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Virtual Compton Scattering (VCS: $\gamma^* + p \rightarrow \gamma + p'$) in the Bjorken regime ($Q^2 \rightarrow \infty$, Q^2/s finite) has been the subject of increasing interest since it has been suggested by Ji[1] and Radyushkin[2] that this reaction could be used to measure a new type of parton distributions, generally referred to as Off Forward Parton Distributions (OFPD's). The latter do not contribute to forward Compton scattering, and hence to inclusive lepton scattering, because they are multiplied by the momentum transfer to the nucleon. This is analogous to the magnetic moment contribution to the electromagnetic current.

The OFPD's satisfy a number of sum rules which either provide tests of the theoretical analysis or can shed a new light on the spin structure of the nucleon[1]. Therefore it seems appropriate to investigate whether these new observables can actually be measured.

Figure 1

The VCS process on the proton can be accessed through the (e, e', γ) reaction, $e + p \rightarrow e' + p' + \gamma$, whose invariant cross section is (see Fig.1 for the definition of the kinematics)

$$d\sigma = \frac{(2\pi)^{-5}}{32\sqrt{(p \cdot k)^2 - m^2 m_e^2}} \frac{d\vec{k}' d\vec{p}' d\vec{q}'}{k'_0 p'_0 q'_0} \delta^4(p + k - p' - k' - q') |T^{VCS} + T^{BH}|^2 \quad (1)$$

where T^{BH} is the amplitude of the Bethe-Heitler process which corresponds to the emission of the real photon by the lepton lines (Fig.1 b,c). It cannot be separated from the VCS process (Fig.1 a) since it has the same final state. In the one photon exchange approximation the BH amplitude depends only on the elastic form factors of the nucleon and so contains no new information. When it dominates over the VCS process the experimental study of the (e, e', γ) reaction thus presents in general little interest. Let us stress that trying to subtract the (theoretical) BH cross section from the experimental (e, e', γ) cross section would be hazardous since this requires a perfect control of the higher order radiative processes.

To lowest order in α_{QED} the expression for T^{BH} is

$$T^{BH} = \frac{-e^3}{t} \epsilon_{\mu}^{I*} L^{\mu\nu} \bar{u}(p') \left[F_1(t) \gamma_{\nu} + i F_2(t) \frac{\sigma_{\nu\rho} (p' - p)^{\rho}}{2m} \right] u(p) \quad (2)$$

with

$$L^{\mu\nu} = \bar{u}(k') \left[\gamma^{\mu} \frac{1}{\gamma \cdot (k' + q') - m_e + i\epsilon} \gamma^{\nu} + \gamma^{\nu} \frac{1}{\gamma \cdot (k - q') - m_e + i\epsilon} \gamma^{\mu} \right] u(k). \quad (3)$$

where $F_{1,2}(t)$ are the usual form factors of the proton normalized to $F_1(0) = 1$, $F_2(0) = 1.79$, $e \simeq \sqrt{4\pi/137}$ and ϵ_{μ}^{I*} is the photon polarization. Note that the virtual photon momentum is $q = k - k'$ for the VCS process while it is $q - q'$ for the BH process. For definiteness all our

formulae correspond to a negatively charged lepton beam and to a proton target. For a positively charged beam one must change the relative sign of the BH and VCS amplitudes.

To the same order one has

$$T^{VCS} = \frac{e^3}{q^2} \epsilon_{\mu}^{\prime\star} H^{\mu\nu} \bar{u}(k') \gamma_{\nu} u(k), \quad (4)$$

with

$$iH^{\mu\nu} = \int d^4x e^{-iq \cdot x} \langle p' | T[J^{\nu}(x), J^{\mu}(0)] | p \rangle, \quad (5)$$

where J is the electromagnetic current carried by the quarks.

