The Relative Ages of Galactic Globular Clusters

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\textbf{ABSTRACT}

We present a review of the present state of knowledge regarding the relative ages of Galactic globular clusters. First, we discuss the relevant galaxy formation models and describe the detailed predictions they make with respect to the formation timescale and chemical evolution of the globular clusters. Next, the techniques used to estimate globular cluster ages are described and evaluated with particular emphasis on the advantages and disadvantages of each method. With these techniques as a foundation, we present arguments in favor of the following assertions: 1) The age of a globular cluster is the likeliest candidate to be the global second parameter, which along with metal abundance, controls the morphology of the horizontal branch. 2) A total age range of as much as \(\sim 5\) Gyr exists among the bulk of the Galactic globulars. 3) There is a significant relation between age and metallicity among the Galactic globular clusters if the slope of the \(M_V(RR) - [Fe/H]\) relation is less than \(\sim 0.23\). These conclusions along

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with other supporting evidence favor a formation scenario in which the inner regions of the Galactic halo collapsed in a monotonic fashion over a short time period much less than 1 Gyr. In contrast, the outer regions of the halo fragmented and collapsed in a chaotic manner over several Gyrs.

1. Introduction

The relative ages of the Galactic globular clusters are an important signpost on the path to understanding the formation of the Milky Way. However, what they tell us about the earliest epochs of the Galaxy has been interpreted in a number of different ways by various investigators. This is compounded by the fact that we have age information for only one-third of the approximately 150 known globular clusters in the Milky Way. The most recent review concerning the relative ages of Galactic globular clusters was presented by Stetson et al. (1996, hereafter SVB). In addition, there has been a flurry of papers written on this topic over the past year. As a result, we felt it appropriate to write another review updating and clarifying some of the points made by SVB and others.

To motivate the study of relative globular cluster ages, we first present a summary of the two competing models for the formation of the Galaxy. The next section describes the various techniques used to estimate relative ages. We show how these ages along with the corresponding abundances of the Galactic globulars can provide insight into which of these models is most relevant to the early formation history of the Milky Way. We then attempt to synthesize a formation scenario that is consistent with the observational data.

2. Milky Way Formation Models

It is generally believed that the Milky Way began as a single large gas cloud with perhaps some angular momentum. Once it decoupled from the overall Hubble flow, this gas cloud began to collapse and thus form the Galactic halo and disk that we know today. Precisely how this collapse progressed is an issue of some debate at the moment. Models for the formation of the Milky Way’s halo can be classified into two categories: those that predict a rapid collapse of the Galactic halo (<<1 Gyr) and those that predict a more gradual and more chaotic collapse (several Gyr).

The principal representative of the rapid collapse picture is the work of Eggen et al. (1962, hereafter ELS). They studied a set of 221 solar neighborhood stars with a range of metallicities. ELS observed that lower metallicity stars, inferred to be older, are on more eccentric orbits than those with higher metallicity. In addition, ELS found that lower metallicity stars seemed to have been formed at a range of heights above the galactic plane whereas those with higher metallicity were formed preferentially near the galactic plane. When interpreted in the context of their model for the Galaxy, these findings lead ELS to conclude that the Milky Way began as a spherical
rotating gas cloud with a radius of at least 100 kpc. This cloud collapsed on a free-fall timescale of a few $10^8$ years and spun up while the first generation of stars and globular clusters formed. These earliest stars were relatively metal-poor and on highly eccentric orbits caused by the rapidly changing gravitational potential of the gas cloud. ELS pointed out that the short timescale of this collapse explains why all of the globular clusters for which accurate photometry was available - M3, M5, M13, M15, and M92 - are so nearly of the same age. One caveat that is immediately obvious from the work of ELS is that their kinematical data only sampled the halo out to a maximum of $\approx 10$ kpc above the galactic plane. As a result, their conclusions are necessarily only applicable to this portion of the halo.

The gradual collapse scenario has been principally advocated by Searle & Zinn (1978, hereafter SZ). However, before describing the work of SZ, it is beneficial to discuss the developments between the time of ELS and SZ. In particular, Sandage & Wildey (1967, see also van den Bergh 1967) noticed that M3, M13, and NGC 7006 have very similar metallicities but quite different HB morphologies - the HB of M13 is completely blueward of the RR Lyrae instability strip, that of NGC 7006 is almost completely redward, and M3 has both a blue and red HB. They knew that the morphology of the HB is determined primarily by metal abundance (the first parameter) based on the theoretical work of Faulkner (1966) and Faulkner & Iben (1966), i.e. high metallicity leads to a red HB morphology and vice versa. However, here were three clusters with the same metallicity but vastly different HB morphologies. Thus, they concluded that a second parameter must be at work affecting the appearance of the HB, giving rise to the ‘Second Parameter Effect.’ SZ were searching for this second parameter. They analyzed the metallicites of globular clusters as a function of their Galactocentric distances. From this, they concluded that outside of $\approx 8$ kpc, the abundance distribution of globular clusters is independent of galactocentric distance. This is in contrast to the predictions of the ELS model which would predict a dependence of abundance on radial distance. Next, SZ constructed the [Fe/H] : HB-type diagram for globulars in different ranges of Galactocentric distance. From this they noticed that the clusters of the inner halo ($R_{GC} < 8$ kpc) are not influenced by the second parameter whereas those outside of 8 kpc are so influenced. They asked the question: which of the many second parameter candidates can vary with Galactocentric distance in the most plausible manner? They argued that the cluster age is the most likely candidate to be the second parameter, implying a range of ages among the globular clusters outside 8 kpc. As a result, the following picture of halo formation emerges. The Galaxy began as a spherical gas cloud. The inner portions (inside 8 kpc) collapsed quickly ($< 1$ Gyr), but the outer parts fragmented in a chaotic fashion. These fragments were on eccentric orbits and sometimes collided with each other losing their angular momentum. After a few Gyr, the gaseous components of these fragments merged with the inner Galaxy while the stars and globular clusters formed in these fragments retain the orbits they possessed at the time of formation.

These then are the two basic formation scenarios of the Milky Way. An important part of galaxy formation models relates the dynamics of the collapsing gas cloud to the history of metal enrichment in the forming galaxy (see e.g. Larson 1975; Tinsley 1975; Ostriker & Thuan 1975;
These are relevant in understanding the formation of the Galactic halo and the age-metallicity relation (Larson 1990). As such, each formation model makes specific testable predictions. In particular, the ELS model predicts a small ($<< 1$ Gyr) age spread among the Galactic globular clusters and a strict one-to-one correspondence between the age of a cluster and its metallicity; there is no implicit prediction regarding the nature of the second parameter. The SZ model predicts an age range of a few Gyr among the Galactic globulars and is predicated on the assumption that age is the second parameter. In the present review, we will examine the present state of the observational data relevant to these predictions. First however, we summarize the techniques used to estimate the globular cluster ages, which are crucial to testing the predictions.

3. Age Determination Methods

Our ability to estimate the ages of star clusters is founded on one basic principal. Given a chemical composition, the theory of stellar evolution makes definite predictions about the rate of evolution of the luminosity and radius of a stellar model.

Thus in principle, with perfect theoretical isochrones, and with the knowledge of the cluster chemical composition parameters, one could derive unambiguously its distance modulus and its age. This is true provided one could observe either the full CMD (including the HB) of a star cluster, and place it in the theoretical ($M_{bol}$ : $logT_{eff}$) plane, or derive its luminosity function based on a complete sample down to magnitudes well below the main sequence turnoff.

