Against Measurement? – On the Concept of Information

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Abstract

In his last article Against ‘Measurement’ J. S. Bell sums up his well known critique of the problem of explaining the measurement process within the framework of quantum theory. In this article I will discuss the measurement process by analysing the concept of measurement from the epistemological point of view and I will argue against Bell that it belongs to the preconditions of experience to necessarily end up with a “reduction of the wavefunction”. I will consider the “chain of reduction” in detail – from pure states of \( S \otimes A \) (system \( S \) and measuring apparatus \( A \)) via different kinds of mixtures to pure states of \( A(S) \). It turns out that decoherence is not sufficient to explain reduction, but that this can be done in terms of the concept of information within a transcendental approach.

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1 Introduction

Quantum theory is the ‘hard core’ of physics today. Its experimental success is overwhelming. But although the mathematical framework and its pragmatical application is non-controversial, it suffers from an obstinate interpretation problem: what does the formalism really mean philosophically? One part of this open question is the measurement problem. In this article I first like to discuss the measurement problem in terms of measurement theory and in comparison with some actual interpretation approaches. Since John S. Bell in his famous article Against ‘Measurement’ (Bell 1964) gives a very sophisticated (and also humorous) analysis of the problems of quantum theory according to measurement theory, I like to deal with some of his clever arguments – including the usage of his abbreviation

\[ FAPP = \text{for all practical purposes.} \]

The final aim is to stress the epistemological point of view in discussing the measurement problem. This will be done using a transcendental argument in the spirit of Kant. It turns out that in opposition to Bell the concept of information plays a key role in quantum theory.

2 The Measurement Problem

The measurement problem is caused by the universality of quantum theory, which means that there exists no inherent reason not to apply it to any physical system or object. In the theory of measurement the world is divided into distinct parts: a system \( S \), on which the measurement is performed, a measuring apparatus \( A \), and the ‘rest of the world’ \( R \). Because of its universality all parts must in principle be describable by quantum theory.

Let us briefly review the steps of the measurement process. I will denote the quantum states of \( S \) by \( |\psi\rangle \) and the states of \( A \) by \( |\chi\rangle \). The observable \( \hat{A} \) satisfies the eigenvalue equation

\[ \hat{A}|\psi_i\rangle = a_i|\psi_i\rangle. \]  

To perform a measurement the Hilbert space of \( S \) must be enlarged to the Hilbert space of the compound system \( S \otimes A \) by forming the tensor product which is spanned by the states

\[ |\Phi\rangle = |\psi \otimes \chi\rangle = \sum_i c_i|\Phi_i\rangle \]  

with

\[ |\Phi_i\rangle = |\psi_i \otimes \chi\rangle. \]  

The tensor product also contains interference terms, which represent the typical quantum correlations between \( S \) and \( A \). The state \( |\Phi\rangle \) is a pure state with a projection operator

\[ \hat{P}_\Phi = |\Phi\rangle\langle\Phi|. \]
After the measurement interaction $\hat{H}_{int}$ between $S$ and $A$ we obtain

$$|\Phi\rangle = e^{i\hat{H}_{int}} |\Phi\rangle$$

with states

$$|\Phi'_i\rangle = \left|\psi'_i \otimes X'(\psi_i)\right\rangle \equiv \left|\psi_i \otimes X(\psi_i)\right\rangle$$

instead of $|\hat{\phi}\rangle$. In case of an ideal measurement we can replace $|\psi'\rangle \equiv |\psi\rangle$. Note that now the states $|X'(\psi_i)\rangle$ of the measuring apparatus are not independent from those of the system $|\psi\rangle$.

After the measurement interaction the compound system $|\hat{\phi}\rangle$ must be separated into the subsystems $S$ and $A$ in order to read $A$. This is done by a cut. Firstly, $\hat{P}_\psi$ transforms into the mixed states of $S$ and $A$ with the density operators $\hat{\rho}_\psi$ and $\hat{\rho}_\chi$.

$$\hat{P}_\psi \rightarrow \hat{\rho}_\psi = \sum_{i,k} w_{ik} |\psi_i\rangle \langle \psi_k|.$$  

Due to decoherence the interference terms will become immensely small (but do not vanish exactly).

