Masses, Decays and Mixings of Gluonia in QCD

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This is a review of the estimate of the gluonia masses, decay and mixings from QCD spectral sum rules and
low-energy theorems. Some phenomenological maximal gluonium-quarkonium mixing schemes in the scalar sector
are presented. This talk is a compact version of the work in Ref.\cite{1}.

1. Introduction

In addition to the well-known mesons and
baryons, one of the main consequences of the non-
perturbative aspects of the QCD theory is the
possible existence of the gluon bound states (glu-
onia or glueballs) or/and of a gluon continuum.
Since the pioneer work of Fritzsch and Gell-Mann \cite{2}, a lot of theoretical and experimental
efforts have been devoted to study the gluonia
properties. In this talk, we present an update of
the predictions for the masses, decay constants
and mass-mixing from QCD spectral sum rules
(QSSR) \cite{3}. We also report some
low-energy theorem (LET) \cite{5} predictions for the
widths of unmixed scalar gluonia and present (al-
most) complete mixing schemes for explaining the
complicated spectra of the observed scalar reso-
nances below 2 GeV.

2. The gluonic currents

In this paper, we shall consider the lowest-
dimension gauge-invariant gluonic currents that
can be built from two gluon fields:
\begin{align*}
J_s &= \beta(\alpha_s)G_{\alpha\beta}G^{\alpha\beta}, \\
\theta_{\mu\nu}^s &= -\frac{1}{4}\theta_{\mu\nu}G^{\alpha\beta}G_{\alpha\beta}, \\
Q(x) &= \left(\frac{\alpha_s}{8\pi}\right)\text{tr} G_{\alpha\beta}G^{\alpha\beta},
\end{align*}

and three-gluon ones:
\begin{equation}
J_{3G} = g^3 f_{abc} G_{\alpha}^a G_{\beta}^b G_{\gamma}^c
\end{equation}

where the sum over colour is understood;
$\beta(\alpha_s)$ is the QCD $\beta$-function, while $\tilde{G}_{\mu\nu} \equiv (1/2)\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$, which have respectively the
quantum numbers of the $J^{PC} = 0^{++}$, $2^{++}$ and
$0^{++}$ gluonia for the two-gluon fields, and to the
$0^{++}$ one for the three-gluon fields. The former
two enter into the QCD energy-momentum tensor $\theta_{\mu\nu}$, while the third one is the U(1)$_A$ axial-

3. QCD spectral sum rules

The analysis of the gluonia masses and couplings
will be done using the method of QSSR. In so
doing, we shall work with the generic two-point
correlator:
\begin{equation}
\psi_G(q^2) \equiv i \int d^4 x \ e^{iqx} \langle 0 | T J_G(x) (J_G(0))^\dagger | 0 \rangle,
\end{equation}

built from the previous gluonic currents $J_G(x)$, which obeys the well-known Källen–Lehmann dis-

\begin{equation}
\psi_G(q^2) = \int_0^\infty \frac{dt}{t - q^2 - i\epsilon} \frac{1}{\pi} \text{Im} \psi_G(t) + ..., \quad (4)
\end{equation}

where $\ldots$ represent subtraction points, which are polynomials in the $q^2$-variable. This sum rule
expresses in a clear way the duality between the integral involving the spectral function $\text{Im} \psi_G(t)$
(which can be measured experimentally), and the
full correlator $\psi_G(q^2)$, which can be calculated
directly in QCD.

For a recent review on the sum rules, see e.g. \cite{4}.
3.1. The two-point correlator in QCD

In addition to the usual perturbative contribution from the bare loop, the non-perturbative contributions can be parametrized by the vacuum condensates of higher and higher dimensions in the Wilson expansion [3] ²:

\[ \psi_G(q^2) \simeq \sum_{D=0,2,4,...} \frac{1}{(-q^2)^{D/2}} \times \sum_{dimO=D} C^{(J)}(q^2, \nu)(O(\nu)), \quad (5) \]

provided that \(-q^2\) is much greater than \(\Lambda^2\); \(\nu\) is an arbitrary scale that separates the long- and short-distance dynamics; \(C^{(J)}\) are the Wilson coefficients calculable in perturbative QCD by means of Feynman diagrams techniques.

- The dominant condensate contribute in the chiral limit \(m_t = 0\) is due to the dimension-four gluonic condensate \(\langle \alpha_s G^2 \rangle\), introduced by SVZ [3], and which has been estimated recently from the \(e^+e^- \rightarrow I = 1\) hadron data [6] and from the heavy quark-mass splittings [7]:

\[ \langle \alpha_s G^2 \rangle \simeq (0.07 \pm 0.01) \text{ GeV}^4. \quad (6) \]

- The first non-leading contribution comes from the triple gluon condensate \(g f_{abc} \langle G^a G^b G^c \rangle\), whose direct extraction from the data is still lacking. We use its approximate value from the dilute gas instanton model [3]:

\[ g f_{abc} \langle G^a G^b G^c \rangle \approx (1.5 \pm 0.5) \text{ GeV}^2 \langle \alpha_s G^2 \rangle, \quad (7) \]

within a factor 2 accuracy.