Practice differs from this ideal situation. On the theoretical side, although we can calculate the evolution of $L$ with time, there remains uncertainties in the run of $R$ vs. time and in the conversion from $T_{eff}$ to color (Demarque et al. 1988; Bergbusch & VandenBerg 1992). This affects not only the morphology of the theoretical CMD (isochrone), but also, in a much lesser way, the predicted luminosity function in a given spectral band. In addition, the theoretical HB luminosity may be suspect (Da Costa & Armandroff 1990), requiring an empirical distance calibration for $M_v(RR)$.

On the observational side, there are errors in the determination of the CMD (in the photometry, particularly at the faint end), and in the cluster chemical composition, primarily [Fe/H] and [O/Fe]. Interstellar reddening can be another source of error which matters in some dating methods. In addition, when using luminosity functions, there are additional problems with completeness which can be very serious at the faint end (Bolte 1994; Sandquist et al. 1996). Finally, the choice of $M_v(RR)$ is a major source of age uncertainty (Renzini 1991; Chaboyer et al. 1996a).

In order to get around these problems, a number of methods for globular cluster age determinations have been devised over the years, which made best use of the information available at the time. It is important to note that there is nothing immutable about any of these approaches. As additional knowledge becomes available (either improved stellar models, or higher quality observations), new and better methods should be used (e.g increased use of luminosity functions), and approaches previously abandoned should be reconsidered as they may become practical again.
3.1. Main Sequence Fitting

Main sequence fitting was the preferred method some thirty years ago. This technique is used to derive the distance to a globular cluster, and its age may be determined from the absolute magnitude of the turnoff ($M_V$(TO)). Much work was done both theoretically and empirically to understand the dependence of the ZAMS position on chemical composition. One of the important issues early-on was the uncertainty in the helium abundance of halo stars (Demarque 1960). In this case, the observed unevolved main sequence in the CMD is superimposed on a fiducial ZAMS appropriate to the chemical composition of the star cluster. In principle, the most straightforward way to establish the position of this fiducial ZAMS is to use field stars with reliable trigonometric parallaxes. Before the advent of the HIPPARCOS database, this approach had been utilized sparingly because too few precise parallaxes for metal poor field dwarfs have been available (Carney 1980; see also the review by Sandage 1986). One could also adopt the position of the better established metal rich ZAMS (the Hyades main sequence), and apply corrections to it for metallicity differences (Sandage & Eggen 1959; Eggen & Sandage 1962). This method, originally based on corrections derived from the measured ultra-violet excess $\delta(U-B)$ (a metallicity index), only provides an approximate index of the cluster metallicity and associated ZAMS calibration (Wildy et al. 1962). To determine the ZAMS position, additional evolutionary corrections to the position of the empirical main sequence must be applied, which are themselves uncertain since they require an estimate of the ages of individual field subdwarfs (Eggen 1973). A final problem with the main sequence fitting method is that it is difficult to separate reddening corrections due to interstellar absorption from the ultra-violet excess, both of which must be known separately.

For all these reasons, main sequence fitting has been generally abandoned in recent years. But the availability of high quality parallaxes from the HIPPARCOS satellite, supplemented by some HST parallaxes has revived interest in the method (Chaboyer et al. 1997; Gratton et al. 1997; Pont et al. 1997; Reid 1997), since it is in principle the most straightforward and fundamental approach. However, even relatively small errors in the reddening ($\pm$0.02 mag) lead to rather large errors in the distance modulus derived from main sequence fitting ($\pm$0.10 mag). Thus, main sequence fitting is only suitable for clusters with well determined (generally small) reddenings. In addition, deep main sequence photometry is required, which restricts the use of main sequence fitting to well studied clusters. Finally, even with the release of the Hipparcos database, the number of metal-poor single stars with well determined parallaxes ($\sigma_\pi/\pi < 0.10$) is rather small (less than 15). For the above reasons, the recent papers utilizing main sequence fitting have concentrated on the absolute ages of a few globular clusters, and have been unable to address the question of the relative ages of Galactic globular clusters as a whole.

Two other globular cluster distance indicators of great potential importance should be mentioned. One is the position of the white dwarf cooling sequence; some white dwarfs have now been observed with HST in the globular clusters M4 (Richer et al. 1995) and NGC 6752 (Renzini et al. 1996). The other would be the discovery of eclipsing binary systems near the main sequence in globular clusters (Paczynski 1997). Both of these techniques allow one to determine a precise
distance to a cluster. From this, the absolute magnitude of the turnoff may be determined, and used to estimate the age of a cluster.

### 3.2. The $\Delta V$ Method

The $\Delta V$ technique, where $\Delta V = V(MSTO) - V(HB)$ measured at the main sequence turnoff color, is illustrated in Fig. 1. The main advantage of the $\Delta V$ technique is that it is insensitive to interstellar reddening. The quantity $\Delta V$ can be directly measured from the CMD, provided the horizontal branch is sufficiently well defined at the color of the main sequence turnoff. In its original form (Sandage 1982) it relied on an empirical or semi-empirical determination of the absolute magnitude $M_v(RR)$ of the RR Lyrae variables. In recent years, the $\Delta V$ method has been considered the most precise and consistent technique for dating purposes, and various forms of the method have been developed (Iben & Renzini 1983; Buonanno et al. 1989; Sarajedini & King 1989; Chaboyer et al. 1996a). In theory, it is also independent of distance since both the turnoff and horizontal branch luminosities can now be derived theoretically, given the chemical composition (Sweigart 1987,1994; Yi et al. 1993). This is possible because evolutionary calculations to the helium flash and on to the HB have been carried out for the case of no or moderate mass loss (Mengel & Sweigart 1981; Cole et al. 1985), applicable to the HB stars that evolve from red to blue. The origin of the extreme mass loss, which is responsible for the HB stars that evolve from the blue end of the HB (sbB and sdO), is still a matter of controversy and several explanations have been proposed (Mengel et al. 1976; Bailyn & Pinsonneault 1995; D’Cruz et al. 1996).

In practice, the $\Delta V$ technique suffers from the difficulty in locating precisely the turnoff luminosity (defined here as the bluest point) because the CMD sequence is nearly vertical at this point and difficult to measure precisely. This difficulty may be overcome by using a point on the subgiant branch (SGB), as discussed by Chaboyer et al. 1996b. This SGB point is +0.05 mag redder than the turnoff and represents a location where the CMD morphology is more horizontal than the turnoff making the magnitude measurement much more precise. Another difficulty with the $\Delta V$ technique concerns the horizontal branch luminosity [usually one adopts $M_v(RR)$]. As mentioned above, the HB luminosity can in principle be calculated theoretically since the helium core mass at the helium core flash is known, and such calculations are becoming increasingly reliable. But in recent years, the determination of $M_v(RR)$, particularly its dependence on [Fe/H], has been the subject of a lively debate. It is convenient to write:

$$M_v(RR) = \alpha [\text{Fe/H}] + \beta$$

where the slope $\alpha$ has important ramifications for the relative ages of globular clusters in the Galactic halo, and the constant $\beta$ in addition affects their absolute ages (Zinn 1985; Sarajedini & King 1989). Sandage (1981a, 1981b) first derived the coefficients in eq.(1) empirically, and estimated a “steep” slope $\alpha$ of 0.35. From a theoretical calibration based on synthetic HB population models, Lee et al. (1990, 1994, hereafter LDZ) derived a “shallower” $\alpha$ in the range 0.17-0.19. A shallow
slope in good agreement with LDZ, has also been found using the semi-empirical Baade-Wesselink method $\alpha = 0.16 \pm 0.03$ (Jones et al. 1992) and $\alpha = 0.21 \pm 0.05$ (Skillen et al. 1993). These shallow slopes have been questioned by Feast (1997), who argued that the usual practice of treating [Fe/H] as the dependent ($x$) variable was subject to bias. Feast (1997) performed the inverse regression ($M_v$ as the dependent variable) and found $\alpha = 0.37$. However, in performing his regression, Feast (1997) did not include the observed errors. We have repeated the analysis of Feast (1997), using the same data set but including the quoted errors in the regression. Using a variety of regressions (including the BCES estimator of Akritas & Bershady 1996; FITEXY routine from Press et al. 1992; and the routine from Ripley & Thompson 1987), we have found the regressions (both forward and inverse), do not support the high value of $\alpha$ claimed by Feast (1997). Using the same 28 data points as Feast (1997), our regressions (both inverse and direct) yielded $\alpha = 0.22$. The error in the slope varied from $\pm 0.04$ (BCES) to $\pm 0.06$ (FITEXY).