In principle the mixed state $\hat{\rho}_\psi$ allows an infinite number of possible decompositions into states $|\psi_i\rangle$ of $S$ (resp. into states $|X(\psi_i)\rangle$ of $A$). By picking out one special decomposition the mixed state transforms into a mixture of states $\hat{\gamma}_\psi$.

$$\hat{\rho}_\psi \rightarrow \hat{\gamma}_\psi = \{ (w_i, \hat{P}_{\psi_i}) \}.$$  

As a last step the system is in a new state $\hat{P}_{\psi_i}$.

$$\hat{\gamma}_\psi \rightarrow \hat{P}_{\psi_i} = |\psi_i\rangle \langle \psi_i|.$$  

Likewise the measuring apparatus will show one result and is therefore in a definite pointer state $\hat{P}_{X(\psi_i)}$. Finally, at the end of the whole measuring act, the systems $A$ and $S$ are in new pure states.

### 3 The Chain of Reduction

As a result of the preceding section we see that strictly speaking the so called ‘reduction of the wavefunction’ has to be considered as a chain of reduction of at least three steps:

$$\hat{\rho}_\psi \xrightarrow{\text{step 1}} \hat{\gamma}_\psi \xrightarrow{\text{step 2a}} \hat{P}_{\psi_i} \xrightarrow{\text{step 2b}} \hat{P}_{X(\psi_i)}$$

Let us consider this chain step by step.

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3In German: "Gemisch" (mixed state) vs. "Gemenge" (mixture of states) (Eisenberg 1957, p. 43), (Sussmann 1958).
Step 1. Due to the measurement interaction and due to the coupling between systems $S$, $A$, and $R$ this step leads to the cancellation FAPP of the interference terms between $S$ and $A$, i.e. the typical quantum correlations disappear (FAPP, but not exactly). Thus, step 1 can be looked upon as a unitary temporal development according to some wave equation.

Step 2. Due to the objectification the mixed state must be replaced by a mixture $\hat{\gamma}$ according to (8). It is first of all important to note that the quantum theoretical framework gives no possibility to describe this step as a pure quantum process (i.e. as a time evolution by a unitary operator). Bell points out that, logically speaking, a system described by $\hat{\rho}$ is in a state

$$\Psi_1^* \text{ and } \Psi_2^* \text{ and } \ldots ,$$

whereas $\hat{\gamma}$ refers to a state

$$\Psi_1^* \text{ or } \Psi_2^* \text{ or } \ldots .$$

Although step 2 is certainly not involved in step 1 it is indeed astonishing that this problem is very often not discussed in the common measurement theory – or as Bell puts it: "The idea of elimination of coherence, in one way or another, implies the replacement of ‘and’ by ‘or’, is a very common one among the solvers of the ‘measurement problem’. It has always puzzled me.” (Bell 1990, p. 36).

Step 2 can be split into two logical steps 2a and 2b (Bell makes no explicit distinction between them):

Step 2a. A mixed state $\hat{\rho}$ represents a class of equivalent mixtures $\hat{\gamma}$ – such that from the mathematical point of view no particular mixture is distinguished. Step 2a stresses the point that one particular has to be chosen.

Step 2b. At the end of the whole measuring act the apparatus must show exactly one result (on a display for instance) – what else could be the meaning of the term ‘measurement’? Step 2b stresses the point that any satisfying theory of measurement must explain the final occurrence of exactly one measuring outcome out of the set of the many possible ones.

To make a distinction between steps 2a and 2b could be confusing. The mixture $\hat{\gamma}$ can be considered as a statistical description of different wavefunctions $\Psi_i$ – such as the density matrix in classical statistical thermodynamics. This means, ontologically speaking, that the system already exists in an actual state $\Psi_i$, but it is not known (only with probabilities) to the observer. Thus, the so called ignorance interpretation is valid for $\hat{\gamma}$ but not for $\hat{\rho}$. From this viewpoint step 2b would just describe the reading of the measuring device by an observer and would be of no philosophical interest. But it should be emphasized that one cannot distinguish
between $\hat{\rho}$ and $\hat{\gamma}$ by any observation. With respect to this the distinction between step 2a and 2b indicates the logical difference between ‘picking out a certain mixture $\hat{\gamma}$’ and ‘actually being in one certain state’ (the final pointer state of $\mathcal{A}$ for instance).

4 Against Measurement? – Some Interpretations

What do the most prominent interpretation programs of quantum theory today answer to the above steps in the chain of reduction? I will briefly discuss some of them.

The Decoherence Program. Nowadays, the concept of decoherence to explain step 1 is very popular with physicists. It seems indeed very useful to scrutinize the conditions and orders of magnitude under which quantum systems decohere, but one has to keep in mind that decoherence is essentially FAPP. The question arises whether a FAPP description will be a satisfactory explanation. Bell was obviously not satisfied – although he and nobody, I presume, would deny the "... absence FAPP of interference between macroscopically different states" (Bell 1964, p. 36). All in all decoherence is sufficient to explain step 1, whereas step 2 is by no means explained.