- In addition to these terms, the UV renormalon and some eventual other effects induced by the resummation of the QCD series, and not included in the OPE, can contribute to the correlator as a term of dimension 2 [8]. We consider that their effects can be safely taken into account in the estimate of the errors from the last known term of the truncated perturbative series [3].

- It has also been argued [10], using the dilute gas approximation, that in the gluonia channels, instanton plus anti-instanton effects manifest themselves as higher dimension \((D = 11)\) operators. However, at the scale \((\text{gluonia scale})\) where the following sum rules are optimized, which is much higher than the usual case of the \(\rho\) meson, we can safely omit these terms ⁴, like any other higher-dimensional operators beyond \(D = 8\).

Through this paper, we shall use for three active flavours, the value of the QCD scale [11]:

\[ \Lambda = (375 \pm 125) \text{ MeV}. \quad (8) \]

3.2. The spectral function and its experimental measurement

It can be best illustrated in the case of the flavour-diagonal light quark vector current, where the spectral function \(\text{Im}\Pi(t)\) can be related to the \(e^+e^- \rightarrow I = 1\) hadrons data via the optical theorem as:

\[ \sigma(e^+e^- \rightarrow \text{hadrons}) = \frac{4\pi^2\alpha}{t} \frac{1}{\pi} \text{Im}\Pi(t), \quad (9) \]

or, within a vector meson dominance assumption, to the leptonic width of the \(\rho\) resonance:

\[ \Gamma_{\rho \rightarrow ee^-} \simeq \frac{2}{3} \pi \alpha^2 M_\rho \frac{1}{27\rho^2}, \quad (10) \]

via the meson coupling to the electromagnetic current:

\[ \langle 0|J^\mu(\rho) = \frac{M_\rho^2}{2\gamma_\rho} e^\mu. \quad (11) \]

More generally, the resonance contribution to the spectral function can be introduced, via its decay constant \(f_G\) analogous to \(f_x = 93.3\) MeV:

\[ \langle 0|J_G|G\rangle = \sqrt{2}f_G M_G^2 ..., \quad (12) \]

where ... represent the Lorentz structure of the matrix elements.

3.3. The form of the sum rules

The previous dispersion relation can be improved from the uses of an infinite number of derivatives and infinite values of \(q^2\), but keeping their ratio

²In the present analysis, we shall limit ourselves to the computation of the gluonia masses in the massless quark limit \(m_t = 0\).

³See e.g. the estimate of the errors in the determination of \(\alpha_s\) from \(\tau\) decays [9].

⁴Their quantitative estimate is quite inaccurate because of the great sensitivity of the result on the QCD scale \(\Lambda\), and on some other less controllable parameters and coefficients.
fixed as $\tau \equiv n/q^2$. In this way, one obtains the Laplace sum rules [3,12,13]:

$$\mathcal{L}_G(\tau) = \int_{t_c}^{\infty} dt \exp(-\tau t) \frac{1}{\pi} \text{Im}\psi_G(t),$$

(13)

where $t_c$ is the hadronic threshold. The advantage of this sum rule with respect to the previous dispersion relation is the presence of the exponential weight factor, which enhances the contribution of the lowest resonance and low-energy region accessible experimentally. For the QCD side, this procedure has eliminated the ambiguity carried by subtraction constants (arbitrary polynomial in $q^2$), and has improved the convergence of the OPE by the presence of the factorial dumping factor for each condensates of given dimensions. The ratio of sum rules:

$$\mathcal{R}_G \equiv \frac{d}{d\tau} \log \mathcal{L}_G,$$

(14)

or its slight modification, is a useful quantity to work with, in the determination of the resonance mass, as it is equal to the mass squared, in the simple duality ansatz parametrization:

$$\text{”one resonance”} \delta(t - M_\Delta^2) + \text{”QCD continuum”} \Theta(t - t_c),$$

(15)

of the spectral function, where the resonance enters by its coupling to the quark current; $t_c$ is the continuum threshold which is, like the sum rule variable $\tau$, an a priori arbitrary parameter.

3.4. Conservative optimization criteria

Different optimization criteria are proposed in the literature, which, to my opinion, complete one another, if used carefully. The sum rule window of SVZ is a compromise region where, at the same time, the OPE makes sense while the spectral integral is still dominated by the lowest resonance. This is indeed satisfied when the Laplace sum rule presents a minimum in $\tau$, where there is an equilibrium between the non-perturbative and high-energy region effects. However, this criterion is not yet sufficient as the value of this minimum in $\tau$ can still be greatly affected by the value of the continuum threshold $t_c$. The needed extra condition is to find the region where the result has also a minimal sensitivity on the change of the $t_c$ values ($t_c$ stability). The $t_c$ values obtained in this way are about the same as the one from the so-called heat evolution test of the local duality FESR [14]. However, in some cases, this $t_c$ value looks too high, compared with the mass of the observed radial excitation, and the procedure tends to overestimate the predictions. More precisely, the result obtained in this way can be considered as a phenomenological upper limit. Therefore, in order to have a conservative prediction from the sum rules method, one can consider the value of $t_c$ at which one starts to have a $\tau$-stability up to where one has a $t_c$ stability. In case there is no $t_c$ stability nor FESR constraint on $t_c$, one can consider that the prediction is still unreliable. In this paper, we shall limit ourselves to extracting the results satisfying the $\tau$ (Laplace) and $t_c$ stability criteria.

