Electron Capture in an Electron Plasma Wave *

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Abstract

We study of the influence of the radial electric field of a plasma wave on the accelerated electrons

In the following paper, we study the influence of the radial electric field of an electron plasma wave (EPW) on the electrons that are accelerated by the EPW. The laser beam has a gaussian profile, and is not guided. The electric field of the EPW is computed in the linear approximation.

(De)focusing

The expressions for the radial and longitudinal electric field of the (EPW) are given in ref.[1]

\[ E_r = -\frac{4r}{w^2} \exp \left( -\frac{2r^2}{w^2} \right) \cos (\omega_p t - k_p z) A \] (1)

\[ E_z = -k_p \exp \left( -\frac{2r^2}{w^2} \right) \sin (\omega_p t - k_p z) A \] (2)

with \( A = \sqrt{\pi} \omega_p \tau_0 \exp \left( -\frac{\omega_p^2 \tau_0^2}{4} \right) I_{\text{max}} e^{(2\tau_0 m_c \omega^2)} \). \( I_{\text{max}} \) is the maximum laser intensity. The two fields are phase-shifted by \( \pi/2 \). We get a four-fold segmentation of the phase axis:

<table>
<thead>
<tr>
<th>( \varphi )</th>
<th>0</th>
<th>( \pi/2 )</th>
<th>( \pi )</th>
<th>( 3\pi/2 )</th>
<th>2( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>accel.</td>
<td>accel.</td>
<td>decel.</td>
<td>decel.</td>
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</tr>
<tr>
<td></td>
<td>defoc.</td>
<td>foc.</td>
<td>foc.</td>
<td>defoc.</td>
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</tbody>
</table>

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As $E_r$ is proportional to $r$ for small $r$, each slice of the plasma wave is equivalent to a lens. The focusing strength of the electrons is

$$K_r = -\frac{F_r}{\gamma mc^2 r} = -\frac{2\sqrt{\pi}}{\gamma} \frac{a_0^2}{u_0^2} \exp\left(-\frac{2r^2}{u_0^2}\right) \cos(\omega_{ph} r - k_r z) \frac{1}{[1 + (z/z_0)^2]^2}$$

(3)

At low field, $r$ is not modified during the way through the plasma wave, and we get the focal length of the EPW by integration $f_r = 1/ \int K_r \, dz$. For constant $\gamma$, i.e. $\varphi = (0, \pi)$, for which the radial force takes its maximum, we have then:

$$|f_r| = \frac{e\gamma_0 u_0^3}{2a_0^2 \sqrt{\pi}} \frac{1}{z_0} \frac{1}{\int [1 + u^2]^2} = \frac{\gamma_0}{2\pi^2 \delta} \lambda,$$

(4)

where $\delta$ is the amplitude of the EPW. For the wakefield experiment at Ecole Polytechnique [2] with an injection at 3 MeV, the focal length of the EPW is very short, $f_r = 0.24 \mu m/\delta$.

The approximation is valid as long as $r$ is not modified during the propagation through the radial field, that is $f_r > z_0$. This condition is satisfied when $\delta$ is smaller that a limit value $\delta_l = \gamma_0 \lambda/(2\pi^2 z_0)$. For a 80 mm diameter beam focused with a 1.4 m lens[2], we obtain $z_0 = 411 \mu m$ and $\delta_l = 7.6 \times 10^{-4}$.

![Fig. 1. Absolute value of the focal length in $\lambda_p$ units, for an injection at $\varphi = 0$ (solid curve), $\varphi = \pi$ (dashed-dotted), and in the low amplitude approximation (eq. 4, dashed). The values of the parameters of ref. [2] were used ($\lambda_p = 200 \mu m$).](image-url)
Indeed for $\delta < \delta_c$, the focal length is well described by eq. 4. (fig 1).

For higher values of the amplitude and a defocusing field ($\varphi = 0$), $r$ increases rapidly, and that makes the radial force increase more. The electron is expelled well before the waist, and the corresponding value of the focal length is very small.

In a LWF acceleration experiment, one is mainly interested in electrons that undergo a maximum longitudinal electric field along the main part of their path in the EPW. The radial field is then minimum. Furthermore $\gamma$ increases during propagation, and the focusing strength decreases. At last, the electrons that reach the focal plane with the optimal phase wrt to the field see the radial field changing sign, so that the integral cancels. For these electrons, the actual value of the focal length is therefore much larger than the one given by eq.4.