The Bohm-de Broglie Program or the Hidden Variable Program. It is the first of Bell’s favourites in his article. Of course a hidden variable argument is an objection which can always be raised: there could be something we do not know today! But, as a consequence of Bell’s own invention – his famous inequalities (Bell 1964) – local hidden variables are nowadays experimentally excluded. But does a theory with non-local hidden variables really show a conceptual difference to common quantum theory? New arguments support the idea that this is indeed not the case (Englert et al. 1992) – would Bell have believed in them?

The Ghirardi-Rimini-Weber Model. This is the second of Bell’s favourites in his article. Its starting point is the ad-hoc-assumption that the wavefunction will 'collapse' after a given small and stochastic time interval by a spontaneous localization process. The parameters of the model are chosen for it to be in good correspondence with the ordinary quantum predictions. The model ‘explains’ step 1 in the bandwidth of its parameters – i.e. very well, but FAPP. Insofar the spontaneous processes are stochastic, the central questions behind step 2a and, most of all, 2b remain unanswered – thus the situation resembles the decoherence program. The question persists how a mere ad-hoc-model could be a satisfactory explanation of the deep problem of quantum measurement.

The Many Worlds Interpretation. This interpretation leads essentially to quantum cosmology, i.e. the reduction problem is considered for the universe as a whole. It does ‘explain’ – in
a very broad-minded meaning of this word – step 2, but it does not explain step 1. Consider for instance the following open questions: At what time steps does the ‘branching’ of the universes occur and how many universes do occur at each time step? Today many authors combine the idea of decoherence with the concept of many worlds. But nevertheless the ‘ontological costs’ of assuming many universes are very high! Should this really be the right answer? In any case, even Bell, also in combination with the BOHM-DE BROGLIE theory, ”... did not like it” (Bell 1976).

The Copenhagen Interpretation (CI). This is the orthodox interpretation of quantum theory, as far as it refers to its founders WERNER HEISENBERG and, most of all, Niels Bohr. Since there does not exist a kind of ‘codification’ of CI, there is still a certain confusion about its basic concepts. I like to propose the following as the central assumption of CI: The outcomes of measurements must be described in classical terms, i.e. the measuring apparatus must be described classically#4. What is the meaning of this assumption? It certainly means not that apparatuses and measuring devices are non-quantum systems. But it means that the apparatuses must necessarily be described classically in order to give an appropriate description of the outcomes of a measurement. ‘Appropriate’ here means that the outcomes have to be communicable and understandable to each observer. E.g., the idea that the pointer of a device should be in a superposition of different pointer positions is obviously senseless and non-communicable. In this sense experimental data must be described classically.

Moreover, CI can be read as just offering the minimal semantics to quantum theory, i.e. semantics which is necessary in order to apply the theory to reality (Gornitz and Weizsacker 1991). Generally speaking, a physical theory contains two parts: the mathematical structure of the formalism and the related physical concepts. The minimal semantics of CI is:

<table>
<thead>
<tr>
<th>Mathematical structure</th>
<th>Physical concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hilbert space $\mathcal{H}$</td>
<td>object (system)</td>
</tr>
<tr>
<td>(topological) tensor product $\mathcal{H}_1 \otimes \mathcal{H}_2$</td>
<td>composition of the objects 1 and 2</td>
</tr>
<tr>
<td>(self-adjoint) operator</td>
<td>observable</td>
</tr>
<tr>
<td>unitary $U(1)$ transformation</td>
<td>temporal development</td>
</tr>
<tr>
<td>vector $</td>
<td>\Psi\rangle \in \mathcal{H}$</td>
</tr>
<tr>
<td>scalar $p =</td>
<td>\langle \Psi</td>
</tr>
</tbody>
</table>

The last row refers to the central concept of CI: probability. The scalar product of two states gives the amplitude of the transition probability between them. Thus, the Hilbert space is provided with a probability metric.

A further remark should be made: probability can be seen as the mathematical quantification of ‘possibility’. Interestingly, the many worlds interpretation is in some sense not richer than CI,

#4 Compare Bohr who emphasized that ”... however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms” (Bohr 1949, p. 209).
since there exists a one-to-one terminological mapping between both interpretations: the many world interpreters have simply replaced the term `possibility' by the term `world'. It seems that `many worlds' is just a fancy way of saying something very trivial, namely `many possibilities' (i.e. probability).

But a crucial question remains: how can a probability theory be understood without the concept of measurement? How can physics be "against measurement"?