4. Masses and decay constants of the unmixed gluonia

The different expressions of the sum rules for each channels have been given in [1]. Applying the previous stability criteria, we obtain the spectra given in Table 1. Our results satisfy the mass hierarchy $M_S < M_P \approx M_T$, which suggests that the scalar is the lightest gluonium state as also expected from lattice calculations [15] and QCD inequalities [16]. However, the consistency of the different subtracted and unsubtracted sum rules in the scalar sector requires the existence of an additional lower mass and broad $\sigma$-meson coupled strongly both to gluons and to pairs of Goldstone bosons (similar to the $\eta'$ of the $U(1)_A$ channel), whose effects can be missed in a one-resonance parametrization of the spectral function, and in the present lattice quenched approximation. One should also notice that the values of $\sqrt{t_c}$, which are about the mass of the next radial excitations, indicate that the mass-splitting between

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5Also called Borel or exponential.
6QCD finite energy sum rules (FESR) [14] lead to equivalent results and complement the Laplace sum rules.
7As tested in the meson channels where complete data are available, this parametrization gives a good description of the spectral integral for the sum rule analysis.
8Many results in the literature on QCD spectral sum rules are obtained using only the first condition.
the ground state and the radial excitations is relatively much smaller (30%) than in the case of ordinary hadrons (about 70% for the ρ meson), such that one can expect rich gluonia spectra in the vicinity of 2–2.2 GeV, in addition to the ones of the lowest ground states. The upper bounds on the gluonium mass squared given in Table 1 have been obtained from the minimum (or inflexion point) of the ratios of sum rules, after using the positivity of the spectral functions.

5. Natures of the ζ(2.2) and E/υ(1.44)

- The ζ(2.2) is a good 2^{++} gluonium candidate because of its mass (see Table 1) and small width in ππ (≤ 100 MeV). However, the value of t_c can suggest that the radial excitation state is also in the 2 GeV region, which should stimulate further experimental searches.

- The E/υ (1.44) or other particles in this region [18] is too low for being the lowest pseudoscalar gluonium. One of these states are likely to be the first radial excitation of the η′ as its coupling to the gluonic current is weaker than the one of the η′ and of the gluonium (see Table 1).

6. Decay widths of the scalar gluonia

6.1. σ_B and σ'_B couplings to ππ

For this purpose, we consider the vertex:

\[ V(q^2) = \langle \pi_1 | \theta_{\mu}^a | \pi_2 \rangle, \quad q = p_1 - p_2, \]  

where: \( V(0) = 2m_\pi^2 \). In the chiral limit (\( m_\pi^2 \simeq 0 \)), the vertex obeys the dispersion relation:

\[ V(q^2) = \int_0^\infty \frac{dt}{t - q^2 - i\epsilon} \frac{1}{\pi} \text{Im} V(t), \]  

which gives the 1st NV sum rule [5]:

\[ \frac{1}{4} \sum_{S=\sigma_B,\sigma'_B,G} g_{S\pi\pi} \sqrt{2} f_S \simeq 0. \]  

Using the fact that \( V'(0) = 1 \) [19], one obtains the second NV sum rule:

\[ \frac{1}{4} \sum_{S=\sigma_B,\sigma'_B,G} g_{S\pi\pi} \sqrt{2} f_S/M_S^2 = 1. \]  

Identifying the G with the G(1.5 \sim 1.6) at GAMS (an almost pure gluonium candidate), we can neglect then its coupling to ππ, and deduce:

\[ g_{\sigma_B \pi\pi} \approx \frac{4}{\sqrt{2} f_{\sigma_B}} \left( 1 - M_{\sigma_B}^2 / M_{\sigma'_B}^2 \right) \]  

\[ g_{\sigma'_B \pi\pi} \approx g_{\sigma_B \pi\pi} \left( f_{\sigma_B} / f_{\sigma'_B} \right). \]  

Using \( M_{\sigma'_B} \approx 1.37 \text{ GeV} \), one can deduce the width into ππ (\( \pi^+\pi^- \) and 2π0) given in Table 2. Our result indicates the presence of gluons.

\[ \text{Table 1} \]