The shallow slope is also supported by HST observations of globular cluster HBs in M31, where $\alpha = 0.13 \pm 0.07$ (Fusi Pecci et al. 1996). Using the relation between Fourier decomposition and luminosity for RRab stars in GCs Kovács & Jurcsik (1996) determined that $\alpha$ is less than 0.20. It would appear that the evidence favors somewhat shallow slopes ($\alpha \leq 0.25$). It is important to note at this point that $\alpha$ is not a universal constant; theory predicts that stars of the same metallicity evolve through the RR Lyrae instability strip at different luminosities depending on whether they originate on the red or blue side of the instability strip. Thus $M_v(RR)$ depends on HB morphology, and RR Lyrae variables are more luminous in clusters with blue HB morphology types than in red HB morphology type, for a given metallicity (Lee 1991). Synthetic HB population models show that the sensitivity of $M_v(RR)$ on HB morphology is most pronounced at very low metallicity. This explains why globular cluster samples which contain only very blue HB types at the lowest metallicities, can yield a large value of $\alpha$. In addition, the value of $\beta$ is also quite uncertain, and this affects sensitively the determination of globular cluster absolute ages (Iben & Renzini 1983; Chaboyer et al. 1996a,1996b). As a result, a semi-empirical version of the $\Delta V$ technique has been used in most recent discussions of globular cluster ages, in which the turnoff luminosity is calibrated on theoretical isochrones, and $M_v(RR)$ is empirically derived (Chaboyer et al. 1996a).

### 3.3. The $\Delta(B - V)$ Method

The quantity $\Delta(B - V)$ is defined as the color difference between the turnoff and the base of the giant branch (Sarajedini & Demarque 1990; VandenBerg et al. 1990, VBS; Sarajedini 1991). It is illustrated in Fig. 1. Because it requires a well defined turnoff/subgiant branch and a fine grid of isochrones for the chemical composition of the cluster, it has only been introduced recently in globular cluster dating. Since the position of the giant branch in the CMD is to first order only a function of chemical composition, $\Delta(B - V)$ decreases with increasing age. In practice, the method is calibrated using the $\Delta V$ method, but in principle, a set of theoretical isochrones could provide an absolute age calibration.
The $\Delta(B - V)$ method is excellent for measuring the relative ages of clusters with similar metallicities. For absolute ages, its calibration suffers from uncertainties in the radii of stellar models which are sensitive to the depth of the convection zone and to the efficiency of helium and heavy element diffusion in the envelope during evolution (Deliyannis et al. 1990; Demarque et al. 1991; Chaboyer et al. 1992). Furthermore, unlike the $\Delta V$ method whose calibration with age is largely insensitive to the particular isochrones used, ages determined with $\Delta(B - V)$ can vary widely depending on which isochrones are used for the calibration.

A slight variation on the “classical” $\Delta(B - V)$ method has recently been introduced by Saviane et al. (1997). They measure the $V - I$ color difference between the main sequence turnoff and the red giant branch (RGB) at a level 2.2 magnitudes brighter than the MSTO. It is unclear what advantages this has over $\Delta(B - V)$. However, because it combines the concept of the $\Delta(B - V)$ method with knowledge about the magnitude of the MSTO, the method of Saviane et al. (1997) may be subject to additional observational and theoretical errors.

3.4. The Use of Luminosity Functions

In principle, luminosity functions could be used to determine the cluster age uniquely, without any external knowledge of the cluster distance or interstellar reddening, provided the chemical composition is known. It is in practice difficult to achieve; accurate photometry is needed down to faint magnitudes well below the turnoff in order to determine precisely the change in slope of the luminosity function due to the turnoff, and for the same reason completeness of the data is also essential down to faint magnitudes. Because of these difficulties, luminosity functions have until recently not been used for dating globular clusters. Some recent studies of globular cluster giant branch luminosity functions (Jimenez & Padoan 1996) can provide useful self consistency checks of evolutionary rates along the giant branch, but not an absolute calibration of cluster ages. However, with the availability of high precision photometry, luminosity functions near the turnoff and down the main sequence will become increasingly useful for globular cluster dating. This is particularly so because the turnoff slope change in luminosity functions is less sensitive to details of the theoretical shape of the isochrone in the CMD, which are affected by convection zone depth and element diffusion rates.

3.5. How Good Are Stellar Models Near The Main Sequence Turnoff?

Our best information comes from the Sun. The extraordinary precision of the Global Oscillations Network Group (GONG) data indicate that the standard solar model predicts the correct run of the sound speed in the Sun to one part in one thousand, provided the effects of helium and heavy element diffusion are included in the models. Perhaps more importantly in our context, the small spacings of the solar p-modes now permit for the first time the determination of the solar age
from seismology (4.5 Gyr) (Guenther & Demarque 1997); and it agrees to within 0.1 Gyr with the radioactive age of the Sun (Guenther 1989; Bahcall et al. 1995). This means that in the case of the Sun and its early evolution near the main sequence, the standard theory predicts the solar age to within two per cent, and it will soon be possible to do much better. One might mention also that in the case of two nearby field disk population binary systems, Procyon (α Canis Minoris) and α Centauri, conventional stellar models are consistent with the available observations on masses and chemical compositions to better than ten per cent. It is reasonable to expect that as for the Sun, seismology will provide very stringent tests of the internal structure, distances, and ages of these two binary systems and of a few nearby dwarfs and subgiants in the field (e.g. the recent observations of η Bootes by Kjeldsen et al. 1995).

Thus recent results for the Sun are very encouraging, but at this date stellar seismology is still in the future. Thus until we can rely on good quality seismic data for a number of nearby dwarfs and subgiants, the ages of old open star clusters must be determined by conventional theoretical isochrone fitting, and can be determined at best to within ten per cent. Observationally, the errors are due to the difficulty in defining the main sequence precisely because of small numbers and cluster membership problems, and errors in distances and interstellar reddening. On the theoretical side, uncertainties are primarily due to convection, convective core overshoot and the depth of the convective envelope.

The problem of the depth of the convection zone, coupled with the uncertainties of the treatment of convection in model atmospheres for halo stars, and the associated uncertainties in the transformation from theoretical temperatures and gravities to observed colors are significant caveats to our reliance on the detailed shapes of theoretical isochrones near the main sequence turnoff. Helium and heavy element diffusion are also known to play a role in affecting the shape of the turnoff (Noerdlinger & Arigo 1980; Proffitt & Michaud 1991; Chaboyer et al. 1992). Much basic work remains to be done in the area of numerical simulations of radiative hydrodynamics as applied to stellar envelopes and atmospheres in low metallicity stars (Fuhrmann et al. 1993; van’t Veeren-Menneret & Megessier 1996). In all these areas, progress will be aided by recent advances in helio- and astero-seismology (e.g. Demarque et al. 1997a). These uncertainties argue strongly in favor of using the ∆V method, which is less dependent on the least known aspects of stellar evolution.

