Some Phenomenological Properties of the Chiral Transition in QCD

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Abstract

On the basis of the NJL model as an effective theory of QCD and analogies with condensed matter physics, we extract simple physical pictures of the chiral phase transition at finite temperature $T$ and/or chemical potential $\mu$ and predict interesting phenomena associated with the phase transition which may be seen in relativistic heavy-ion collisions to be performed in RHIC or LHC. We indicate that the macroscopic equation describing non-equilibrium phenomena of the chiral transition like DCC might be given by a diffusion-like equation rather than a wave equation especially when the baryon density is present.

1 Introduction

The aim of the work to be reported here is to extract simple physical pictures of the chiral phase transition at finite temperature $T$ and/or chemical potential $\mu$ and to predict interesting phenomena associated with the phase transition which may be seen in relativistic heavy-ion collisions to be performed in RHIC or LHC. The approach adopted will be not a fundamental nor very microscopic one like lattice QCD reported by Kanaya[1]; we shall make semi-phenomenological arguments based on some effective theory of QCD, the NJL model[2], and also analogies with possible counterparts of condensed matter physics. A part of the present report is based on a review paper[3] based on the NJL model. A remarkable point is that the basic picture presented in [3] has been being confirmed in subsequent studies with lattice QCD [4] and in particular instanton liquid model[5]. In this report, we shall also present further developments and predictions obtained in the approach mentioned above.

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2 The dependence of thermodynamical quantities on the number of active flavors and the vector coupling

The lattice simulations [1, 4] show that the order and even the existence of the phase transition(s) are largely dependent on the number of the active flavors especially when the physical current quark masses are used: For \(m_u \sim m_d \sim 10 \text{MeV} \ll 100 \text{MeV} \ll m_s\), the phase transition may be weak 1st order or 2nd order or not exist.

The gross feature of the \(T\) dependence and the striking difference between the condensates of \(u\) (d) quark and the \(s\) quark are well described by the NJL model[3, 6, 7]. A minimal model which describes the low energy phenomena related with the chiral symmetry may be given by the generalized Nambu-Jona-Lasinio (NJL) model with the anomaly term[8]:

\[
\mathcal{L} = \bar{q}(i \gamma \cdot \partial - m)q + \frac{g_2}{2} \left( (\bar{q} \lambda_a q)^2 + (\bar{q} i \lambda_a \gamma_5 q)^2 \right) + g_D [\det(1 - \gamma_5)q] + h.c.,
\]

where \(\lambda^a\) \((a=0\sim 8)\) are the Gell-Mann matrices with \(\lambda_0 = \sqrt{\frac{2}{3}} \mathbf{1}\). \(SU_V(3)\) symmetry is explicitly broken by the current quark masses, \(\mathbf{m} = \text{diag}(m_u, m_d, m_s)\). The determinantal term has the \(SU_L(3) \otimes SU_R(3)\) invariance but breaks the \(U_A(1)\) symmetry; this term is a reflection of the axial anomaly in QCD and induced by instantons[9]. The anomaly term causes a mixing in flavors: In the chirally broken phase, it induces effective 4-fermion vertices such as \(\langle \bar{d}d \rangle \langle \bar{u}u \rangle \langle \bar{s}s \rangle\) and \(-\langle \bar{d}d \rangle \langle \bar{u}i\gamma_5 u \rangle \langle \bar{s}i\gamma_5 s \rangle\), where the former (latter) gives rise to a flavor mixing in the scalar (pseudo-scalar) channels.

The numerical calculation of the thermodynamical potential \(\Omega(\langle \langle \bar{u}u \rangle \rangle, \langle \langle \bar{s}s \rangle \rangle)\) as a function of the quark condensates \(\langle \langle \bar{q}q_i \rangle \rangle\) \((i = u, s)\) shows the following[10]: In the chiral limit, the phase transition is of second order, although the thermodynamical potential is asymmetric with respect to the zero condensate due to the cubic term coming from the determinantal term. With realistic current quark masses \((m_u = m_d = 5.5 \text{MeV}, m_s = 135 \text{MeV})\), \(\Omega(\langle \langle \bar{u}u \rangle \rangle, 0)\), i.e., in the \(u\)-d sector, has an asymmetric double well, while \(\Omega(0, \langle \langle \bar{s}s \rangle \rangle)\), i.e., in the strangeness sector, has only single minimum for all temperatures, which implies that the phenomenon of the disoriented chiral condensate (DCC)[11] could not be expected in the strangeness sector in contrast to the non-strange sector.

It is also noteworthy that the entropy \(S(T)\) calculated with the NJL model shows that \(S(T)\) goes up with with increasing \(T\) in accordance with the fact that the chiral restoration is a phase transition from an ordered phase to a disordered phase [7]; the increase of the entropy may imply [12] that the NJL model also qualitatively reproduce the ratios \(\epsilon/T^4, (\epsilon - 3P)/T^4\) etc obtained in the lattice QCD.