<table>
<thead>
<tr>
<th>J^{PC}</th>
<th>Name</th>
<th>Mass [GeV] Estimate</th>
<th>Upper Bound</th>
<th>f_G [GeV]</th>
<th>\sqrt{t_c} [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0^{++}</td>
<td>G</td>
<td>1.5 ± 0.2</td>
<td>2.16 ± 0.22</td>
<td>390 ± 145</td>
<td>2.0 \sim 2.1</td>
</tr>
<tr>
<td></td>
<td>\sigma_B</td>
<td>1.00 (input)</td>
<td>1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>\sigma'_B</td>
<td>1.37 (input)</td>
<td>600</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3G</td>
<td>3.1</td>
<td>62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2^{++}</td>
<td>T</td>
<td>2.0 ± 0.1</td>
<td>2.7 ± 0.4</td>
<td>80 ± 14</td>
<td>2.2</td>
</tr>
<tr>
<td>0^{--}</td>
<td>P</td>
<td>2.05 ± 0.19</td>
<td>2.34 ± 0.42</td>
<td>8 \sim 17</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>E/υ</td>
<td>1.44 (input)</td>
<td>7</td>
<td>J/ψ \rightarrow \gamma \nu</td>
<td></td>
</tr>
</tbody>
</table>
inside the wave functions of the broad $\sigma_B$ resonance below 1 GeV and of the $\sigma'(1.37)$, which can decay copiously into $\pi\pi$.

### 6.2. $G(1.5)$ coupling to $\eta'$

Analogous low-energy theorem (see NV) gives:

$$\langle \eta | \theta_\mu | \eta \rangle = 2M^2_{\eta},$$

where $\eta$ is the unmixed $U(1)$ singlet state of mass $M_\eta \simeq 0.76$ GeV [20]. Writing the dispersion relation for the vertex, one obtains the $\eta$ sum rule:

$$\frac{1}{4} \sum_{S=\sigma_B,\sigma'_B, G} g_{\eta \eta \eta} \sqrt{2} f_S = 2M^2_{\eta},$$

which, by assuming a $G$-dominance of the vertex sum rule, leads to:

$$g_{\eta \eta \eta} \approx (1.2 \sim 1.7) \text{ GeV}.\quad (23)$$

Introducing the “physical” $\eta'$ and $\eta$ through:

$$\eta' \sim \cos \theta \eta \eta - \sin \theta \eta \eta'$$
$$\eta \sim \sin \theta \eta \eta + \cos \theta \eta \eta',$$

where [21,22] $\theta \eta \eta \approx -(18 \pm 2)\degree$ is the pseudoscalar mixing angle, one obtains the width given in Table 2. The previous scheme is also known to predict (see NV and [23]):

$$r \equiv \frac{\Gamma_{\eta \eta \eta'}}{\Gamma_{\eta \eta}} \simeq 0.22, \quad g_{\eta \eta' \eta} \approx \sin \theta \eta \eta' g_{\eta \eta'},\quad (25)$$

which leads to the width of 60-138 MeV, much larger than the one into $\eta \eta$ and $\eta' \eta$ in Table 2. This feature seems to be satisfied by the states seen by GAMS and Crystal Barrel. Our previous approaches show the consistency in interpreting the $G(1.6)$ seen at GAMS as an “almost” pure

### Table 2

Unmixed scalar gluonia and quarkonia decays

<table>
<thead>
<tr>
<th>Name</th>
<th>Mass [GeV]</th>
<th>$\pi^+ \pi^-$ [GeV]</th>
<th>$K^+ K^-$ [MeV]</th>
<th>$\eta\eta$ [MeV]</th>
<th>$\eta'\eta'$ [MeV]</th>
<th>$(4\pi)\gamma$ [MeV]</th>
<th>$\gamma\gamma$ [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gluonia</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_B$</td>
<td>0.75 $\sim$ 1.0</td>
<td>0.2 $\sim$ 0.5</td>
<td>$SU(3)$</td>
<td>$SU(3)$</td>
<td>$\sim 0.2$</td>
<td>$\sim 0.3$</td>
<td></td>
</tr>
<tr>
<td>$\sigma'_B$</td>
<td>1.37</td>
<td>0.5 $\sim$ 1.3</td>
<td>$SU(3)$</td>
<td>$SU(3)$</td>
<td>43 $\sim$ 316</td>
<td>0.7 $\sim$ 1.0</td>
<td></td>
</tr>
<tr>
<td>$G$</td>
<td>1.5</td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
<td>1.1 $\sim$ 2.2</td>
<td>5 $\sim$ 10</td>
<td>60 $\sim$ 138</td>
<td>0.2 $\sim$ 1.8</td>
</tr>
<tr>
<td>Quarkonia</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_2$</td>
<td>1</td>
<td>0.12</td>
<td>$SU(3)$</td>
<td>$SU(3)$</td>
<td>0.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S'_2$</td>
<td>1.3 $\sim \pi'$</td>
<td>0.30 $\pm$ 0.15</td>
<td>$SU(3)$</td>
<td>$SU(3)$</td>
<td>4 $\pm$ 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_3$</td>
<td>1.47 $\pm$ 0.04</td>
<td>73 $\pm$ 27</td>
<td>15 $\pm$ 6</td>
<td>0.4 $\pm$ 0.04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S'_3$</td>
<td>$\approx 1.7$</td>
<td>112 $\pm$ 50</td>
<td>$SU(3)$</td>
<td>1.1 $\pm$ 0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Footnote: The decays of the physically observed states will be discussed later on.