5 Information and the Transcendental Approach

As shown in section 3, the `reduction of the wavefunction’ is logically more than one single step. CI gives a necessary minimal semantics to quantum theory, but the alternative interpretive attempts in section 4 do not go beyond it to provide a satisfactory understanding of the entire chain of reduction. Thus, maybe a philosophical invention would be helpful: the transcendental point of view. As I like to argue in the following, this will not contradict CI, but it will extend the epistemological aspect of quantum theory.

At the end of section 4 probability was identified as the central concept of quantum theory – at least according to CI, its minimal semantics. "Against Bell" I like to stress the point that in any probability theory one can neither renounce the concept of measurement, nor the concepts microscopic, macroscopic, reversible, irreversible, observable, or observer – some of the "... worst terms" in Bell’s understanding. Moreover, I like to propose information – decidedly against Bell a really good word – as the central concept behind it all (lyre 1998). In view of the main question, what the quantum wavefunction $\Psi$ really stands for, the following basic assumption should be made

$\Psi$ is information.

What are the reasons for believing this? Probability means – due to a logarithm – the same as the syntactic aspect of information $I_{\text{syn}} \sim - \ln p$. There also exist a semantic and a pragmatic aspect. Thus, probability can be seen as a sub-concept of information. Information, moreover, involves the terms subject and object. Let us try to answer Bell (1990, p. 34): "Information? Whose information?" Information for any subject with conceptual and empirical competence! "Information about what?" Information about empirically knowable objects, which are constituted just by the information which can be gained from them! In short: Objects are constituted by information which is available to subjects.

In order to make the above answers plausible the direction of arguments must be changed. This can be done by using a transcendental argument in the manner of Immanuel Kant: the foundations of empirical science are based on the preconditions of experience (Kant 1781) – and, certainly nowadays, the foundations of empirical science are the foundations of quantum theory (Drieschner 1979). The key idea is that since experience, and moreover empirical science,
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is obviously possible and successful, certain preconditions of experience must hold, which, in the last analysis, make experience possible. These preconditions will of course never fail in experience – per definition they can never become empirically falsified. The crucial question then is: What are good candidates for preconditions of experience? I like to propose just these two: distinguishability and temporality. Why? Without the possibility of making distinctions we could never be able to have speech, concepts, and communicable thoughts. If science is possible distinguishability is one of its most rudimentary methodological preconditions. Further on, without implicitly using the already known difference between past and future we could never give any meaning to the word experience – or as Carl Friedrich von Weizsäcker puts it: "... experience means to learn from the past for the future, then any empirical science presupposes an understanding of past and future" (Gornitz and Weizsäcker 1994). This understanding may be called temporality.

Translated into an information theoretic language the difference between past and future can be expressed as the difference between actual and potential information. More detailed than the above statement we can say that the wavefunction or, in general, density operators represent potential information. Still more detailed, it is quantum information, since quantum bits are composed by the tensor product of two dimensional Hilbert spaces, i.e. quantum bits are indistinguishable in contrast to bits of classical potential information. In an earlier paper (Lyre 1997) I have argued that the so called complete concept of information, i.e. the syntactic, semantic, pragmatic, and temporal aspect of information, can conceptually be deduced from distinguishability and temporality alone. In this sense information can be based transcendentally, i.e. concerning the preconditions of experience.

6 The Measurement Problem – Revisited

What are the advantages in expanding CI strictly towards an information theoretic interpretation of quantum theory? In this last section I like to revisit the measurement problem – and especially the chain of reduction. I will do this in a list of eight theses:

1. Thesis. The chain of reduction is not a chain of physical interaction steps but of methodological ones. The usual aim of measurement theory is to describe the measurement process as a quantum physical process by itself, i.e. to describe it as an interaction between $S$ and $A$. Surely, first of all the physical interaction $\hat{H}_{\text{int}}$ as expressed in (31) establishes the measuring act. But this has nothing to do with the problem of reduction! Otherwise there should exist a unitary transformation for each step of the chain of reduction. But as analysed in section 5 step 1 can only be considered as a time development FAPP, whereas step 2 certainly cannot. For the whole procedure described in section 5 we should better speak of measuring ‘act’ instead of ‘process’ (to speak with Bell: ‘process’ is a bad word at this stage).
2. **Thesis.** According to step 1: *Quantum theory does not predetermine the cut between \( S \) and \( A \), nevertheless the cut is necessary in order to apply quantum theory to reality.* Because of the universality of quantum theory, any system can in principle be described by quantum theory and, consequently, each measuring apparatus as well. As it was explained in section \( 4 \), this is thoroughly compatible with CI. But in CI it is clearly seen that the universality of quantum theory is in a certain contradiction to its meaning as a theory of empirical science. Therefore from the CI viewpoint the idea of a wavefunction of the universe is a physically senseless extrapolation of the mathematical formalism. Is is, in principle, not forbidden to apply quantum theory to the world as a whole, but this would be no information for anybody since no subject is left. Thus, for any measurement the cut is a necessary precondition.