Halo stars in the field and in globular clusters are generally more distant than disk stars, and we do not know as much about their individual properties. There are however a few factors which facilitate the precise dating of globular clusters. Observationally, because of large numbers and no cluster membership problems, the main sequence can be remarkably well delineated (see e.g. NGC 6752, Rubenstein & Bailyn 1997), and with some effort, reliable luminosity functions can be derived (Sandquist et al. 1996). On the theoretical side, low metallicity means better determined opacities and equation of state than in the Sun and Sun-like stars, and therefore probably more reliable isochrones. Despite these advantages, we must remember that empirical data on the masses and luminosities of individual metal-poor dwarfs are sorely needed. The Hipparcos mission has provided high quality parallaxes for ~15 individual subdwarfs in the field, and parallaxes from
HST may increase this number in the future. Perhaps good fortune will also lead to the discovery of eclipsing binaries near the turnoff of globular clusters.

The different age determination methods make use of different sets of information from GC photometry, and by and large make use of only part of the information contained in the data. The result is that different methods can yield different age calibrations. The discrepancies are indicators of the uncertainties in the stellar models and the observations, and much can be learned from them about the details of stellar physics. For example, uncertainties in stellar radii will affect the $\Delta V$, $\Delta (B-V)$ and luminosity function calibrations in different ways. As the data become more complete and more accurate to faint magnitudes, they will provide increasingly stringent tests of stellar evolution theory.

These then are the various techniques used to estimate the ages of globular clusters. We now proceed with our discussion of what the globular cluster ages tell us about the formation of the Galactic halo. First, the nature of the second parameter effect is examined. We then go on to discuss the age range among the Galactic globulars and the existence of an age-metallicity relation.

4. The Concept of Age as the Global Second Parameter

The SZ scenario for the formation of the Galactic halo is based upon the premise that cluster age is the global second parameter, but what exactly does ‘global second parameter’ mean? The work of SZ and, more recently, Lee et al. (1994) has shown that the second parameter effect varies with Galactocentric distance. Globular clusters inside 8 kpc of the Galactic center are minimally affected by the second parameter, if at all, while those outside 8 kpc are significantly affected. That is to say, whatever this second parameter actually is, it varies on a global scale. Some have argued that there may not be a single second parameter affecting the HB morphology (SVB). In this scenario, the global variation in HB-type is caused by different parameters acting in precisely the proper amount to yield the observed variation of HB-type. This situation seems rather contrived and is therefore unlikely to be correct. The large scale (global) variation of the second parameter effect argues in favor of a global second parameter.

Let us assume now that there is a global second parameter, which along with metallicity, influences the HB morphologies of globular clusters. We must also assert that it is highly unlikely that there exist two second parameters, which affect the HB morphology to precisely equal degrees. What is more likely is that there is the first parameter (metallicity), the second parameter, the third parameter, and so on. As a result, it makes little sense to speak of ‘a’ second parameter, which implies that more than one parameter of equal significance is at work.
4.1. The Case of NGC 288 and NGC 362

Ever since Sandage & Wildey (1967) proposed that the second parameter is helium abundance and SZ made convincing arguments in favor of age as the second parameter, a number of investigators have endeavored to reveal the nature of the global second parameter. The primary question that is asked is: “Can a difference in age account for the difference in HB-types of clusters at a given metal abundance?” As noted above, M3 and M13 were recognized early-on as being a ‘second parameter pair’ in which two clusters have similar metallicities but different HB-types. However, these were not the first such clusters to be studied in detail. The first pair of clusters to receive in-depth scrutiny are NGC 288 and NGC 362. These have \([Fe/H] \approx -1.3\), but NGC 288 has a completely blue HB whereas NGC 362 has an HB that is predominantly red.

Bolte (1989) presented the first detailed comparison of the main sequence turnoff (MSTO) regions of NGC 288/362. He constructed precise B–V color-magnitude diagrams (CMDs) for these clusters down to 3 magnitudes below the MSTO. Then, by shifting them by the relative reddenings and distance moduli, both of which are small, Bolte (1989) showed that the MSTO of NGC 362 is brighter than that of NGC 288. This leads directly to an age difference of \(\approx 3\) Gyr with NGC 288 being older than NGC 362. This finding, along with the work of King et al. (1988) and Dickens et al. (1991), revealed direct unequivocal evidence that the HB of NGC 288 is bluer than that of NGC 362 because the former is older than the latter.

A number of other authors have reached the same conclusion (e.g. Demarque et al. 1989; Sarajedini & Demarque 1990; VandenBerg et al. 1990). Most notable among these is the work of Green & Norris (1990, hereafter GN90). They followed an approach similar to that of Bolte (1989) except that they chose to work in the B–R CMD arguing that changes in the morphology of the CMD near the MSTO are more pronounced in B-R as compared with B–V. The conclusion reached by GN90 is identical to that of Bolte (1989), namely that NGC 288 is older than NGC 362 by 3 Gyr. In contrast, VandenBerg & Durrell (1990) used the magnitude of the first ascent red giant branch tip to estimate the relative distance moduli of NGC 288 and NGC 362 to allow a comparison of the MSTO’s of these clusters. From such a comparison, which included the adoption of relative reddenings, they concluded that there is no detectable age difference between these clusters. However, in considering the significance of this result, one must weigh the following points. GN90 note that the brightest first ascent giant star in NGC 288 should have a similar color to an analogous star in NGC 362 if the metallicities of the clusters are indeed similar. However, GN90 argue that the colors of the stars are in fact significantly different; this and other evidence presented by GN90 indicates that VandenBerg & Durrell have chosen a star that is several tenths of a magnitude too faint to be at the giant branch tip. Da Costa & Armandroff (1990) have reached a similar conclusion after considering the bolometric magnitudes of the brightest stars on the RGB of NGC 288. Therefore, it seems that if one modifies the analysis of VandenBerg & Durrell to account for the improperly chosen giant branch tip magnitude of NGC 288, one again reaches the conclusion that NGC 288 is older than NGC 362 by \(\approx 3\) Gyr.
Until recently, it seemed that a general consensus was building regarding the relative age of NGC 288/NGC 362. The review presented by SVB includes a new analysis of the entire question using NGC 1851, a cluster with both a red and blue HB, as a ‘bridge’ between NGC 288 and NGC 362. In the spirit of VandenBerg & Durrell (1990), they once again concluded that an age difference between these clusters is not proven. They begin by assuming that all three clusters (NGC 288, NGC 362, NGC 1851) have the same age. This permits the registration of their main sequence/subgiant branch (SGB) fiducial sequences using the nearly horizontal portion of the SGB as shown in Fig. 2a. This procedure yields magnitude and color offsets required to bring the three sequences into coincidence at the SGB. If any age differences exist between the clusters, then using these magnitude and color offsets on the brighter portion of the CMD should cause their HBs to be misaligned. Fig. 3 shows a comparison of the NGC 1851 fiducial sequence from Walker (1992) with the photometric data of NGC 362 (Harris 1982) and data for NGC 288 from Bolte (1992) and Bergbusch (1993). The latter has been offset to the photometric scale of Bolte (1992). The comparisons in Fig. 3 indicate that while the fiducial sequence of NGC 1851 lines up adequately with the photometric data for NGC 362, it does not faithfully reproduce the position and shape of the NGC 288 RGB and HB. In particular, the red end of the blue HB is fainter in NGC 1851 as compared with NGC 288, and the entire blue HB of NGC 1851 is systematically brighter/bluer than that of NGC 288. This is contrary to the conclusions drawn by SVB from a similar comparison. The difference in results is due to the fact that SVB included NGC 1851 RR Lyraes in their definition of the position of this cluster’s blue HB thus extending the red end of the blue HB into the RR Lyrae instability strip. In fact, assuming $E(B - V) = 0.02$ (Da Costa & Armandroff 1990), the intrinsic color of the blue edge of the instability strip in NGC 1851 is inferred to be $(B - V)_0 = 0.23$ from the blue HB envelope drawn by SVB in their Fig. 7. From the work of Sandage (1990), this is approximately 0.05 mag too red since he places the blue edge of the strip at $(B - V)_0 = 0.18$. Furthermore, it turns out to be extremely difficult to find magnitude and color offsets that allow the entire fiducial sequence of NGC 1851 to fit the photometric data of NGC 288. We return to this point below. As for the apparent alignment of the NGC 362 and 1851 HBs in Fig. 3, what is not shown in Fig. 6 of SVB is the poor correspondence of the NGC 362 and NGC 1851 unevolved main sequences between $V \approx 20.2$ and $V \approx 21.5$ now seen in Fig. 2a. If we adjust the offset for NGC 362 to optimize the fit in this magnitude range, we arrive at the comparison shown in Fig. 2b. This clearly shows that when the main sequences of NGC 288 and 362 are aligned, the latter is younger than the former. In addition, with this new offset for NGC 362, its red HB will be brighter than that of NGC 1851 as it should be given that the luminosity of the red HB is dependent on age as well as metallicity (see Sarajedini et al. 1995, hereafter SLL). It appears therefore that the work of many previous investigators presenting evidence in favor of an age difference between NGC 288 and NGC 362 continues to stand on a robust foundation.