11The decays of the physically observed states will be discussed later on.

compared with the GAMS data [18] $r \simeq 0.34 \pm 0.13$, and which implies the width $\Gamma_{\eta \eta}$ in Table 2. This result can then suggest that the $G(1.6)$ seen by the GAMS group is a pure gluonium, which is not the case of the particle seen by Crystal Barrel [18] which corresponds to $r \approx 1$.

### 6.3. $\sigma_B(1.37)$ and $G(1.5)$ couplings to $4\pi$

Within our scheme, we expect that the $4\pi$ are mainly $S$-waves initiated from the decay of pairs of $\sigma_B$. Using:

$$\langle \sigma_B | \theta_\mu | \sigma_B \rangle = 2M^2_{\sigma_B},\quad (26)$$

and writing the dispersion relation for the vertex, one obtains the sum rule:

$$\frac{1}{4} \sum_{i=\sigma_B,\sigma'_B, G} g_{\sigma \sigma \sigma} \sqrt{2} f_S = 2M^2_{\sigma_B}.\quad (27)$$

We identify the $\sigma'_B$ with the observed $f_0(1.37)$, and use its observed width into $4\pi$, which is about (46 $\sim$ 316) MeV [21,18] ($S$-wave part). Neglecting, to a first approximation, the $\sigma_B$ contribution to the sum rule, we can deduce:

$$g_{G\sigma \sigma_B} \approx (2.7 \sim 4.3) \text{ GeV},\quad (28)$$

and which leads to the width of 60-138 MeV, much larger than the one into $\eta \eta$ and $\eta' \eta$ in Table 2. This feature seems to be satisfied by the states seen by GAMS and Crystal Barrel. Our previous approaches show the consistency in interpreting the $G(1.6)$ seen at GAMS as an “almost” pure
the "constituent" quark mass, which we shall take to be $\eta_f$ quarkonia width: $\Gamma(\eta_f)$ smaller (as expected) than the well-known $\eta_c$. From which we deduce the couplings $\sigma_1^q$ with the scalar-$L_0$ and the fact that its RHS is $O(10^{-3})$.

6.5. $J/\psi \to \gamma S$ radiative decays

As stated in [19], one can estimate this process, using dispersion relation techniques, by saturating the spectral function by the $J/\psi$ plus a continuum. The glue part of the amplitude can be converted into a physical non-perturbative matrix element $\langle 0|\sigma_1 G^2 |S\rangle$ known through the decay constant $f_S$ estimated from QSSR. By assuming that the continuum is small, one obtains [14]:

$$\Gamma(J/\psi \to \gamma S) \approx \frac{\alpha^3 \pi}{\beta_f^2 \delta_{656100} 100} \left( \frac{M_{J/\psi}}{M_c} \right)^4 \left( \frac{M_S}{M_c} \right)^4 \left( 1 - \frac{M_S^2}{M_{J/\psi}^2} \right)^3 \frac{f_S^2}{\Gamma(J/\psi \to e^+e^-)}.$$

This leads to (in units of $10^{-3}$) [15]:

$$B(J/\psi \to \gamma S) \times B(S \to all) \approx 0.4 \sim 1.$$ (36)

for $S \equiv \sigma_B, \sigma_B', G$. These branching ratios can be compared with the observed $B(J/\psi \to \gamma f_2) \approx 1.6 \times 10^{-3}$. The $\sigma_B$ could already have been produced, but might have been confused with the $\pi\pi$ background. The "pure gluonium" $G$ production rate is relatively small, contrary to the naive expectation for a glueball production. In our approach, this is due to the relatively small value of its decay constant, which controls the non-perturbative dynamics. Its observation from this process should wait for the $\tau$CF machine. However, we do not exclude the possibility that a state resulting from a quarkonium-gluonium mixing may be produced at higher rates.

7. Properties of the scalar quarkonia

7.1. Mass and decay constants

We shall consider the $SU(2)$ singlet $S_2(uu + dd)$ and the $SU(3)$ $S_3(\bar{s}s)$ states. We consider the former state as the $SU(2)$ partner of the $a_0(0.98)$ associated to the divergence of the charged vector current of current algebra:

$$\partial_\mu V^\mu(x) \equiv (m_u - m_d + \bar{u}(i\gamma_5)d).$$ (37)

12$F^{\mu\nu}$ is the photon field strength, $Q_\eta$ is the quark charge in units of $e$, $-\beta_1 = 9/2$ for three flavours, and $m_\eta$ is the "constituent" quark mass, which we shall take to be $m_u \approx m_c \approx M_{J/\psi}/2$, $m_s \approx M_f/2$.

13Here and in the following, we shall use $M_{\sigma_B} \approx (0.75 \sim 1.0)$ GeV.

14We use $M_c \approx 1.5$ GeV for the charm constituent quark mass and $-\beta_1 = 7/2$ for six flavours.