3. **Thesis.** According to step 2a: *The measuring apparatus must be suitable as such.* E.g., the pointer states must be orthogonal. Once a system is chosen to act as a measuring apparatus the decomposition of the density operator into the spectrum of the pointer variable is fixed. This is involved in the CI's central assumption of describing the measuring apparatus classically.

4. **Thesis.** According to step 2b: *Measurements must lead to irreversible facts.* Irreversibility in this context characterizes any documents of the past, e.g., pointer devices, printer outputs, computer memories, human brain states. Facticity means that a measurement can lead to one and only one outcome. Thus, facts are classical (at least FAPP).

5. **Thesis.** *‘Measurement’ is a necessary term in any empirical science.* It is related to information as the key concept of quantum theory. Experience means to learn from the facts of the past for the possibilities of the future. In empirical science this will be done by measurements. Thus, in empirical science the term ‘measurement’ is methodologically irreducible. Quantum physics, the hard core of empirical science, describes possibilities of the future in terms of potential information (*"\( \Psi \) is information*). Potential information is information which can be gained, i.e., can become actual, if a measurement is performed.

6. **Thesis.** A measurement represents the transition from potential to actual information. This thesis can mathematically be quantified. Quantum information is measured in terms of the entropy of the density operator \( S[\hat{\rho}] = -k_B \text{Sp}(\hat{\rho} \ln \hat{\rho}) \). A pure state contains no potential information, i.e., the initial as well as the final state of the chain of reduction represents \( S[\hat{P}] = 0 \) bit. During step 2 the potential information amounts \( S[\hat{\xi}] > 0 \) bits. Since a measuring act represents the transition from potential to actual information, the reduction is a presupposition of the measurement.

\(^5\text{Compare Heisenberg: "... wenn man das ganze Universum in das System einbezieht – dann ist ... die Physik verschwunden und nur noch ein mathematisches Schema geblieben" (Heisenberg 1950, p. 44), "... if the whole universe were to be included into the system then physics would vanish and just a mathematical scheme remains" (translation by the author).}
7. Thesis. The subject is an irreducible element in any empirical science. According to the logic of the transcendental argument subjects in empirical science must be equipped at least with conceptual and empirical competence – a "PhD" (Bell 1990, p. 34) is of course not a necessary precondition of experience! It should be noted that being a subject in this sense and being conscious is not necessarily the same. Thus, the assertion is not the reduction taking place in the consciousness of the observer as in the London-Bauer or Wigner approach, since in these approaches human consciousness is excluded from (quantum) physical description. But this contradicts the universality of quantum theory. The idea of the proposed transcendental approach is not to exclude anything from quantum theory, i.e. any subject can in principle be described by quantum theory – but then, of course, as an object for and from another subject. This is the key point of the argument: subjects can be described as objects, but not all of them at the same time! Physics without any subjects would be meaningless. In that sense the subject is irreducible.

8. Thesis. Physics is essentially FAPP, but ‘FAPPress’ is no sufficient explanation to the measurement problem. Objects are constituted by information which exists for subjects. Any (object) information presupposes a certain semantics under which the information can be understood. But for the same reason the information invested in the semantics needs other semantics before and so on. In a finite world, constituted by a huge but finite amount of information this leads to an inherent circularity (Lyke 1997). Therefore, empirical science is by no means as exact as its mathematical framework suggests. Strictly speaking there exist no isolated quantum objects. Quantum theory is a holistic theory which, in the empirical application, must necessarily be FAPP. But ‘FAPPress’ is no sufficient explanation to the measurement problem. It explains step 1, whereas my proposal is that step 2 should be seen under the transcendental approach.

All in all the measurement ‘problem’ could be soluble or even vanishes, if it is not seen as a problem on the intrinsic-physical level, i.e. described as a physical interaction process, but on the meta-level, i.e. seen on the basis of the methodological and epistemological presuppositions of physics. Thus, Bell’s list of bad words in fact appears to be a list of necessary terms of any empirical science – most of all the terms ‘measurement’ and ‘information’.

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6Recent research even shows the universality of quantum theory leading to problems of self-referentiality for inner observers. E.g., an inner observer A cannot distinguish between a pure state and a mixture of S ⊕ A, i.e. they are indistinguishable FAPP. Moreover, self-referentiality seems to be connected to incompleteness of quantum physics (Breuer 1997, Mittelstaedt 1998).
References