As mentioned above, it is difficult to satisfactorily align the entire fiducial sequence of NGC 1851 from the RGB to the HB to the main sequence with the photometry for NGC 288. It is unclear whether this is due to an age difference, a chemical abundance difference, a combination of these two effects, or something completely unknown. What is clear is that NGC 1851 is an extremely
unusual cluster in that its HB morphology is strongly bimodal. In addition, Catelan (1996, private communication) has found that the RR Lyraes in NGC 1851 possess unusual period shifts.

4.2. The Case of M3 and M13

The difference in HB-types between M3 and M13 is not as extreme as the difference between NGC 288 and NGC 362. As a result, if age is the global second parameter, then we would expect the age difference between M3 and M13 to be somewhat less than \( \approx 3 \) Gyr. One of the first modern attempts to answer this question was provided by VandenBerg et al. (1990). Although their photometry was of sufficient precision, they could not unequivocally address the age difference question because there were only a handful of stars on the subgiant and red giant branches in the CMDs making the measurement of \( \Delta (B - V) \) uncertain. Their quoted age differences, though fraught with uncertainties, did indicate that M3 may be younger than M13 by 1 to 2 Gyr. This is consistent in sign and magnitude with what we would expect based on the comparison between NGC 288 and 362.

Catelan & de Freitas Pacheco (1995) applied the \( \Delta V \) method to M3 and M13. They combined published values of \( \Delta V \) for each cluster with synthetic HBs designed to estimate the RR Lyrae luminosity. This latter approach is especially important in the case of M13 which has a very blue HB morphology and few RR Lyrae variables. Based on this analysis, they are able to set an upper limit of \( \approx 3 \) Gyr on the age difference between M3 and M13. Such a limit is not a useful constraint when one expects an age difference less than \( \approx 3 \) Gyr. Another factor to consider when evaluating the work of Catelan & de Freitas Pacheco (1995) is that they utilized cluster photometry with a precision that is inadequate to answer the question they posed. For example, in the case of M3, they relied on the \( \Delta V \) value estimated by Buonanno et al. (1989) which actually dates back to the photographic photometry of Buonanno et al. (1986) and some apparently unpublished data. Catelan & de Freitas Pacheco examined two different possibilities for the \( \Delta V \) value of M13 both of which are problematic. In one case, photographic photometry of only 3 RR Lyraes from Pike & Meston (1977) is used to estimate \( V(HB) \), and in the other case, the CCD photometry of Guarnieri et al. (1993), which exhibits significant photometric scatter, is utilized to yield the turnoff magnitude. Thus, the results of their analysis do not serve to shed light on the question of whether there is an age difference between M3 and M13 because the quality of the observational inputs is inadequate.

4.3. The Case of the Galactic Globular Clusters as a Whole

In addition to examining the relative ages of second parameter cluster pairs, it is also important to investigate the Galactic globular cluster system as a whole. In this way it is possible to see if the age explanation is adequate to explain the global behavior of the second parameter effect. A number
of investigators have considered the variation of cluster age with HB morphology over a narrow range in metallicity. Most of this work has used the $\Delta V$ age diagnostic. For example, Sarajedini & King (1989) and Chaboyer et al. (1992) conclude that there is a statistically significant correlation between age and HB type with older clusters having bluer HB types. Carney et al. (1992a) have performed a similar analysis using different observational and theoretical inputs and reached the same conclusion though at a lower level of significance.

Zinn (1993) has adopted a different approach to this question. He used the $[Fe/H]$:HB type diagram to divide the Galactic globulars into “old halo” and “younger halo” subsamples based on their position in this diagram (see his Fig. 1). He found a correlation between cluster age (as derived from both the $\Delta V$ and $\Delta (B-V)$ methods) and HB type with the younger clusters having redder HBs. Chaboyer et al. (1996a) have taken Zinn’s (1993) analysis further by determining new and improved $\Delta V$ ages for 43 Galactic globulars and computing the mean ages of the old halo and younger halo samples. These age designations were made based on the expectation that redder HB clusters are younger, and indeed the $\Delta V$ ages of Chaboyer et al. (1996a) indicate that the younger halo clusters are some 2 to 4 Gyr younger than the old halo clusters with statistical significances in the $4\sigma$ to $8\sigma$ range. Even when the globular clusters belonging to the Sagittarius dwarf galaxy and those with very young ages are all removed, the age difference between the younger halo and the old halo falls in the range 1.5 to 2.5 Gyr with a mean significance level of $3.4\sigma$.

We can take these analyses still further by avoiding the step of converting measured $\Delta V$s to ages and simply examining the behavior of $\Delta V$ with HB type. The top panel of Fig. 4 shows the canonical $[Fe/H]$:HB type diagram for the Galactic globular clusters using data from Lee et al. (1994, hereafter LDZ94). The solid lines are the results of synthetic HB calculations (LDZ94) and indicate the expected locations of clusters for different relative ages based on the assumption that age is the sole global second parameter. In the lower panel of Fig. 4, we investigate the behavior of $\Delta V$ with HB type for two metallicity ranges: $-1.6 \leq [Fe/H] \leq -1.4$ (filled circles) and $[Fe/H] \leq -2.0$ (open circles) utilizing the observational database of Chaboyer et al. (1996a). Weighted least squares fits to the filled circles yield highly significant relations (probabilities $>99\%$) even if we randomly remove single points from the fits. This is also true when fits are performed to the open circles. The solid line in the lower panel is the expected behavior of $\Delta V$ as inferred from the synthetic HB models (top panel) for clusters with $[Fe/H] = -1.5$ while the dashed line is the expected behavior for clusters with $[Fe/H] = -2.2$. These have been shifted vertically to match the mean locations of the observed $\Delta V$ values. These lines reproduce the general trends quite well providing further evidence that age is the strongest candidate to be the global second parameter.