15From the previous results, one can also deduce the corresponding stickiness defined in [25].
We expect from the good realization of the $SU(2)$ symmetry that they are degenerate in mass, where we shall use the QSSR prediction [4]:

$$M_{a_0} \approx (1 \sim 1.05) \text{ GeV},$$

(38)

in good agreement with the observed $a_0$ mass. The continuum threshold at which the previous prediction has been optimized can roughly indicate the mass of the next radial excitation, which is about the $f_0(1.37)$ mass [4]:

$$M_{S_2^*} \approx \sqrt{f_c} \approx (1.1 \sim 1.4) \text{ GeV} \approx M_{c'}. \quad (39)$$

In order to compute the mass of the $S_3(\bar{s}s)$ state, we work with the double ratio of Laplace transform sum rules:

$$\frac{R_{\bar{s}s}}{R_{\bar{s}s}} \approx \frac{M_{\bar{s}s}^2}{M_{K^*_0(1.43)}^2}, \quad (40)$$

where $R_{\bar{s}s}$ has been defined in Eq. (14) and corresponds to the two-point correlator:

$$\psi_{\bar{s}s}(q^2) \equiv i \int d^4x e^{i\bar{s}s} \langle 0|TJ_{\bar{s}s}(x)(J_{\bar{s}s}(0))^\dagger|0\rangle, \quad (41)$$

associated to the scalar current:

$$J_{\bar{s}s}(x) = (m_\bar{s} + m_s)\bar{s}s, \quad q = d, s. \quad (42)$$

At the stability point, one obtains\(^\text{16}\):

$$\frac{M_{\bar{s}s}/M_{K^*_0(1.43)}}{\approx 1.03 \pm 0.02} \Rightarrow \frac{M_{\bar{s}s}}{M_{s}s} \approx (1474 \pm 44) \text{ MeV}, \quad (43)$$

confirming the earlier QSSR estimate in [4]. The result indicates the mass hierarchy:

$$M_{S_2=\bar{s}s+dd} < M_{K^*_{\bar{s}s}} < M_{\bar{s}s}. \quad (44)$$

The $SU(3)$ breaking obtained here is slightly larger than the naive expectation, as in addition to the strange-quark mass effect, the $\langle \bar{s}s \rangle$ condensate also plays an important role in the splitting.

### 7.2. Hadronic and $\gamma\gamma$ widths

- The hadronic and electromagnetic couplings of the lowest ground states $S_2$ and $S_3$ have been estimated using vertex sum rules. The $S_2$ coupling to pair of pions in the chiral limit is [27]:

$$g_{S_2\pi+\pi-} \approx \frac{16\pi^3}{3\sqrt{3}} \langle \bar{u}u \rangle \tau e^{M_{S_2}^2 \frac{\pi^2}{3}} \simeq 2.46 \text{ GeV}, \quad (45)$$

\(^{16}\)We have used $\langle \bar{u}u \rangle (1 \text{ GeV}) \simeq (150 \sim 190) \text{ MeV}$ correlated to the values of $\Lambda$ [26].

for the typical value of $\tau \approx 1 \text{ GeV}^{-2}$, in good agreement with the $SU(3)$ expectations. We thus deduce the width in Table 2. Using $SU(3)$ symmetry, one can also expect:

$$g_{S_2K^++K^-} \approx \frac{1}{2} g_{S_2\pi+\pi-}. \quad (46)$$

Analogous analysis for the $\gamma\gamma$ width leads to the predictions in Table 2.

- The estimates of the $\gamma\gamma$ and hadronic widths of the radial excitations $S_2^*$ and $S_3^*$ are more uncertain. In so doing, we use the phenomenological observations that the coupling of the radial excitation increases as the ratio of the decay constants $r \equiv f_{S_2}/f_{S_3}$. Therefore, we expect:

$$\frac{\Gamma(S_2^* \rightarrow \gamma\gamma (\pi\pi))}{\Gamma(S_2 \rightarrow \gamma\gamma (\pi\pi))} \approx r^2 \left( \frac{M_{S_2^*}}{M_{S_2}} \right)^{3(1)}, \quad (47)$$

which, by taking $r \approx (M_{S_2^*}/M_{S_2})^{(n=2-1)}$, like in the pion and $\rho$ meson cases [4] gives the result in Table 2.

- To a first approximation, we expect that the decay of the $S_2^*$ into $4\pi$ comes mainly from the pair of $\rho$ mesons, while the one from $\sigma\rho\sigma\rho$ (gluonia) is relatively suppressed like $\alpha^2_{\sigma}$ using perturbative QCD arguments.

### 8. Gluonium-quarkonium mass mixings

This quantity can be obtained from the QSSR analysis of the off-diagonal quark-gluon two-point correlator. It has been obtained for different channels [28,17,27]. The results show that the mixing angle is tiny (less than 12\(^\circ\)) and justify a posteriori that the masses of the observed gluonia are approximately given by the theoretical estimate of the gluonia masses obtained with taking into account a such term. Note that the mass-mixing between the 3- and 2-gluon bound states is also small [29].

