

\textbf{$K^+$ production in baryon-baryon and heavy-ion collisions}

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Abstract

Kaon production cross sections in nucleon-nucleon, nucleon-delta and delta-delta interactions are studied in a boson exchange model. For the latter two interactions, the exchanged pion can be on-mass shell, only contributions due to a virtual pion are included via the Peierls method by taking into account the finite delta width. With these cross sections and also those for pion-baryon interactions, subthreshold kaon production from heavy ion collisions is studied in the relativistic transport model.
Kaon production in heavy-ion collisions at subthreshold energies continues to attract great interest, as it is expected to carry useful information not only on the initial collision dynamics but also on the nuclear equation of state at high density [1–5] and the kaon properties in dense matter [6–8]. However, to extract these information from the experimental data requires the use of transport models, in which elementary kaon production cross sections in hadron-hadron interactions are needed. At SIS energies (\(\sim 1\) GeV/nucleon) the colliding system consists mainly of nucleons, delta resonances, and pions, so one needs kaon production cross sections in interactions between these particles. For kaon production in the nucleon-nucleon (NN) interaction, there are only experimental data at high energies. Thus various parameterizations have been introduced to make predictions at low energies. Furthermore, for processes involving baryon resonances, such as \(N\Delta \rightarrow NYK\) and \(N\Delta \rightarrow NYK\), which are unique in heavy-ion collisions and play important roles in subthreshold kaon production [1–6], no experimental information is available. The usual prescription used in transport models is to assume that these cross sections are either the same as those in NN interactions at same center-of-mass energies or somewhat smaller using the scaling ansatz of Randrup and Ko based on isospin considerations [9].

There are attempts to evaluate the kaon production cross sections using the one-boson exchange model [10–13], which has also been used frequently in calculating the pion [14], eta [15], phi [16], and dilepton [17,18] production cross sections in NN interactions. The extension of this model to particle production in nucleon-delta (N\(\Delta\)) and delta-delta (\(\Delta\Delta\)) interactions are straightforward, except for the complication that the exchanged pion can be on-mass shell, which then leads to a singular cross section. Since the two-step process, which involves the decay of a delta into a nucleon and a pion and the subsequent interaction of the pion with another nucleon to produce a kaon, has already been included in the transport model, one should not include in \(N\Delta\) and \(\Delta\Delta\) interactions the contribution due to an on-shell pion. In Ref. [12], a complex pion self-energy based on a schematic delta-hole model has been introduced to evaluate the off-shell contribution. In a similar study of eta meson production from the \(N\Delta\) interaction [15], the imaginary part of the pion self-energy
is determined from the pion-nucleon scattering cross section. Although the resulting cross sections are finite at finite densities, they remain diverge as the density approaches zero.

In heavy ion collisions, the delta-induced reaction is part of a more complex process that involves first the production of a delta in an $NN$ interaction, which can be either on or off its energy shell, and then its interaction with another nucleon. For kaon production, it was shown in Ref. [19] that the contribution from an off-shell delta is much smaller than that from an on-shell one. Since an on-shell delta has a finite lifetime, its mass is thus complex with the imaginary part given by its width. Then energy-momentum conservation leads to a complex four momentum for the exchanged pion, which moves the singularity to the complex plane and thus makes the contribution from the exchange of an off-shell pion finite. Such a method was first introduced by Peierls [20] in understanding higher resonances production from the pion-nucleon reaction by first producing a delta and then followed by $\pi\Delta \rightarrow \pi N^*$ via a $t$-channel nucleon exchange. The singular amplitude at the nucleon pole was regularized by including the complex delta mass. In Ref. [16], this method was used to evaluate the phi meson production cross section in $N\Delta$ and $\Delta\Delta$ interactions. This physical region singularity in the scattering of particles with finite lifetimes appears also in muon collider physics [21], where a singularity appears in the process $\mu^+\mu^- \rightarrow e\bar{\nu}W^+$, as the exchanged neutrino can be on-shell due to the decay $\mu \rightarrow e\bar{\nu}\nu\mu$. In Ref. [21], this singularity has also been regularized via the Peierls method by including the width of the muon.