The temperature dependence of the quark condensates shows that at high temperatures, the flavor \(SU(3)_f\)-symmetry becomes worse [7], which may reflect in the baryon and the vector meson spectra, because they are well described by the constituent quark models.

So much for the case with \(\mu = 0\). How about the phase transition at finite density? A
numerical calculation[7] shows that at low temperatures lower than about 50 MeV, the phase transition is of strong first order in the density direction. Note that our model Lagrangian has no vector term like $g_v (\bar{q} \gamma_\mu q)^2$. As a matter of fact, the strength and even the existence of the 1st order transition are strongly dependent on the strength of the vector coupling $g_v$[13]; the vector term prevents a high-density state; see also §5.

3 Mass shift and character change of hadrons

The NJL model [15, 16] predicts that the mass of the sigma meson decreases as $T$ is raised till $T_c$, a “critical temperature”: $m_\pi$ is found to be constant as long as $T < T_c$; $T_c$ may be defined as the temperature at which $m_\pi$ starts to go high. This may be in accordance with the lattice results on the screening masses[17]. This suggests that at high temperatures the decay $\sigma \rightarrow 2\pi$ would get suppressed and finally hindered, and then only the electro-magnetic process $\sigma \rightarrow 2\gamma$ is allowed. It means that the sigma meson may show up as a sharp resonance with the mass $m_\sigma \approx 2m_\pi$. Thus we propose to observe $\pi^+\pi^-$, $2\pi^0$, $2\gamma$ and construct the invariant mass and examine whether there is a bump in the mass region 300 to 400 MeV. One can expect that in the charged system, the process $\sigma \rightarrow \gamma \rightarrow 2$ leptons become possible, because $\pi^+$ and $\pi^-$ have different chemical potentials with each other[18].

The behavior of kaon at finite temperature was examined by the present author using the $SU(3)$-NJL model Eq.(1) [7]. It was found that as long as the system is in the NG phase, the mass of kaon $m_K(T)$ keeps almost a constant, the value at $T = 0$.

One can argue that the vector mesons $\rho, \omega$ and $\phi$ decrease their masses as the chiral symmetry gets restored; the constituent quark model gives for instance, $m_\phi \simeq 2M_s$ with $M_s$ being the constituent strange quark mass which may be identified with the dynamical one generated by the chiral symmetry breaking[7]. Other QCD-motivated models and theories also predict the decrease of the vector meson masses at finite $T$ [19], although it is not necessarily clear how the decrease of the quark condensates is reflected in the mass shift in such theories.

If the strength of the anomaly term decreases, a character change of $\eta$ and $\eta'$ mesons is expected. A model calculation[20] shows that the $\eta$-$\eta'$ system tends to be well described by the non-strange-strange bases (quark bases) rather than the flavor octet-singlet ones at high temperatures. This has been confirmed also by instanton liquid model[5].
4 Precursory soft modes of chiral transition and channel dependence of hadronic correlations at $T > T_c$

If a phase transition is of second order or weak first order, there should exist precursory soft modes in the symmetric phase prior to the phase transition to a disordered phase; the soft modes become tachyonic on the unstable “vacuum” after the critical point. This is well known in condensed matter physics and nuclear physics, and there are lots of examples of such soft modes; see [21] for an example in nuclear physics. The soft modes are actually fluctuations of the order parameter of the phase transition. In the chiral transition, thus the soft modes corresponding to the fluctuations $\langle (\bar{\psi}\psi)^2 \rangle$ and hence $\langle (\bar{\psi}i\gamma_5\gamma_\tau\psi)^2 \rangle$ due to the chiral symmetry are expected to exist in the symmetric phase or the high-temperature phase. The NJL model was used to demonstrate explicitly that this is the case [14, 15]. The subsequent lattice simulations on screening masses[17] and studies of the instanton molecules[5] support the existence of the soft modes.

Remarkably, the lattice simulations [17, 4] seemed to show that vector-mesonic and baryonic modes were also obtained in the high-$T$ phase. Actually, the screening masses of the vector modes coincide with $2\pi T$ within the error bars, which may simply indicate that the interactions between $q\bar{q}$’s in this channel are absent or greatly suppressed. The nucleons exist as a parity doublet, which was subsequently showed to be compatible with chiral symmetry [23]: A linear sigma model with parity doubling can be constructed which is not in contradiction with the low-energy phenomena including the $\pi$-N coupling constant.

