The Role of Monopoles for Color Confinement

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We study the role of the monopole for color confinement by using the monopole current system. For the self-energy of the monopole current less than ln(2d − 1), long and complicated monopole world-lines appear and the Wilson loop obeys the area law, and therefore the monopole current system almost reproduces essential features of confinement properties in the long-distance physics. In the short-distance physics, however, the monopole-current theory would become nonlocal due to the monopole size effect. This monopole size would provide a critical scale of QCD in terms of the dual Higgs mechanism.

1. Kosterlitz-Thouless Type Transition in Monopole Current Dynamics

Color confinement can be interpreted using the dual version of the superconductivity[1]-[5]. In this picture, condensation of color magnetic monopoles is the key concept and leads to squeezing of the color-electric flux between quarks through the dual Meissner effect. QCD is reduced into an abelian gauge theory with monopoles in the 't Hooft abelian gauge[2]-[9]. As for the maximally abelian (MA) gauge, recent lattice studies suggest that only abelian gauge fields with monopoles are essential for description of nonperturbative phenomena in the infrared region[6]-[8].

We study the role of monopoles for confinement using a monopole action. In the confinement phase, only short-range interactions remain among monopoles due to the screening effect[10] by the dual Higgs mechanism, and therefore the infrared monopole-current dynamics would be described by a simple action[3,11]. \( S = \alpha^{\text{lat}} \sum_{\mu} k_{\mu}^{\text{lat}}(s)^2 \) in the lattice formalism. Here, \( k_{\mu}^{\text{lat}}(s) \equiv e/(4\pi) \cdot a^2 k_{\mu}(s) \in \mathbb{Z} \) and \( \alpha^{\text{lat}} \equiv \alpha/2 \cdot (4\pi/ea)^2 \) denote the magnetic current and its self-energy on the lattice, respectively.

The monopole current system is generated with the current conservation condition, \( \partial_{\mu} k_{\mu}^{\text{lat}} = 0 \). Here, we update the link of the monopole current using the Metropolis method.

We show in Fig.1 the monopole density \( \rho_M \equiv \frac{1}{V} \sum_{s} |k_{\mu}^{\text{lat}}(s)| \) with the lattice volume \( V=5^4 \) and the clustering parameter \( \eta \equiv \sum L_i^2 / (\sum L_i)^2 \), where \( L_i \) denotes the loop length of the \( i \)-th monopole cluster. With increasing \( \alpha \), the monopole density \( \rho_M \) is reduced gradually for \( \alpha < \alpha_c \). The clustering parameter is drastically changed at \( \alpha_c \). Quantitatively, the critical value of monopole self-energy, \( \alpha_c \approx 1.85 \), is close to ln7±1.95, which is derived from the entropy factor \( (2d-1)^{L} = \tau^{L} \) in the 4-dimensional self-avoiding random walk[11]. Such a transition is quite similar to the Kosterlitz-Thouless type transition in 1+2-dimensional superconductors. Vortex condensation at high temperature in the superconductor corresponds to monopole condensation in the strong coupling region of QCD.

2. Dual Field Formalism and Role of Monopoles for Confinement

In this section, we study confinement properties in the monopole current system with \( j_{\mu} \neq 0 \) and \( j_{\mu} = 0 \), which is the dual version of QED. To estimate the abelian Wilson loop, we introduce the dual gauge field \( B_\mu(x) \), since the gauge field \( A_\mu(x) \) inevitably includes the singularity as the Dirac string in the presence of magnetic monopoles. Using the relations \( \partial_{\mu} * F_{\mu\nu} = k_{\nu} \) and \( * F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \), the dual gauge field \( B_\mu \) can be described from the monopole current \( k_{\mu} \).

\[
B_\mu(x) = \partial^{-2} k_\mu(x) = -\frac{1}{4\pi^2} \int d^4y \frac{k_\mu(y)}{|x-y|^2}, \tag{1}
\]
in the dual Landau gauge $\partial_\mu B_\mu = 0$. The Wilson loop is obtained as $\langle W \rangle = \langle \exp \{ i e \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \int d^4x (\partial_\mu B_\nu - \partial_\nu B_\mu) \rangle$. In estimating the integral in Eq. (1) numerically, we should take a finer mesh than original lattice spacing $a$ to extract $B_\mu$ correctly. Otherwise, the unexpected electric currents appear as the lattice artifact. However, too finer mesh is not necessary, because the original current $k_\mu^{\text{lat}}(s)$ includes the error in the order of $a$. Numerical analyses show that the calculation with $c \leq a/2$ is good enough for the estimation of $B_\mu$, and hence we take $c = a/2$ hereafter. On lattices, the dual gauge field $\theta_\mu^{\text{dual}} = a e B_\mu/2$ is defined in the dual Landau gauge; $\theta_\mu^{\text{dual}}(s + \mu) \equiv 2\pi \sum_{s'} \Delta(s-s') k_\mu^{\text{lat}}(s')$ using the lattice Coulomb propagator $\Delta(s)$. The dual version of the abelian field strength $F_{\mu\nu}^{\text{dual}} \equiv e a^2 F_{\mu\nu}/2$ is defined by $\theta_\mu^{\text{dual}}(s) \equiv \partial_\mu \theta_\nu^{\text{dual}}(s) - \partial_\nu \theta_\mu^{\text{dual}}(s)$, where $\partial_\mu$ denotes the backward derivative.