### 9. “Mixing-ology” for the decay widths of scalar mesons

In the following, we shall be concerned with the mixing angle for the couplings, which, in the same approach, is controlled by the off-diagonal non-perturbative three-point function which can (a
priori) give a large mixing angle. However, a QCD evaluation of this quantity is quite cumbersome, such that in the following, we shall only fix the decay mixing angle from a fit of the data.

9.1. Mixing below 1 GeV and the nature of the $\sigma$ and $f_0(0.98)$

We consider that the physically observed $f_0$ and $\sigma$ states result from the two-component mixing of the $\sigma_B$ and $S_2 \equiv \frac{1}{\sqrt{2}} (uu + dd)$ unmixed bare states:

$$|f_0\rangle \equiv -\sin \theta_S |\sigma_B\rangle + \cos \theta_S |S_2\rangle$$

$$|\sigma\rangle \equiv \cos \theta_S |\sigma_B\rangle + \sin \theta_S |S_2\rangle.$$  

(48)

Using the prediction: $\Gamma(\sigma_B \rightarrow \gamma\gamma) \simeq (0.2 \sim 0.3)$ keV, and the experimental width $\Gamma(f_0 \rightarrow \gamma\gamma) \approx 0.3$ keV, one obtains [17]:

$$\theta_S \approx (40 \sim 45)^\circ,$$  

(49)

which indicates that, in this scheme, the $f_0$ and $\sigma$ have a large amount of gluons in their wave functions. This situation is quite similar to the case of the $\eta'$ in the pseudoscalar channel (mass given by its gluon component, but strong coupling to quarkonia). Using the previous value of $\theta_S$, the predicted value of $g_{S_2KK}$, the approximate relation $g_{S_2KK} \simeq \frac{1}{2} g_{S\pi\pi}$, and the almost universal coupling of the $\sigma_B$ to pairs of Goldstone bosons, one can deduce (in units of GeV):

$$g_{f_0\pi^+\pi^-} \simeq (0.1 \sim 2.6),$$

$$g_{f_0sKK} \simeq -(1.3 \sim 4.1)$$

$$g_{\sigma\pi^+\pi^-} \simeq g_{\sigma sKK} \simeq (4 \sim 5),$$  

(50)

which can provide a simple explanation of the exceptional property of the $f_0$ (strong coupling to $KK$ as observed in $\pi\pi$ and $KK$ data [21]), without appealing to the more exotic four-quark and $KK$ molecules natures of these states [17]. Using the previous predictions for the couplings, and for $\theta_S$, we obtain the results in Table 3.

9.2. Nature of the $f_0(1.37)$

Among the observed widths of the $f_0(1.37)$, we shall mainly be concerned with the ones into $\gamma\gamma$ and $(4\pi)_S$ [21,18] showing that the $f_0(1.37)$ has amusingly the combined properties of the scalar quarkonium $S'_2$ from its $\gamma\gamma$ width and of scalar gluonium $\sigma'_B$ from its decay into $(4\pi)_S$ through the pair of $\sigma$ states.

9.3. 3x3 mixing and nature of the $f_0(1.5)$

In order to explain the nature of the $f_0(1.5)$, we need to consider the 3x3 mixing matrix in Eq. (51). The mixing angles in the first line of the matrix have been fixed by using the negligible $KK$ and $\pi\pi$ widths of the $f_0(1.37)$ given in Table 3. For the second line of the matrix, we use the observed width of the $f_0(1.5)$ into $\pi\pi$. The first (resp. second) numbers in the matrix correspond to the case of large (resp. small) widths from the data. From the previous schemes, we deduce the predictions in Table 3 [18]. The orthogonal $f_0(1.6)$ state is too broad for being considered as a resonance [19]. Despite the crude approximation used and the inaccuracy of the predictions, these results are in good agreement with the data (especially from the Crystal Barrel collaboration), and suggest that the observed $f_0(1.37)$ and $f_0(1.5)$ come from a maximal mixing between the gluonia ($\sigma'_B$ and $G$) and the quarkonium $S_3$ states. The mixing of the $S_3$ and $G$ with the quarkonium $S'_2$, which we have neglected compared with the $\sigma'_B$, can restore the small discrepancy with the data. One should notice, as already mentioned, that the state seen by GAMS is more likely the unmixed gluonium state $G$ (dominance of the $4\pi$ and $\eta\eta'$ decays, as emphasized earlier in NV), which can be due to some specific features of the production at the GAMS experiment, but not present in the Crystal Barrel and Obelix ones.