In the Bjorken regime the four fold integral of eq.(5) is dominated by the light cone singularity of the product $J^{\nu}(x)J^{\mu}(0)$. Therefore an operator product expansion around this limit is a priori possible (see ref.[3]). Here we admit as in [1] that in the forward direction ($\hat{p}' = \hat{p}$) the leading twist contribution is given by the hand bag diagram. To quantify the degree of forwardness we can impose that the momentum flows through the photon lines so as to allow a factorization between the hard and soft parts of the amplitude. This amounts to impose that $\theta_{\gamma\gamma^*} \ll m^2/Q^2$ if $\theta_{\gamma\gamma^*}$ is the laboratory angles between the photons. For instance at $\theta_{\gamma\gamma^*} = 0$ the energy momentum transfer to the proton is $(q^0 - |\vec{q}'|) \approx mx_B^2/2(1 - x_B)$, $|\vec{q}| - |\vec{q}'| \approx mx_B + mx_B^2/2(1 - x_B)$ which is soft. Though this point may need a more sophisticated analysis we take it as our working hypothesis.

A standard calculation (see for instance ref.[4]) then gives the following leading twist contribution to $H^{\mu\nu}$:

$$\begin{aligned} H^{\mu\nu} &= \frac{1}{2}(\bar{n}^{\mu}n^{\nu} + \bar{n}^{\nu}n^{\mu} - g^{\mu\nu})H^{+} + \frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\bar{n}_{\alpha}n_{\beta}H^{-} \\ H^{+} &= \int dx \left(\frac{1}{x - \xi/2 + i\epsilon} + \frac{1}{x + \xi/2 + i\epsilon} \right) \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{\psi}(-\lambda n/2) \gamma \cdot n Q^2 \psi(\lambda n/2) | p \rangle \\ H^{-} &= \int dx \left(\frac{1}{x - \xi/2 + i\epsilon} - \frac{1}{x + \xi/2 + i\epsilon} \right) \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{\psi}(-\lambda n/2) \gamma \cdot n \gamma_5 Q^2 \psi(\lambda n/2) | p \rangle \end{aligned} \quad (6)$$

where Q is the charge matrix of the quarks and the light-like vectors ($\bar{n}$, n) are defined by

$$n = \frac{1}{2\Lambda} \begin{pmatrix} 1 \\ \vec{0}_{\perp} \\ -1 \end{pmatrix}, \quad \bar{n} = \Lambda \begin{pmatrix} 1 \\ \vec{0}_{\perp} \\ 1 \end{pmatrix} \quad (7)$$

in a frame where q and $\tilde{P} = (p + p')/2$ are both along the z direction. Λ is an arbitrary energy scale. The physical momenta decompose as follows

$$\tilde{P} = \bar{n} + \frac{\bar{m}^2}{2} n, \quad q = -\xi \bar{n} + \frac{Q^2}{2\xi} n, \quad \Delta = p' - p = -\xi \left(\bar{n} - \frac{\bar{m}^2}{2} n \right) + \Delta_{\perp}, \quad (8)$$

with ξ defined by

$$\frac{1}{\xi} - \xi \frac{\bar{m}^2}{Q^2} = \frac{1}{\tilde{x}_B}, \quad \tilde{x}_B = \frac{Q^2}{2\tilde{P} \cdot q}. \quad (9)$$

and $\bar{m}^2 = m^2 - t/4$. where $t = \Delta^2$. Note that $\xi \rightarrow \tilde{x}_B$ when $Q^2 \rightarrow \infty$ at fixed t .

The OFPD's $(H, E, \tilde{H}, \tilde{E})$ of the *proton* are then defined through [1]

$$\begin{aligned} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{\psi}(-\lambda n/2) \gamma^\mu \mathcal{Q}^2 \psi(\lambda n/2) | p \rangle &= \bar{u}(p') \left[H(x, \xi, t) \gamma^\mu + i E(x, \xi, t) \frac{\sigma^{\mu\nu} \Delta_\nu}{2m} \right] u(p) \\ \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{\psi}(-\lambda n/2) \gamma^\mu \gamma_5 \mathcal{Q}^2 \psi(\lambda n/2) | p \rangle &= \bar{u}(p') \left[\tilde{H}(x, \xi, t) \gamma^\mu \gamma_5 + i \tilde{E}(x, \xi, t) \frac{\gamma_5 \Delta_\mu}{2m} \right] u(p). \end{aligned} \quad (10)$$