Is there soft modes in the strangeness sector associated with the chiral transition? Soft modes are expected to exist when the phase transition is of second order or weak first order, so that the restoring force for fluctuations of the order parameter becomes small gradually. In the strangeness sector, the thermodynamic potential has single minimum and shows an only a shift of the minimum point, which suggests that there is no necessity of a softening of the excitation energy for fluctuations of the order in the strangeness sector. A numerical calculation shows that this is the case in contrast with the non-strange sector[27].

5 Fate of vector couplings and correlations in the vector channel at high temperature

One may say that there is a reason d’etre of the pionic and sigma-mesonic excitations near the transition point even in the high-$T$ phase; they are fluctuations of the order parameter of the chiral restoration. Then, is there any fundamental reason why hadronic excitations in the vector channel? The approach of the hidden local symmetry[24] seems to claim that the existence of the vector mesons are intimately related with the chiral symmetry and its spontaneous breaking. We show that the great suppression seen in the screening mass is

\footnote{DeTar also conjectured the existence of such modes in a quite different context[22].}
consistent with the $T$-dependence of the quark-number susceptibility $\chi_q(T)$ [25] obtained by the lattice simulations[26].

The quark-number susceptibility $\chi_q$ is the measure of the response of the quark number density to infinitesimal changes in the chemical potentials $\mu_i (i = u, d)$ if we confine ourselves to the two-flavor case[26, 28]; thus

$$\chi_q(T, \mu) = \frac{\partial}{\partial \mu} \rho_q = \beta \int dx \langle \langle \bar{q}(0, x) \gamma_0 q(0, x) \bar{q}(0, 0) \gamma_0 q(0, 0) ra \rangle \rangle,$$  \hspace{1cm} (2)

where $\rho_q = \langle \langle 3\hat{N}_B \rangle \rangle / V$ is the quark-number density with $\hat{N}_B$ being the baryon number operator and $\beta = 1 / TV$ the volume of the system. We remark that the number susceptibility $\chi_q$ at finite density is directly related to the (iso-thermal) compressibility $\kappa_T$ as $\kappa_T = \chi_q / \rho^2$. Thus one sees that if $\chi_q$ of a system is large, the system is easy to compress, which may be a reflection of a weak repulsion between the constituents of the system.

The lattice simulations [26] show that as $T$ is raised $\chi_q$ at $\mu_q = 0$ increases very rapidly around the critical point of the chiral transition: The ratio of the values in the high-$T$ and low-$T$ phase reads $\chi_q(T_c + \epsilon, 0) / \chi_q(T_c - \epsilon, 0) = 4 \sim 5$ with the light two flavors, where $\epsilon$ is a small number, say $0.03T_c$.

One can easily verify that $\chi_q(T)$ of the free quark gas increases as $M(T)$ decreases and reaches $N_f T^2$ at $M(T) = 0$: The enhancement is, however, found to be merely about 1.6 with $M(T)$ as described in the effective theory Eq.(1). Thus there must be an additional mechanism to increase $\chi_q$ to realize the anomalous (relative) enhancement obtained in the lattice QCD. One may first note that $\chi_q$ is the density-density correlation which is nothing but the 0-0 component of the vector-vector correlations or fluctuations [28]. Accordingly, $\chi_q$ is further related with the retarded Green’s function or the response function in the vector channel, the poles of which give the masses of the vector mesons. Thus one recognizes that the quark-number susceptibility is intimately related with the properties of the vector mesons and fluctuations in the vector channel.

To demonstrate the relevance of the vector correlations to $\chi_q$, a model calculation with a dynamical model was performed [25] with a simple NJL model having a vector coupling. Although one only needs the case with a vanishing chemical potential to compare the results with the lattice simulations, it is noteworthy that when $\mu_q \neq 0$ there arises a coupling between $\chi_q$ and the scalar-density susceptibility $\chi_s$ owing to the non-vanishing “vector-scalar susceptibility” $\chi_{VS}$, which are defined by

$$\chi_s = -\frac{d\langle \bar{\psi} \psi \rangle}{dm} = \beta \int dx \langle \langle \bar{\psi}(0, x) \psi(0, x) \bar{\psi}(0, 0) \psi(0, 0) \rangle \rangle,$$

$$\chi_{VS} = \frac{\partial \langle \bar{\psi} \psi \rangle}{\partial \mu_q} = \beta \int dx \langle \langle \bar{\psi}(0, x) \gamma_0 \psi(0, x) \bar{\psi}(0, 0) \psi(0, 0) \rangle \rangle,$$  \hspace{1cm} (3)

respectively. One may also note that $\chi_s$ is the fluctuation of the order parameter of the chiral transition, and is related with the sigma meson propagator. Thus when $\rho_q \neq 0$, which the lattice QCD is difficult to calculate, the properties of the sigma meson reflects in $\chi_q$. 


This is a good example of mode-mode coupling known in the dynamic critical phenomena in condensed matter physics.