9.3.1. Nature of the $f_J(1.71)$

The narrow $f_J(1.7)$ observed to decay into $KK$ with a width of the order $(100 \sim 180)$ MeV can be essentially composed by the radial excitation $S'_3(1.7 \sim 2.4)$ GeV of the $S_3(\bar{ss})$, as they have about the same width into $KK$ (see Table 2). This feature can also explain the smallness of the

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[17] A QSSR analysis of the $a_0(0.98)$ within a four-quark scheme leads to too low a value of its $\gamma\gamma$ width as compared with the data [30].
Table 3
Predicted decays of the observed scalar mesons

<table>
<thead>
<tr>
<th>Name</th>
<th>$\pi^+\pi^-$ [MeV]</th>
<th>$K^+K^-$ [MeV]</th>
<th>$\eta\eta$ [MeV]</th>
<th>$\eta'\eta'$ [MeV]</th>
<th>$(4\pi^0)S$ [MeV]</th>
<th>$\gamma\gamma$ [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0(0.98)$</td>
<td>$0.2 \sim 134$</td>
<td>Eq. (50)</td>
<td></td>
<td>$\approx 0.3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(0.75 \sim 1)$</td>
<td>$300 \sim 700$</td>
<td>Eq. (50)</td>
<td>$SU(3)$</td>
<td>$0.2 \sim 0.5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_0(1.37)$</td>
<td>$22 \sim 48$</td>
<td>$\approx 0$</td>
<td>$\leq 1.$</td>
<td>$\leq 2.5$</td>
<td>$150$</td>
<td>$\leq 2.2$</td>
</tr>
<tr>
<td>$f_0(5.5)$</td>
<td>$25$</td>
<td>$3 \sim 12$</td>
<td>$1 \sim 2$</td>
<td>$68 \sim 105$</td>
<td>($exp$)</td>
<td>$\leq 1.6$</td>
</tr>
<tr>
<td>$f_J(1.71)$</td>
<td>$\approx 0$</td>
<td>$112 \pm 50$</td>
<td>$SU(3)$</td>
<td>$\approx 0$</td>
<td>$1.1 \pm 0.5$</td>
<td></td>
</tr>
</tbody>
</table>

The $f_J(1.7)$ width into $\pi\pi$ and $4\pi$. Our predictions of the $f_J(1.71)$ width can agree with the result of the Obelix collaboration [18], while its small decay width into $4\pi$ is in agreement with the best fit of the Crystal Barrel collaboration [18], which is consistent with the fact that the $f_0(1.37)$ likes to decay into $4\pi$. However, the broad $f_0(1.6)$ and the $f_J(1.71)$ can presumably interfere destructively for giving the dip around $1.5 \sim 1.6$ GeV seen in the $\bar{K}K$ mass distribution from the Crystal Barrel and $pp$ annihilations at rest.

9.3.2. Comparison with other scenarios

Though the relative amount of glue for the $f_0(1.37)$ and $f_0(1.5)$ is about the same here and in [31], one should notice that, in our case, the $\pi\pi$ partial width of these mesons come mainly from the $\sigma_B$, a glue state coupled strongly to the quark degrees of freedom, like the $\eta'$ of the $U(1)_A$ anomaly, while in [31], the $S_2$ which has a mass higher than the one obtained here plays an essential role in the mixing. Moreover, the $f_J(1.71)$ differs significantly in the two approaches, as here, the $f_J(1.71)$ is mainly the $\bar{s}s$ state $S_2$, while in [31], it has a significant gluon component. In the present approach, the eventual presence of a large gluon component into the $f_J(1.71)$ wave function can only come from the mixing with the broad $f_0(1.6)$ and with the radial excitation of the gluonium $G(1.5)$, which mass is expected to be around 2 GeV as suggested by the QSSR analysis. However, the apparent absence of the $f_J(1.71)$ decay into $4\pi$ from Crystal Barrel data may not favour such a scenario.

10. Conclusions

We have reviewed:

- The QCD spectral sum rule (QSSR) predictions of the masses and decay constants of gluonia, and given some interpretations of the nature of the observed $\zeta(2.2)$ and $E_4/\iota(1.44)$ mesons (Table 1).
- Some low energy theorems (LET) and vertex sum rule estimates of the widths of the scalar gluonia quarkonia (Table 2).
- Some maximal quarkonium-gluonium mixing schemes, for explaining the complex structure and decays of the observed scalar mesons (Table 3).

The good agreements between the theoretical predictions and the data are encouraging.

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25. M.S. Chanowitz, Proc. of the VI Int. Workshop on $\gamma\gamma$ collisions, (WSC 1984).


DISCUSSIONS

M. Loewe, Santiago (Chili)

What are the values of the condensates you used in your estimation of the masses of gluonium states?

S. Narison

I use the new values of the gluon condensates (see text) obtained from the $e^+e^-$ data ($\tau$-like sum rule) and the heavy quark-mass splittings.

G. Schuler, CERN (Geneva)

You predict a 3-gluon state at 3.1 GeV:

i) Do you have any ideas about its decay and where one can look for it?

ii) Can this state any of the $J/\psi$ decays?

S. Narison

i) This scalar state can preferably decay into the $U(1)$ channels $\eta'\eta'$, $\eta'\eta$ and $\eta\eta$. It can also decay into $4\pi$ in a S-wave through the pair of broad scalar state $\sigma$. Its hadronic coupling can be suppressed by a factor $\alpha_s$ relatively to the analogue scalar gluonium $f_0(1.5)$ formed by two gluons. If, it also couples weakly to $\pi\pi$ and $KK$, its experimental detection will be difficult.

ii) I do not think so, as, in addition, its production is strongly suppressed by phase space factors.