The elementary kaon production cross sections in baryon-baryon interactions have been studied in Ref. [12] based on the pion and kaon exchange model. By adjusting two cut-off parameters $\Lambda_\pi$ and $\Lambda_K$ in the form factors at the $\pi NN$ and $KNA$ vertices, respectively, very good agreements with experimental kaon production cross sections in the $NN$ interaction have been obtained. In Fig. 1, results for $pp \rightarrow pKA^+$ are compared with the experimental data and various parameterizations. Open circles are old data from the compilation by Baldini et al. [22], while the solid circle is the data from the recent experiment at COSY for proton-proton collisions at 2 MeV above the kaon production threshold [23]. It is seen that our model provides a good description of both old and new data. The parameterization
of Randrup and Ko [9] is shown in the figure by the dashed curve. It describes reasonably well the experimental data at high energies (open circles), which were available when it was introduced. However, it significantly overestimates the COSY data as a result of its incorrect threshold behavior. The parameterization of Schürmann and Zwermann [24] is shown in the figure by the dash-dotted curve. Its assumption of a quartic dependence on \( p_{\text{max}} \) (the maximum momentum of kaon at a given center-of-mass energy) does not agree with the experimental data near the threshold either. The dotted curve in the figure comes from Cassing et al. [25], which is also based on the boson-exchange model.

In evaluating the kaon production cross sections in \( N\Delta \) and \( \Delta\Delta \) interactions by the Peierls method, the pion propagator is modified by the imaginary delta energy due to its width \( \Gamma_{\Delta} \), i.e.,

\[
\frac{1}{(p_{\Delta} - p_N)^2 - m_\pi^2} \rightarrow \frac{1}{(p_{\Delta} - p_N)^2 - m_\pi^2 - \frac{im_{\Delta}(E_{\Delta} - E_N)}{E_{\Delta}}\Gamma_{\Delta}},
\]

where \( p_N \) and \( p_{\Delta} \) are four momenta of the final-state nucleon and initial-state delta resonance, respectively; \( E_N \) and \( E_{\Delta} \) are their energies in the center-of-mass frame. The off-shell pion contribution is then obtained by including only the real part of Eq. (1). In nuclear medium, the delta width is expected to change, and also the exchanged pion can acquire additional imaginary self energy due to interactions. Such medium-dependent effects are interesting but require further study and are thus neglected here.

Results for the isospin-averaged kaon production cross sections for both \( N\Sigma K \) and \( N\Lambda K \) final states are shown in Fig. 2, together with those from the parameterization of Randrup and Ko [9]. Near the threshold, because of their incorrect threshold behavior (see Fig. 1), the Randrup-Ko cross sections are larger than our microscopic results for all the channels. In the energy region most relevant for kaon production at SIS energies, our results for \( N\Delta \) and \( \Delta\Delta \) channels are larger than those of Randrup and Ko.

To apply these cross sections in the transport model, we have parameterized our theoretical results in terms of \( \sqrt{s} - \sqrt{s_0} \), with \( \sqrt{s} \) and \( \sqrt{s_0} \) denoting, respectively, the available energy and the threshold,
\[ \sigma_{BB \rightarrow NYK} = \frac{a(\sqrt{s} - \sqrt{s_0})^2}{b + (\sqrt{s} - \sqrt{s_0})^x} \text{mb.} \] (2)

The fitted parameters \(a\), \(b\) and \(x\) for six channels are listed in Table 1. In addition, we also include kaon production from pion-baryon interactions with cross sections taken from Ref. [26], which were also parameterized in terms of \(\sqrt{s} - \sqrt{s_0}\). Furthermore, we include processes with one or two pions in the final states. The cross sections for these processes are obtained from parameterizations based on experimental data [27]. When medium effects on kaons are considered, the threshold is calculated with in-medium masses. In addition, kaons also propagate in their mean field potentials [28].