### 4.4. Second Parameter Effect Among Red HB Clusters

The principal diagnostic diagram used to gauge the second parameter effect has traditionally been the aforementioned $[Fe/H] : (B - R)/(B + V + R)$ diagram. However, as several authors have noted, this diagram becomes insensitive to the “second parameter” when the number of blue
HB stars becomes zero (i.e. $B = 0$). That is to say, one cannot explore the existence of a second parameter among red HB clusters using the $[Fe/H] : (B−R)/(B+V+R)$ plane. However, one way in which we can explore this issue is by plotting $[Fe/H]$ as a function of the dereddened mean color of the red HB stars ($⟨(B−V)_{o,RHB}⟩$, for clusters with $(B−R)/(B+V+R) < −0.8$. Figure 5a shows such a diagram which includes the observational data analyzed in SLL (filled circles), consisting of Galactic globular clusters as well as the SMC cluster Lindsay 1 and the LMC cluster ESO121-SC03. In addition, the globular cluster Pyxis (Sarajedini & Geisler 1996) and the SMC clusters Kron 3 and NGC 121 (Mighell et al. 1997, all open circles) are also plotted in Fig. 5a. The conversion from $(B−R)_o$ to $(B−V)_o$ for Pyxis has been performed using the empirical transformation devised by Sarajedini & Geisler (1996). The adopted reddenings are taken from the compilation of Harris (1996), and the papers by Rich et al. (1984) for Kron 3, Stryker et al. (1985) for NGC 121, and Sarajedini & Geisler (1996) for Pyxis. Also plotted in Fig. 5 are the mean intrinsic colors of Zero Age Horizontal Branches (ZAHBs) for scaled-solar abundances and masses of $0.90M_⊙$ (left dashed line) and $0.66M_⊙$ (right dashed line) from the work of Dorman (1992). These lines are meant to indicate the general behavior of ZAHB color with $[Fe/H]$ as well as with mean mass.

The dominant influence of the first parameter, metallicity, is evident in Fig. 5a with more metal-rich clusters having generally redder intrinsic HB colors. In addition, there appears to be a nonzero width in the distribution of colors at a given metallicity. This is the classic signature of the presence of a second parameter effect. One weakness of this diagram is the need to adopt a reddening for each cluster. This adds additional uncertainty to the color axis of Fig. 5a. To circumvent this problem, Fig. 5b shows $[Fe/H]$ as a function of the difference in color between the RGB and red HB. The horizontal error bars are estimated by adding, in quadrature, the standard error of the mean HB color and the standard error of the RGB color. This procedure yields a mean error in $d_{B−V}$ of $\approx 0.01$ mag. In accord with the conclusions drawn from Fig. 5a, Fig. 5b also indicates that there is a significant spread in $d_{B−V}$ at a given metallicity, especially in the range $−1.5 < [Fe/H] < −1.0$.

The most straightforward explanation for this nonzero spread in $d_{B−V}$ is the presence of an age range among these red HB clusters. Thus, at a given metal abundance, clusters with smaller values of $d_{B−V}$ (or larger values of $⟨(B−V)_{o,RHB}⟩$) are younger. In fact, SLL demonstrated that age differences estimated from $d_{B−V}$ are fully commensurate with relative ages inferred from the position of the MSTO for clusters with metallicities in the range where their theoretical models were applicable. In the present paper, we can perform a similar analysis looking specifically at the relative values of $⟨(B−V)_{o,RHB}⟩$ between clusters of similar metallicity. We have chosen the cluster NGC 362 with $[Fe/H] \approx −1.3$ (Zinn & West 1984) as the comparison cluster. Fiducial sequences are available for NGC 362 in $B−R$ (Green & Norris 1990) and $B−V$ (Harris 1982; VandenBerg et al. 1990). Figures 6 and 7 show CMD comparisons of six red HB clusters with the fiducial sequence of NGC 362. The photometric data are taken from the following sources: Kron 3 (Rich et al. 1984 $[B−R]$; Mighell et al. 1997 $[B−V]$), NGC 121 (Stryker et al. 1985 $[B−R]$; Mighell et al. 1997 $[B−V]$), Pyxis (Sarajedini & Geisler 1996), Palomar 4 (Christian & Heasley
Each cluster has been registered to the magnitude of the NGC 362 red HB and the color of the RGB at the magnitude of the HB. This is not strictly correct since SLL showed that the luminosity of the red HB is dependent on metallicity and age, but for the purposes of the present comparison, making this correction will not alter the results. It is immediately obvious from Figs. 6 and 7 that there is a correlation between relative HB color and relative age (as judged from the MSTO magnitude) with respect to NGC 362. In particular, the clusters with brighter MSTO mags also have redder HB colors in comparison with NGC 362.

4.5. Exceptions To the Rule?

4.5.1. Arp 2

The globular cluster Arp 2, which is believed to be a member of the Sagittarius dwarf galaxy, has \( \frac{(B-R)}{(B+V+R)} = +0.86 \) (Sarajedini & Layden 1997, hereafter SL97), a predominantly blue HB morphology, even though Buonanno et al. (1995, hereafter BCFRF) claim that its age is some 3 Gyr younger than other clusters at its metallicity ([Fe/H] = -1.80±0.10, SL97). As a result, the properties of Arp 2 clearly contradict the explanation of age as the sole global second parameter. However, we note that the measured \( \frac{(B-R)}{(B+V+R)} \) value of Arp 2 is an upper limit since investigators have been forced to assume \( R = 0 \) because of the severe field contamination present in this part of the CMD. To gauge the importance of this effect, we have counted stars in the red HB portion of the SL97 Arp 2 CMD; we find \( R = 9 ± 3.0 \), assuming Poisson statistics. In an identical region of their ‘off-cluster’ CMD, we count \( 3 ± 1.7 \) stars yielding a net value of \( R = 6 ± 3.4 \). Keeping in mind that \( B = 24±4.9 \) and \( V = 4±2.0 \) (SL97), we calculate \( \frac{(B-R)}{(B+V+R)} = 0.53±0.17 \). This value is not as anomalous as the value obtained under the assumption that \( R = 0 \), and we have shown that, indeed, \( R \) is not likely to be identically zero.

The age of Arp 2 is another matter that requires in-depth scrutiny. BCFRF quote a value of \( \Delta V = 3.29±0.10 \) for the magnitude difference between the MSTO and the HB and \( \Delta(B-V) = 0.248±0.005 \) for the color difference between the MSTO and the lower RGB. When compared with the \( \Delta V \) values of CDS and the \( \Delta(B-V) \) values given by VBS, both of these quantities indicate that Arp 2 is a relatively young cluster. However, we arrive at a different conclusion when we reanalyze their photometry. Figure 8 shows the BCFRF Arp 2 photometry compared with the fiducial sequence of M68 (Walker 1994) registered by matching the magnitude of the HB and the RGB color at the level of the HB. This is done because both clusters have very similar metallicities (M68 has \([Fe/H] = -2.09±0.11\); Zinn & West 1984). First, we note that \( V(HB) = 18.20±0.04 \) for Arp 2, which is composed of \( 18.13±0.04 \) (RR Lyrae mag from SL97) plus \( 0.074±0.002 \) (offset from SL97 zeropoint to that of BCFRF). We adopt \( V(HB) = 15.64 \) for M68 (Walker 1994). What is immediately obvious from Fig. 8 is that the apparent \( \Delta V \) of Arp 2 does not differ drastically from that of M68. The vertical dashed lines in Fig. 8 show the measured \( \Delta(B-V) \) value (as defined
by VBS) quoted by BCFRF. Inspection of Fig. 8 is enough to cast doubt on the finding that Arp 2 is significantly younger than other clusters at its metallicity. A similar conclusion results from a comparison of Arp 2 with the fiducial sequences of M92 and NGC 6397.

It is possible to use the new age estimator described by Chaboyer et al. (1996b) to calculate a more precise age for Arp 2 using the point brighter and +0.05 mag redder than the MSTO \([M_V(BTO)]\). Utilizing the procedure outlined by Chaboyer et al. (1996b), we find \(V(BTO) = 21.25 \pm 0.02\), which, when coupled with the HB mag given above, yields \(\Delta V(BTO) = 3.05 \pm 0.04\). Assuming \([Fe/H] = -1.80 \pm 0.10\) (SL97), the models of Chaboyer et al. (1996b) give an age of 14.4 \pm 0.7 Gyr for Arp 2, which can be compared with 15.2 \pm 0.4 Gyr for their 17 metal-poor globular clusters. These ages agree to within the errors, but there is the possibility that Arp 2 could be slightly younger than other clusters at its metallicity (perhaps \(\approx 1\) Gyr), but certainly not \(\approx 3\) Gyr younger. When considered along with the downward-revised \((B - R)/(B + V + R)\) value, the age of Arp 2 is no longer a glaring exception to the hypothesis that age is the sole global second parameter.