As shown in Fig.2, the Wilson loop $\langle W \rangle$ exhibits the area law, which leads to the linear confinement potential. In Fig.3, we show the string tension $\sigma^{\text{lat}} \equiv \sigma^{\text{phys}} a^2$ in the lattice unit. For $\alpha < \alpha_c$, the string tension is finite due to the long monopole currents, the current simulations for $\alpha > \alpha_c$ indicate that $\sigma^{\text{lat}} \propto 0$ in the system without monopole currents. On the other hand, lattice QCD simulations show that the long monopole currents cover the whole system and string tension is finite in the confinement phase even for the large $\beta$ as $\beta = 2.3 \sim 2.5$ on $16^4$ lattices. Therefore, the monopole self-energy $\alpha$ extracted from QCD must be smaller than $\alpha_c \equiv \ln(2d-1) = \ln 7$ in the confinement phase. Here, to extract the monopole action, one should use the monopole part in QCD, where only monopole currents exist. Since the electric current and the short-range Coulomb force cannot be expressed only by the monopole currents, it is dangerous to compare the monopole action with the full SU(2) QCD (or the abelian part); the former includes monopole currents only, while the latter includes also electric currents and Coulomb force. In any case, the monopole action is to be extracted from the monopole part in QCD and the monopole self-energy $\alpha$ should be smaller than $\alpha_c = \ln 7$ in the confinement phase.

3. Monopole Size Effect and Critical Scale in QCD

The aim of our study is to analyze the theoretical structure of QCD in terms of the monopole degrees of freedom. Our studies indicate that the multi-monopole system has a similar structure to QCD in the MA gauge. However, the monopole in abelian-projected QCD has the underlying structure reflecting QCD. For instance, QCD-monopoles would be as extended objects like hadrons[9], because it is not an elementary particle in QCD, but a collective mode composed by gluons. We compare the monopole current system with QCD [6] especially in terms of the self-energy and the size of monopoles[12].

In the static frame of the monopole with an intrinsic radius $R$, the spherical magnetic field is created around the monopole as $H(r) = \frac{g_m a}{4 \pi r}$ for $r \geq R$ and $H(r) = \frac{g_m a}{4 \pi r} e^{ie(r/R)}$ for $r \leq R$, with QCD running gauge coupling $e(r)[11]$. For $a > R$, the electromagnetic energy observed on the lattice around a monopole is roughly estimated as $M(a) \simeq \int_a^\infty \frac{1}{2} H(r)^2 \simeq \frac{2\pi}{e^2} a$, which largely depends on the lattice mesh $a$. Hence, the monopole contribution to the lattice action reads $S \equiv M(a) a L$, where $L$ denotes the monopole current length measured in the lattice unit $a$. Therefore, $M(a)$ is related to the monopole-current coupling $\alpha^{\text{lat}}$ and $\beta = 2N_c/e^2$, $\alpha^{\text{lat}} \simeq M(a) a \simeq \frac{2\pi}{\beta(e)} \approx \frac{2}{\beta} \beta_{\text{SU}(2)}$. By the screening effect, this relation becomes $\alpha^{\text{lat}} \simeq \frac{2}{\beta} \beta_{\text{SU}(2)}[11]$, which is consistent with the numerical result as shown in Fig.3. Thus, as long as the mesh is large as $a > R$, the lattice monopole action would not need modification by the monopole size effect, and $\alpha^{\text{lat}}$ is proportional to $\beta$.

On the other hand, there exists a critical coupling $\alpha_c \simeq 2\ln 7$ in the current dynamics. Above $\alpha_c$, the lattice current action provides no monopole condensation and no confinement, while $\beta \to \infty$ can be taken in the original QCD keeping the confinement property shown in Fig.3. Such a discrepancy between $\beta$ and $\alpha$ can be naturally interpreted by introducing the monopole size
effect, which should lead the nonlinear correspondence between $\alpha$ and $\beta$. For $\alpha < R$, the extended structure of the monopole can be observed on the lattice, and the monopole creates the electromagnetic energy $M(\alpha) \sim \frac{e^2}{2\pi R} = \frac{2\pi}{\alpha R}$ on the lattice, which is almost $\alpha$-independent. Accordingly, the lattice action should be changed as $S = M(R)\alpha \sum_{k,l} k_\mu k_l - \left( \frac{R^2}{2} - |k - l| \right) \delta_{\mu,\nu}$ because of the self-avoidness originating from the monopole size $R$. Thus, the monopole-current theory becomes nonlocal in the UV region.

The monopole size $R$ provides a critical scale for the description of QCD in terms of the dual Higgs mechanism. In the infrared region as $\alpha > R$, QCD can be approximated as a local monopole-current action, and the QCD vacuum can be regarded as the dual superconductor. On the other hand, in the ultraviolet region as $\alpha < R$, the monopole theory becomes nonlocal and complicated due to the monopole size effect, and the perturbative QCD would be applicable instead.

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REFERENCES

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Figure Captions

Fig.1 The monopole density $\rho_M$ and the clustering parameter $\eta$ as the function of $\alpha$.

Fig.2 The Wilson loop $\langle W(I \times J) \rangle$ on the lattice with the mesh $c = a/2$ for $\alpha = 1.7, 1.8, 1.9$.

Fig.3 The string tension $\sigma_{\text{lat}} \equiv \sigma_{\text{phys}} a^2$ as the function of $\alpha$ in the multi-monopole system. The dotted line denotes the Creutz ratio in the lattice QCD with $\beta = \frac{4}{\pi} a$. 