**Betatron oscillations; Electron guiding by the EPW**

Figure 2 presents the variation of $f_r$ with the injection phase $\varphi$ for several values of $\delta$. The numerical integration described in ref. [3] was used\(^1\). At low amplitude, we find again a variation as $-1/\cos \varphi$. At higher amplitude, and for a defocusing field $\varphi \in [-\pi/2, \pi/2]$, we have a very short negative focal length: the EPW blows up the beam before the waist. For $\varphi \in [\pi/2, 3\pi/2]$, we have long focal lengths with alternating sign: this is the indication of betatron oscillations of the electrons inside the EPW.

The wave-length $\Lambda$ of the betatron oscillation is $\Lambda = 2\pi/\sqrt{K_r}$. We can call “adiabatic” the behaviour of the electrons in the EPW when $\Lambda/2\pi$ is shorter than the characteristic length describing the longitudinal evolution of the EPW, that is than $z_0$.

This is the case as long as the EPW amplitude is larger than a critical value $\delta_c = \gamma o^2/4$, where $o$ is the aperture of the focusing optics. With the values of the parameters given above, we have $\delta_c = 1.2 \times 10^{-3}$. Indeed, the “betatron” regime is reached between $\delta = 0.001$ and $\delta = 0.01$ (fig. 2). Note that $\delta_c = \pi \delta_l/2$.

We re-write the focusing strength as $K_r(z) = \delta(z)/\left(\delta_c z_0^2\right)$.

The number of betatron oscillations, i.e. the phase advance, is $Q = \int dz/\Lambda$, that is

$$Q = \int \frac{dz}{\Lambda} = \frac{1}{2\pi} \int \sqrt{K_r} dz = \frac{1}{2\pi} \sqrt{\frac{\delta_0}{\delta_c}} \int \frac{du}{(1+u^2)^{1/2}}$$

\(^1\) As this phase actually evolves while the electron is moving forward into the EPW, we must define it in an univocal way. We define it as the phase the electron would have at the focal plane if its movement had not been affected by the wave.

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Fig. 2. Absolute value of the focal length in $\lambda_p$ units as a function of $\varphi$, for several values of $\delta$. The sign of $f_r$ changes at each divergence.

Note that $\Lambda \propto 1/\sqrt{\delta}$, that is, for high $z$, $\Lambda \propto z$ : the integral diverges. Actually the integration bounds are approximatively determined by the adiabaticity break-down, $\Lambda/2\pi \approx z_0$, that is $\delta \approx \delta_c$, that is $u = \sqrt{\delta_0/\delta_c} - 1$. $Q$ is given approximatively by:

$$Q = \frac{1}{\pi} \sqrt{\frac{\delta_0}{\delta_c}} \log \left[ \left( \frac{\delta_0}{\delta_c} - 1 \right)^{1/2} + \left( \frac{\delta_0}{\delta_c} \right)^{1/2} \right]$$

The estimation of $Q$ given by eq. 6 is compared to the value observed in figure 2 in table 1 (the number of divergences of $f_r$ is equal to $2Q + 2$), with a satisfying agreement.

We notice on figure 2 that on the plateau, the focal length takes minimum values close to $|f_r|/\lambda_p = 4 \pm 1$, and it does not depend on $\delta$ nor on $\gamma$. In the high amplitude regime, the output angle is of the order of the paraxial angle at adiabaticity break-down. We then get approximatively $|f_r| \approx z_0$, and here $|f_r|/\lambda_p \approx 2.1$. 