4.5.2. IC 4499

Ferraro et al. (1995, hereafter FFFCB) present a CCD CMD of IC 4499 in the \(BV\) passbands. They quote a value of \(\Delta V = 3.25 \pm 0.12\). From this and their measured \(\Delta(B - V)\) value, they conclude that IC 4499 is 3-4 Gyr younger than other globulars at its metallicity, which is \([Fe/H] = -1.75 \pm 0.20\). This large age difference does not seem to match the HB morphology of IC 4499 measured to be \((B - R)/(B + V + R) = 0.08\) by Sarajedini (1993). For comparison, M3 also has \((B - R)/(B + V + R) = 0.08\) and \([Fe/H] = -1.66 \pm 0.06\) (Zinn & West 1984). As a result, there is an apparent conflict between the HB type of IC 4499, which matches that of M3, and its age which appears to be much younger than that of M3.

Using the IC 4499 photometry of Ferraro et al. (1995) divided into radial bins to optimize the definition of the CMD sequences, we measure \(V(BTO) = 20.54 \pm 0.01\). Together with the HB magnitude of \(V(HB) = 17.65 \pm 0.04\), we calculate \(\Delta V(BTO) = 2.89 \pm 0.04\) for IC 4499. This compares favorably with \(\Delta V(BTO) = 2.92\) for M3 estimated from the fiducial sequence of Buonanno et al. (1994). Thus, IC 4499 does not seem to be significantly younger than M3, thereby refuting the existence of any contradiction between the HB and age of IC 4499.

We should point out however that IC 4499 possesses a rather bizarre and intriguing HB. First of all, it has the highest specific frequency of RR Lyrae variables among Galactic globular clusters. Walker & Nemec (1996) found that 66 \pm 5\% of the stars on the HB are RR Lyraes of one form or another. Sarajedini (1993) found that the color extent of the BHB is significantly smaller than those of M3 and NGC 3201. Based on this, he speculated that the mean mass of the BHB stars in IC 4499 is at least \(\approx 0.02M_\odot\) higher than those in more normal clusters such as M3 and NGC 3201.
4.5.3. Bimodal Horizontal Branch Clusters

There are three Galactic globular clusters whose HBs appear to be the result of having combined a purely blue HB, like that in NGC 288, with a purely red HB, like that of NGC 362. These bimodal HB clusters are NGC 1851 (Walker 1992) and NGC 2808 (Byun & Lee 1991; Sosin et al. 1997). In addition, Borissova et al. (1997) have claimed that NGC 6229 also has a bimodal HB. The reason for these peculiar HBs is as yet unclear. Lee et al. (1988) claim that they are the natural result of evolution away from the ZAHB, while others have evoked more exotic phenomena.

More recently, Rich et al. (1997) have obtained CMDs in the cores of several clusters using the Hubble Space Telescope and WFPC2. In the case of three clusters thought to have purely red HB morphologies (NGC 362, NGC 6441, and NGC 6388), they have discovered a small fraction of blue HB stars. The number of blue HB stars is between 10% and 15% of the total number of red HB stars. In contrast, the ground-based CMDs of NGC 6388 (Silbermann et al. 1994) and NGC 362 (Harris 1982), which mostly probe the outer regions of these clusters, exhibit very weak blue HBs, if any, as compared with the HST CMDs. Therefore, this variation in the HB morphology near the cluster center is likely to be a result of some sort of stellar dynamical effect; it may be connected to increased mass loss due to stellar interactions or it may be related to the subdwarf-B phenomenon studied by Bailyn et al. (1992). In either case, their apparent affect on HB morphology, which has in the past introduced some confusion in discussions of the systematics of HB morphology, are now understood to reflect local conditions rather than a stage in the evolution of isolated single stars, and are probably not relevant to the global properties of the globular cluster system.

4.5.4. Other Candidates For the Global Second Parameter

In addition to age, there are a number of other globular cluster star properties that could be considered to be candidates for the global second parameter. These are individually discussed and effectively discounted by the arguments presented in LDZ94. For example, there is absolutely no possibility that a change in the CNO abundance could explain the behavior seen in Figs. 6 and 7. Although increasing the CNO abundance makes the HB morphology redder, it also makes the MSTO redder and fainter, which is the opposite of what is observed. Another example is a decrease in the helium abundance, which makes the HB redder as well, but also causes the MSTO to become redder and fainter, which is also not what is observed. The reader is referred to LDZ94 and Demarque et al. (1989) for more examples.

Buonanno et al. (1997) have recently published the latest installment in a series of papers (Buonanno 1993; Fusi Pecci et al. 1993; Buonanno & Iannicola 1995; Fusi Pecci et al. 1996; see also van den Bergh & Morris 1993) which examine the role of stellar density on the morphology of the HB. Their first conclusion is that clusters having similar metallicity but different central stellar densities tend to exhibit different blue HB morphologies. In particular, Buonanno et al. (1997) claim that clusters with higher central densities are more likely to populate the bluest extremes of
the HB. Their second conclusion is illustrated in Fig. 9. The top panel (9a) shows the variation of $B_2/B + V + R$ with the peak dereddened color of the HB $[(B - V)_{\text{peak}}]$. The HB-type index $B_2/B + V + R$ is composed of the number of stars blueward of $(B - V)_{\text{o}} = -0.02$ (i.e. the blue HB tail), denoted by $B_2$, divided by the sum of the number of stars blueward of the instability strip $(B)$, redward of the instability strip $(R)$, and the number of RR Lyrae variables $(V)$. As would be expected, Fig. 9a shows that when the HB peaks in the blue ($(B - V)_{\text{peak}} < \sim 0.3$), $B_2/B + V + R$ tracks the peak HB color quite well. However, redward of this color, $B_2/B + V + R$ becomes insensitive to the peak HB color because the quantity $B_2$ goes to zero. In order to model the general variation of $B_2/B + V + R$ with $(B - V)_{\text{peak}}$, Buonanno et al. (1997) performed a least squares fit to the filled circles in Fig. 9a; this yielded the solid line shown in the same figure, which likely represents the effect of the first parameter (metallicity) on the HB morphology. Then, they plotted the residuals from this fit versus the log of the central density of each cluster $(\log \rho_o)$ as shown in Fig. 9b. The apparent correlation between the $B_2/B + V + R$ residuals and the central density seems to indicate that at a given $(B - V)_{\text{peak}}$, the strength of the blue HB is controlled by the stellar density in the cluster. This is in-line with the findings of Djorgovski et al. (1997) discussed above. The dashed line in Fig. 9a shows the ‘inverse’ linear fit (i.e. independent and dependent variables are interchanged) to the data points with $(B - V)_{\text{peak}} < \sim 0.3$. This inverse fit appears to represent the general behavior of $B_2/B + V + R$ with $(B - V)_{\text{peak}}$ much better than the direct fit. Fig. 9c illustrates the residuals from this inverse fit as a function of $\log \rho_o$. In this case, there is no compelling reason to believe that these residuals are correlated with the cluster central density. As a result, this casts doubt on the role that central density plays in influencing the HB morphology.

In this section, we have presented arguments in favor of the idea that age is the global second parameter. However, this does not mean that some of the other second parameter candidates play no role whatsoever in influencing the HB morphologies of a minority of clusters. As with all general trends, there are a few exceptions to the rule.

