primarily on the initial configuration of the system) this term becomes sufficiently large, to invalidate our perturbation scheme. On the other hand in the Gaussian approximation the perturbative terms in the propagator remain sufficiently small at all times. We therefore conclude, that as long as our Gaussian approximation is valid, perturbation theory is reliable. Of course, both the Gaussian approximation and perturbation theory is due to break at the long time limit.

What then can we say, about decoherence in position of the electron, outside this approximation. Unfortunately very little, because the term that we would expect to give rise to decoherence (namely $\gamma(t, r)$) is the one that eventually breaks perturbation theory and hence does not allow us to reach any general conclusions. The problem is aggravated, because the only natural length scale in our system is the Compton wavelength of the particle and qualitative arguments of length scale separation are not sufficient to give any insight.

What we can conclude from our analysis is the following. There exists a regime dependent on the spread of the initial state of the electron, that one can safely use the Gaussian approximation. In such a regime no decoherence phenomena appear. As the spread of the initial state $L$ increases, the influence functional phase gets a large real negative contribution which is characteristic of decoherence phenomena, and we are led to conjecture that decoherence is indeed possible.

One can view these results in the light of the following qualitative observations. Decoherence in any system is connected with the propagation of coherent phase from the system to the environment. In our particular case, the carrier of the information is of course the photonic field. Hence decoherence is expected, when the configuration of the state of the particle is suitable to excite a number of photons, which propagate in the environment carrying the phase information. In the Gaussian regime as we have seen the flow of photons (essentially dissipation) is absent, while the $\beta$ term in the propagator- negligible in this case- essentially contains the effects of dissipation.
Now in the electron - photon vacuum system there are essentially two competing processes governing the emission of photons. On one hand the space where the electron wave function has support "contains" a charge distribution, which essentially acts as "screening" medium, that does not allow virtual photons to escape. The vacuum fluctuations here are of an essentially "Coulomb" type and can only give noise, whose effect only effect is randomness in the evolution of the electron. This process is stronger when the size of the wavepacket is small, and it is precisely in this regime that our Gaussian approximation is valid.

As the spread of the wavepacket increases, and in particular its shape deviates more from a spherical distribution another process starts gaining momentum. Higher moments of the charge distribution are coupled to the electromagnetic vacuum and as such they enhance photon emission. In particular, the case where the electron is in a superposition of two spatially localized states, the contribution from the dipole moment (rather than the quadrupole for more localized states) becomes significant and the electron essentially behaves as an oscillating dipole. When this effect becomes strong and sufficient to overcome screening, significant photon production occurs and decoherence is possible.

At this point we should comment on the relation of our results with the existing bibliography. The density matrix propagator is distinctly different from the one obtained in [7]. These authors have been able to derive a Markovian limit of their master equation and obtain the Lorentz equation in the semiclassical limit. The crucial difference is that they have derived their result under the dipole approximation. For this they considered a restoring harmonic potential so that the particle is constrained to lie in a small spatial neighborhood. In our case having derived our master equation from full Quantum Electrodynamics, we have considered an essentially free electron. Since the only natural length scale appearing in our calculations is the Compton wavelength of the particle, and free evolution does not allow for localization of the particle, the dipole approximation should not be expected to be valid here.
5 Conclusions

The density matrix propagator (3.17) and (4.19) for the single particle, we consider to be the most important results in our paper. The former is to be seen as a starting point for many interesting applications (to be outlined below), while the latter provides (we believe) a conclusive demonstration that electromagnetic field vacuum fluctuations are not sufficient to cause decoherence to single free electrons.

It has been rather disappointing for us not to be yet able to say anything conclusive yet about the possibility of dynamically induced superselection rules. As we said earlier the complicated nature of our expressions preclude any possibility of being able to make any statement about generic states. Still we think we have provided the basic tools for a more thorough investigation of these issues.

The study of various regimes seems to be a natural step after this work. An important case can be the case where the field lies either in the \( N \) or the \( M \) particle state. The propagator would then reads \( \mathbf{P}_{NM} \mathbf{J} \mathbf{P}_{NM} \) where \( \mathbf{P}_{NM} \) is defined by \( \mathbf{P}_{NM} \rho = (P_N + P_M)\rho(P_N + P_M) \), \( P_N \) being the projector in the \( N \) particle sector. If the hypothesis of the dynamical origin of charge superselection rule is valid then decoherence phenomena should be definitely clear in this regime.

Another interesting issue towards an answer of which, our formalism can aim is the study of electrons in a background squeezed state. Squeezed states are characterized by negative energy densities, and a connection between them and decoherence, dissipation and noise would be very rewarding.

Finally, the expression (4.10) for the single electron propagator implies a spin - momentum self-coupling due to the vacuum fluctuations. The study of this kind of couplings is a rather unexplored territory and potentially of much interest in the light of quantum optics.

References


[14] L. D. Romero and J. P. Paz, Quant-Ph/9612036 


A Coefficients in sigle-particle propagator

We here give expressions for the functions $\alpha_1$, $\beta_1$, $\gamma_1$ and $A$ in equations (4.17 - 4.19).

\[\alpha_1(t, r) = \frac{1}{4\pi m} \sin^2 mt \sin^2 mr\]  \hspace{1cm} (A.1)

\[\beta_1(t, r) = \frac{1}{16\pi r} \int_0^t ds \left( e^{-im(s-r)} \sin m(s - r) + \cos m(2t - s)e^{im(t-s+r)} ight. \\
- \left. e^{im(s)} \sin ms - \cos m(2t - s + r)e^{-im(t-s)} \right)\]  \hspace{1cm} (A.2)

\[\gamma_1(t, r) = \frac{\sin mr}{16\pi mr} (t \cos mr + \sin m(2t - r) + \sin mr)\]  \hspace{1cm} (A.3)

\[A(t) = \frac{1}{2\pi^2} \int_0^t ds \int_0^s ds' \left( 1 + \cos 2m(s - s') + \cos 2m(t - s - s') \right. \\
+ \cos 2m(s - s') \cos 2m(s + s') \\
+ \left. \cos 2m(s - s') \cos 2m(t - s - s') \right) \\
\times \frac{1}{(s - s')^2 + \epsilon^2}\]  \hspace{1cm} (A.4)

Note, that the term $A$ has a cut-off dependence on $\epsilon$, which becomes smaller and at late times negligible. It can be absorbed into an early time redefinition according to our renormalisation scheme.

Also, we can see that the term $\gamma_1$ is the one whose amplitude increases with time and is to be held responsible for the breakdown of our perturbative approximation and possibly as the one giving rise to decoherence.