We remark that when $\mu_q = 0$, the coupling between $\chi_q$ and $\chi_S$ disappear. Putting $\mu_q = 0$ into the expressions one obtains $\chi_q = \chi_q^{(0)}(T)/(1 + 2 g_V \chi_q^{(0)}(T))$, where $\chi_q^{(0)}(T)$ is the susceptibility for the free-quark gas and $g_V$ the coupling constant in the vector channel as mentioned in §2. The denominator of $\chi_q$ is essentially the inverse of the propagator of the vector meson in the ring approximation, at the vanishing four momenta. One sees that $\chi_q$ is suppressed with the vector coupling; $g_V$ is positive. It reasonably implies that the system becomes uneasy at $\mu_q \neq 0$ to compress with the vector coupling; recall that $\chi_q$ is proportional to $\kappa_T$ when $\mu_q \neq 0$.

It was shown that the lattice results [26] can be accounted for by a decreasing vector coupling as well as the decreasing dynamical quark mass due to the chiral transition[25]. Thus the lattice results may simply suggest that the vector coupling is suppressed or vanishes in the high-$T$ phase, which implies that the vector correlations and hence the collective vector modes would disappear in the chirally restored phase.

It is interesting that this picture is consistent with the observation that the screening masses of the vector modes obtained in the lattice simulations almost coincides with $2\pi T$, the lowest screening mass of the $q\bar{q}$ system in the chiral limit.

6 What can we learn about DCC from physics of superconductors?

Recently much attention has been paid to time-dependent phenomena of the chiral transition, especially in relation to the disoriented chiral condensate (DCC)[11]. One of the basic questions of this problem is what is a macroscopic phenomenological equation which can describe the time-dependent phenomena. Usually, the phenomenological linear sigma model is used. This is, however, not necessarily justified. First we note that the problem is a dynamical critical phenomena of the chiral transition at finite temperature, and there is much resemblance between the chiral transition in QCD and the phase transition to a superconductors[2, 3]. Abraham and Tsuneto[29] examined what type of equation could describe slow and long-wave length phenomena in superconductors; see also [30] for subsequent development. They clarified that the answer depends on the kinematics in which one is interested in: For example, in the vicinity of the critical point, the equation is well approximated by the non-linear diffusion equation, called time-dependent Ginsburg-Landau (TDGL) equation, not a hyperbolic equation as derived from the linear sigma model. The origin of the dumping is the Landau damping. It means that it is an open and hence interesting problem to explore what type of equation is suitable to describe the DCC, and to see what kinematical conditions are relevant to relativistic heavy-ion collisions. A preliminary calculation shows that when baryon density is present, the relaxation effect due to the Landau damping is significant[31]; we notice that the scalar-vector coupling is also important at
finite $\mu$[25].

7 Summary

We have seen that the static properties, as represented by the quark condensates $\langle \langle \bar{u}u \rangle \rangle$, $\langle \langle \bar{s}s \rangle \rangle$, the thermodynamical potential $\Omega(\langle \langle \bar{u}u \rangle \rangle$, $\langle \langle \bar{s}s \rangle \rangle)$, of the chiral phase transition can be understood in terms of a simple model Eq.(1). We have indicated that the flavor dependence should reflect in the phenomena of DCC; DCC may be only relevant to the $u,d$-sector. We have also shown that when the baryon density is present, i.e., $\mu \neq 0$, the strength of the vector coupling may change the nature of the chiral transition substantially.

We have seen that in relation to the mass shift of the sigma meson, the detection of the process $\sigma \rightarrow 2\pi$, $2\gamma$ and $2\ell^\pm$ is interesting. We have also seen that that $\eta-\eta'$ mesons may change their characters at high temperatures.

We have also seen that the non-perturbative effects at $T > T_c$ are channel dependent: the sigma and pionic channel are special because they are intimately related to the chiral symmetry and its spontaneous breaking. On the other hand, the interactions in the vector channel are likely to be weak. It was indicated that the parity doubling in the baryon sector is consistent with chiral symmetry and the low energy phenomena at zero temperature.

We have seen that the quark-number susceptibility is intimately related to fluctuations in the vector channel in the system: We have suggested that the vector correlations seem to be suppressed after the chiral transition. Direct measurements of this behavior of the susceptibility in experiment would be extremely interesting. We have also emphasized that at finite baryon density, there arises a mode-mode coupling between the scalar and the vector channels.

As for DCC, we have indicated on the basis of an analogy with superconductors that the decay process due to the Landau damping is important and it may change the macroscopic equation which describes the DCC from the wave-equation like to a diffusion-like equation; it also depends on the relevant kinematics. We remark that in non-equilibrium phenomena like DCC, effects of the mode-mode coupling will affect the dynamics considerably.

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References

[1] K. Kanaya, these proceedings.