The support of the OFPD's is $[-1, 1]$ and from their definitions one gets the sum rules

$$\begin{aligned} \int_{-1}^1 dx H(x, \xi, t) &= F_p^1(t) + \frac{2}{3} F_n^1(t), \\ \int_{-1}^1 dx E(x, \xi, t) &= F_p^2(t) + \frac{2}{3} F_n^2(t), \\ \int_{-1}^1 dx \tilde{H}(x, \xi, t) &= \frac{1}{6} g_A(t) + \frac{5}{18} g_A^0(t), \\ \int_{-1}^1 dx \tilde{E}(x, \xi, t) &= \frac{1}{6} h_A(t) + \frac{5}{18} h_A^0(t), \end{aligned} \quad (11)$$

where we have assumed that only the (u,d) quarks contribute, which amounts to take $\mathcal{Q} = 1/6 + \tau^3/2$, τ^3 being the isospin of the quark. In eqs (11), $F_{p,n}^{1,2}$ refer to the proton or neutron electromagnetic form factors, (g_A, h_A) are respectively the axial and pseudoscalar isovector form factors, and (g_A^0, h_A^0) their isoscalar equivalents.

To evaluate the VCS amplitude we need a first guess for the OFPD's. Here we assume a simple factorized form

$$\begin{aligned} H(x, \xi, t) &\simeq H(x, t) = q(x)h(t), & E(x, \xi, t) &\simeq E(x, t) = q(x)e(t), \\ \tilde{H}(x, \xi, t) &\simeq \tilde{H}(x, t) = \tilde{q}(x)\tilde{h}(t), & \tilde{E}(x, \xi, t) &\simeq \tilde{E}(x, t) = \tilde{q}(x)\tilde{e}(t), \end{aligned} \quad (12)$$

which should be acceptable in the small t region. The ξ dependence is neglected. This is somewhat justified by the bag model estimate of Ji et al.[5].

From the definition of the OFPD's we see that in the limit $p = p'$, H is the ordinary parton distribution. The phenomenology then suggests[6] for $x \in [0, 1]$

$$q(x) = q_v(x) + q_s(x) = N_v x^{0.5} (1-x)^3 + N_v x^{-1} (1-x)^7 / 8, \quad (13)$$

where for convenience N_v is such that q_v is normalized to 1. We then determine $h(t)$ and $e(t)$ by the sum rules (11). Note that the distribution (13) is only a rough guess which should be improved in the future. Nevertheless, since we normalize the OFPD's by the sum rules the outcome of the calculation is relatively safe as long as it is not sensitive to the end points.

Similarly for $p = p'$, $\tilde{H}$ is the polarized parton distribution. To simplify we assume that the sea is not polarized and so we choose $\tilde{q}(x) = q_v(x)$ and again we determine $\tilde{h}(t)$ and $\tilde{e}(t)$ by the sum rules.

For $F_{p,n}^{1,2}$ and g_A we use the usual dipole parameterization and for h_A we assume pion pole dominance $h_A(t) = -4m^2 g_A(t)/(t - m_\pi^2)$. The isoscalar axial form factor is obtained from the

quark model: $g_A^0(t) = 3g_A(t)/5$, and we neglect $h_A^0(t)$ as there is no pole in this channel in the low t region.

The rest of the calculation is standard. Using ($q(-x) = -\bar{q}(x)$, $\bar{q} = q_s$, $\bar{q}_v = 0$) the x integration from 0 to 1 is transformed to an integration from 0 to 1 and the latter is done by contour deformation in the complex x plane.

Using this calculation¹ we now investigate the experimental possibilities. It is clear that if the BH process dominates or if it interferes strongly with the VCS process, the output of the experiment will be difficult to interpret. The BH amplitude goes like $1/t$ while the VCS goes like $1/Q^2$. Therefore the interesting region, large Q^2 , small t favors the BH process. To overcome this problem the only way is to increase the virtual photon flux of the VCS process by increasing the beam energy. To illustrate this point we show on Fig.2 the cross sections for what we think to be a typical kinematic, $Q^2 = 2 \text{ GeV}^2$, $x_B = 0.3$, in the case of CEBAF and COMPASS (in this latter case we now use a positively charged muon beam of 100 GeV while some results shown in the past were for a negative beam of 200 GeV). In both cases the kinematic is coplanar, the negative angles corresponding to an azimuthal angle $\phi = 180^\circ$.