Kaon properties in dense matter have been extensively studied; for a recent review see Ref. [29]. These studies indicate that the kaon energy in medium can be expressed as

\[ \omega_K = \left[ m_K^2 + k^2 - a_K \rho_S + (b_K \rho_N)^2 \right]^{1/2} + b_K \rho_N, \] (3)

where \(b_K = 3/(8f^2_{\pi}) \approx 0.333 \text{ GeV fm}^3\) represents the vector repulsion, while \(a_K\) determines the strength of the attractive scalar potential. If one considers only the Kaplan-Nelson term, then \(a_K = \Sigma_{KN}/f^2_{\pi}\), with \(\Sigma_{KN}\) being the \(KN\) sigma term. In the same order, there is also an energy-dependent range term which cuts down the scalar attraction [29]. Since both \(\Sigma_{KN}\) and the range term are not very well determined, we treat \(a_K\) as a free parameter. Taking \(a_K \approx 0.22 \text{ GeV}^2 \text{ fm}^3\), then a kaon feels a repulsive potential of about 20 MeV at normal nuclear matter density, in rough agreement with the prediction of impulse approximation using the \(KN\) scattering length in free space [28].

Extensive transport model calculations for subthreshold kaon production have been carried out recently in [25,27,32]. Here we shall concentrate on \(K^+/\pi^+\) ratio in Au+Au collisions at 1 AGeV, as measured by KaoS Collaboration [30,31]. We first show in Fig. 3 the \(\pi^+\) transverse momentum spectra in Au+Au collisions at 1 AGeV. The results from our transport model (histogram) are in very good agreement with the recent data from both the FOPI Collaboration [33] (open circles) and the Kaos Collaboration [31] (open squares).

In the upper panel of Fig. 4 we show the \(K^+\) yield, normalized by the number of participant nucleons \(A_{\text{part}}\), as a function of \(A_{\text{part}}\). In both cases with and without kaon
medium effects, the ratio increases more than a factor of two from peripheral to central collisions. On the other hand, the ratio $\pi^+/A_{\text{part}}$ stays almost a constant of 0.03 in the entire impact parameter range. The stronger dependence of the $K^+$ yield than the pion yield on the centrality of the collision indicates that secondary processes involving pions and delta resonances are extremely important for subthreshold particle production. Indeed, for central Au+Au collisions, the kaon yield from pion-baryon collisions accounts for more than 60% of the total kaon yield, as was also found in Ref. [32]. Furthermore, the $N\Delta$ channel is found to be the most important one among all the baryon-baryon contributions.

In the lower window of Fig. 4 we show the ratio $K^+/\pi^+$ as a function of $A_{\text{part}}$. It is seen that our results with kaon medium medium effects are in agreements with the latest KaoS data shown by solid squares [31]. The results without kaon medium effects overestimate this ratio. Also shown in the figure by open squares are the previous KaoS data published in Ref. [30], which seem to be better described by the scenario without the kaon medium effects (which implies a complete cancellation of the scalar and vector potentials in Eq. (3)).

In summary, extending our previous work on phi meson production cross sections in baryon-baryon collisions [16], we have evaluated the kaon production cross sections in $N\Delta$ and $\Delta\Delta$ interactions using the Peierls method by including the finite delta width. Using them in the relativistic transport model, we have studied kaon production, in particular the $K^+/\pi^+$ ratio, in Au+Au collisions at 1 AGeV. We found that the latest KaoS data on this ratio are consistent with a weak repulsive kaon potential.

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REFERENCES


Table 1 Fitted parameters in Eq. (2).

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<th>$NN \rightarrow N\Sigma K$</th>
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Figure Captions

Fig. 1: Kaon production cross section for $pp \rightarrow p\Lambda K^+$. The solid curve is the results from the boson-exchange model of Ref. [12]. The dashed, dash-dotted, and dotted curves are, respectively, the parameterizations of Randrup and Ko [9], Schürmann and Zwermann [24], and Cassing et al. [25]. Open circles are experimental data from Baldini et al. [22], while the solid circle is from Ref. [23].

Fig. 2: Isospin-averaged kaon production cross sections in $NN$, $N\Delta$, and $\Delta\Delta$ interactions. Solid curves are from this work, and dotted curves are from the Randrup-Ko parameterization [9].

Fig. 3: $\pi^+$ transverse momentum spectra in Au+Au collisions at 1.0 AGeV. The histogram gives our theoretical results, while the circles and squares are the experimental data from the FOPI [33] and KaoS [31] collaborations, respectively.

Fig. 4: Upper window: $\pi^+$ and $K^+$ yields normalized by the participant nucleon number $A_{\text{part}}$ in Au+Au collisions at 1 AGeV. Lower window: $K^+/\pi^+$ ratio as a function of the participant nucleon number.