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Table 1
Variation of $Q$ with $\delta$. The value $\gamma$ used in eq. 6 is $\gamma_0 + \Delta \gamma_{\text{max}}/2$, close to the actual value at the focal plane.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\gamma$</th>
<th>$\delta_e$</th>
<th>“mesured” number</th>
<th>$Q$</th>
<th>$4Q + 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.4</td>
<td>5.3 $10^{-3}$</td>
<td>44</td>
<td>14.4</td>
<td>59.6</td>
</tr>
<tr>
<td>0.1</td>
<td>7.9</td>
<td>1.6 $10^{-3}$</td>
<td>16</td>
<td>6.9</td>
<td>29.7</td>
</tr>
<tr>
<td>0.01</td>
<td>6.1</td>
<td>1.2 $10^{-3}$</td>
<td>6</td>
<td>1.55</td>
<td>8.2</td>
</tr>
<tr>
<td>0.001</td>
<td>5.89</td>
<td>1.2 $10^{-3}$</td>
<td>2</td>
<td>0.</td>
<td>2</td>
</tr>
</tbody>
</table>

Radial extension of the linear region

The paraxial approximation $\theta_{\text{out}} = r_{\text{in}}/f_r$ is only valid close to the axis, at low $r_{\text{in}}$. For $\varphi$ close to $\pi/2$ (on the plateau) and $\delta = 1$, we “observe” that the linear region ranges $r_{\text{in}} < 0.05 - 0.1 \lambda_p = 10 - 20 \mu m$. But the final energy of the accelerated electrons is not degraded on a larger range $r_{\text{in}} \approx (0.25 - 0.35) \lambda_p = 50 - 70 \mu m$.

This is strange, because the laser beam has a narrow radial extent, with a RMS width of $\sigma_0 = u_0/2 = 6 \mu m$. Actually the EPW captures the electron very early, far from the waist, in a region where $\delta \approx \delta_e$, that is $z \approx z_0 \sqrt{\delta/\delta_e}$, here $z \approx 29 z_0 = 12 \text{mm}$. There, the radial extension of the EPW is $\sigma \approx \sigma_0 z/z_0$, that is $\sigma \approx 29 \sigma_0 = 170 \mu m$.

During capture, the size of the electron beam is strongly reduced in the high intensity region, by one order of magnitude with our parameters (fig. 3). This is due to two cumulating factors:

- first, $\sigma \sim \sqrt{\beta \epsilon_e}$, and $\beta \sim 1/\sqrt{K_r}$, so that $\sigma \sim K_r^{-1/4}$, $\epsilon_e$ being the RMS emittance of the electron beam. The associated reduction factor is $(\delta_0/\delta_e)^{1/4} = 5.4$ (“observed” 5).
- second, $\sigma \sim \sqrt{\epsilon_e} \sim \sqrt{\epsilon N/\gamma}$; $\gamma$ at focus is close to $\gamma_0 + (\Delta W/mc^2)/2 = 24.6$.

The associated reduction factor is $\sqrt{\gamma/\gamma_0} = 2.04$, (“observed” 2.).

Capturing emittance

The region of the transverse phase space in which the electrons can gain more than half their maximum energy gain is presented in figure 4. When the coordinates of the incoming electrons are referenced at the focal plane (left, $z = 0$), we notice a strong positive correlation between them. This is the indication of a capture that takes place at about $10 z_0$ upstream from the waist (right, $z = -10 z_0$).

The capturing emittance in figure 4 is 2.4 mm.mrad.
Fig. 3. Variation of \( \gamma \) and of the radial position \( r \) of an electron injected parallel to the axis along its way through the EPW.

**Beam quality**

For a LWA accelerator, we may want that the EPW does not disturb the accelerator optics. This means that the focal length of the EPW must be larger than the Rayleigh length of the electron beam, \( f_r > \beta^* \), and also that the normalised emittance \( \epsilon_N \) is conserved.

Actually we have seen that on a given small segment of the phase axis, the value of the focal length varies strongly in amplitude an in sign. Therefore the conservation of the normalised emittance also requires that \( f_r > \beta^* \).

**Conclusion**

Finally the transverse electric field of the EPW has a rather positive action on the electron beam, providing a capture and a guiding of the accelerated electrons, on one fourth of the phase segment (fig. 5).

The normalised emittance of the captured beam can be conserved in the pro-
Fig. 4. Region of the transverse phase space of the incident electrons for which $\Delta \gamma > \Delta \gamma_{\text{max}}/2$. Left: reference at $z = 0$. Right: reference at $z = -10z_0$.

Fig. 5. Energy spectrum and its phase dependence, with $\alpha = 1$, and an electron beam emittance as used in [2] with $\sigma = 20\mu m$ and $\sigma' = 10\text{mrad}$ [4].

cess, provided the Rayleigh length $z_0$ of the laser beam be larger than the one on the electron beam $\beta^*$. 

References