Obviously CEBAF is not the right place to measure DVCS (See however the last paragraph of this paper). In the forward direction the cross section is dominated by the BH process. To get the signal one is forced to go at large angles where the hypothesis of the calculation are not satisfied and anyway cross section is already too small to be measured.

Figure 2

By contrast COMPASS seems to offer a nice opportunity. At zero angle the VCS dominates over the BH process by more than one order of magnitude and the cross section looks large

¹ After the correction of a numerical error, there is a small difference between the results shown here and those shown in the DIS97 proceedings.

enough. This encourages us to study the COMPASS case in more details. The results shown on Fig.2 suggests that the relevant region is $\phi = 180^\circ$. Therefore $\phi = 0^\circ$ region which is rapidly contaminated by the BH process will no longer be shown. We have computed the cross sections for $Q^2 = 2, 4 \text{ GeV}^2$ and $x_B = 0.1, 0.3, 0.5, 0.7$ which is hopefully in the Bjorken region. From the results shown in Fig.3, it is apparent that a quantitative study of VCS should be possible at COMPASS. The BH contamination is negligible except for $x_B = 0.1$ which is already very small. This could be improved by increasing the beam energy² to 200 GeV. The problem is now to check that the counting rates are large enough.

Figure 3 Cross sections for COMPASS (100 GeV, positive muons). The axis labels are the same as in Fig. 2

Since the available energy is crucial to favor the VCS over the BH process, it is interesting to have a glance at what could be done with the HERA collider. So we show on Fig.4 some cross sections computed in collider mode (800 GeV protons+27.5 GeV positrons). Note that here we use the angle of the final photon with respect to the electron beam. Due to the large proton

² Due to a labelling mess the case $Q^2 = 4 \text{ GeV}^2$, $x=0.7$ shown in the DIS97 proceedings corresponds actually to a beam energy of 200 GeV. This explains the difference with the curves shown here.

energy the small t region is concentrated near 180° , that is along the proton beam. The region shown corresponds to $t < 1 \text{ GeV}^2$ and the calculation is for $\phi = 180^\circ$. As expected the VCS cross sections are large and completely dominates the BH process. However it is not clear that the photon can be efficiently detected when it is emitted so close to the proton beam, in particular for the large x_B region. To have the photon emitted farther from the proton beam one needs either to increase Q^2 , which decreases the VCS cross section, or to go to very small x_B where the model used here certainly breaks.

Figure 4 Cross sections for HERA (27.5 GeV positrons+820 GeV protons). The vertical axis labels is the same as in Fig. 2

Finally, though CEBAF does not seem to be the adequate facility to study the VCS in the Bjorken regime, we may take advantage of its polarized beam to open a (small) window on this new domain. The point is that, in the leading twist approximation, the single spin asymmetry comes only from the interference between the BH and the VCS process. Therefore, even if the BH process

dominates, the asymmetry still measures a linear combination of the OFPD's. The price to pay is that, due to time reversal invariance and the reality of the BH amplitude, only the imaginary part of the VCS amplitude contributes to the asymmetry. This implies that the OFPD's are probed only along the line $x = \pm\xi/2$ while the sum rules require the measurement at any x for fixed ξ . So the use of the asymmetry may be an interesting possibility, for instance to test the scaling properties of the VCS amplitude, but it should be considered only as a first limited exploration.

With this restriction in mind we can check that the asymmetry is large enough. On Fig.5 we show the results for a typical kinematic of CEBAF. Of course the azimuthal angle must be non zero since by parity the asymmetry vanishes in coplanar kinematics. Even though the cross section is dominated by the BH process in the forward region, the asymmetry reaches 40% at $\theta = 8^\circ$ and therefore should be measurable at CEBAF.

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