5. The Age Range

Detailed studies of individual GCs have demonstrated rather convincingly that a few GCs are substantially younger (or older) than other clusters with a similar metallicity (e.g. Rup 106, Buonanno et al. 1993; Pal 12, Stetson et al. 1989; NGC 362, see previous section). However, a few anomalously young (or old) GCs do not answer the question of whether the bulk of the outer halo formed as the result of a rapid collapse (ELS), or over an extended period of time (SZ). This requires a study of the age range among the bulk of the Galactic GCs. The $\Delta(B - V)$ technique cannot be reliably applied to clusters of differing metallicity, so one must examine GC ages obtained via $\Delta V$ in order to explore the existence of an age range among the bulk of Galactic GCs. We performed such a study of the $\Delta V$ ages of 43 GCs in CDS, and concluded that a total ($\pm 2\sigma$) age range of 5 Gyr exists among the bulk of the Galactic GCs.
This work was severally criticized by SVB, who suggested that CDS underestimated the Gaussian 1-σ error bars associated with the ΔV measurements. Although observers typically quote errors in their derived ΔV (or V(TO), V(HB)) values, they never state if these are to be interpreted as Gaussian 1-σ error bars. Indeed, it is very difficult for an observer who makes a single measurement of ΔV to estimate the true Gaussian 1-σ error associated with the measurement. Walker (1992) provides an example of the difficulty in estimating the error; he originally estimates ΔV = 3.42 ± 0.05 (page 582), “but given the difficulty in measuring the turnoff to an accuracy better than ±0.10 mag” gives ΔV = 3.42 ± 0.10 mag in his abstract. CDS used the larger error bar in their data table.

CDS attempted to directly estimate the Gaussian 1-σ error bar associated with measurements of ΔV by examining independent measurements of ΔV made by various authors. An analysis of these repeated observations led CDS to conclude that the spread among these observations was much smaller than would be expected if the observers were quoting Gaussian 1-σ error bars. A reasonable estimate of the Gaussian error could be obtained by multiplying the quoted errors by 0.61. However, SVB pointed out that a few of the ‘independent’ ΔV measurements actually could be traced back to similar sources. For this reason, we have re-checked all of the original references, removed those that were in error and repeated our analysis. This re-analysis is presented in Appendix A, where it is determined that multiplying the quoted errors by 0.61 does indeed lead to a reasonable estimate of the true 1-σ error associated with the ΔV measurements tabulated by CDS.

An independent estimate of the Gaussian error associated with measuring ΔV can be made by considering all of the old, metal-poor clusters. One would expect that these clusters should all have the same ΔV. Hence, the standard deviation about the mean should be similar to the average Gaussian error in the individual measurements. An old, metal-poor GC sample may be selected independent of ΔV by selecting GCs which have [Fe/H] ≤ −1.6; have not been shown to be young using Δ(B − V) and are not suspected of being young based on HB morphology (i.e. belonging to the Old Halo group of Zinn 1993). Applying these selection criteria to the ΔV measurements in Table 2 of CDS along with the additional requirement that the original authors have included an estimated error in their ΔV measurement results in a list of 12 metal-poor, old globular clusters with ΔV measurements: NGC 2298, 5024, 5897, 6205, 6254, 6341, 6397, 6535, 6809, 7078, 7099, and 7492. The average ΔV error quoted by the observers is ±0.158 mag. However, the standard deviation of the points about the mean is σ = 0.080 mag, calculated using the standard formulae $\sigma^2 \equiv \Sigma(x_i - \bar{x})^2/(N - 1)$. Alternatively, one may use the small number statistics of Keeping (1962), whose formulae also yield σ = 0.080 for the 12 metal-poor clusters. This standard deviation is remarkably similar to the average 1-σ ΔV error bar used by CDS (±0.083 mag), suggesting that CDS used the appropriate 1-σ error in their analysis.

For these old, metal-poor clusters, the standard deviation about the mean is significantly smaller (factor of 2) than the average error quoted by the observers, suggesting that observers are not quoting Gaussian 1-σ error bars. This may be quantified by comparing the observed distribution
of measured $\Delta V$ values to an expected distribution. The expected distribution is constructed by randomly generating 1000 $\Delta V$ values for each input $\Delta V$ assuming the mean value given by the observer, and using a Gaussian distribution with the $\sigma$ of the Gaussian taken to be the observer’s error bar. In this test, the expected distribution has $\sigma = 0.17$ and the $F$-test (Press et al. 1992) finds that there is less than a 1% probability that the observed distribution has the same standard deviation as the expected distribution. When the observed error bars are multiplied by 0.61 (as advocated by CDS), then the expected distribution has $\sigma = 0.12$ (still larger than the observed $\sigma$). There is a 10% probability that the observed and expected distributions have the same standard deviation. Thus, the measured $\Delta V$ values for old, metal-poor GCs strongly support the conclusion of CDS, that a reasonable estimate of the Gaussian 1-$\sigma$ error associated with the measurement of $\Delta V$ can be estimated by assuming the errors quoted by observers correspond to $\sim 1.64\sigma$ error bars. This essentially implies that the quoted error ranges correspond to 90% confidence levels, as opposed to the 68% confidence level for a Gaussian 1-$\sigma$ error.

As an additional justification of their criticism of CDS, SVB give some anecdotal remarks about the observational difficulties associated with measuring $\Delta V$. We fully agree with this, as mentioned in §2.2. Even with good photometry, the turnoff region can be nearly vertical for $\sim 0.2$ mag. However, this does not imply that the Gaussian 1-$\sigma$ error in the measured turnoff must be of order 0.1 mag. The average 1-$\sigma$ $\Delta V$ error bar used by CDS was $\pm 0.083$ mag. Assuming that the error in $\Delta V$ is due in equal parts to the error in determining the HB and TO levels, this implies that 68% of the time, observers will be able to determine the magnitude of the TO within $\pm 0.06$ mag.

Finally, SVB based part of their criticism on a quote from Grundahl (1996, PhD thesis), who compared the $\Delta V$ ages of CDS with the $\Delta (B - V)$ ages of Richer et al. (1996). Richer et al. (1996) determined relative GC ages within a given metallicity group using $\Delta (B - V)$, and derived the absolute age of the different metallicity groups using a reference cluster in each group whose age was derived from isochrone fits to the observed color magnitude diagram. According to SVB, Grundahl concluded that there was a poor match between the Richer et al. (1996) ages and the CDS ages, and the likely cause were the errors in the adopted $\Delta V$ values from the literature. However, it is not at all clear how Grundahl performed such a comparison, as Richer et al. (1996) did not give any errors in their derived ages. The Richer et al. (1996) ages are based on fitting isochrones to observed color magnitude diagrams. As discussed in Section 3 above, the colors predicted by stellar models and isochrones are very uncertain, due to the difficulties in treating convection, and our incomplete knowledge of stellar atmospheres. Thus, ages derived by fitting isochrones to GC color magnitude diagrams will have very large uncertainties attached to them. It is not surprising then, that Grundahl (1996) did not find a good match between the Richer et al. (1996) ages and the CDS ages.

Salaris et al. (1997, hereafter SW) have recently completed a study which relies upon relative $\Delta (B - V)$ ages within 4 metallicity groups and uses $\Delta V$ ages to define the absolute age of a reference cluster in each metallicity group. The study of SW has an additional advantage over the work of
Richer et al. (1996) of quoting errors in their derived ages. Comparing the 24 CDS and SW ages in common\(^4\) reveals a linear Pearson R correlation coefficient of 0.707, with a probability of correlation of 99.99\%. This clearly reveals an excellent correlation between the ∆V ages of CDS and the mainly ∆(B − V) ages of SW. SW derived rather young ages, and the mean ages of the two samples are different. However, when examining the question of an age range, it is the relative ages that are important. A linear fit between the SW and CDS ages (which takes into account the errors in both variables, Press et al. 1992) yields a reduced \(\chi^2\) of 0.85, which, if anything suggests that the error bars are overestimates. If one uses the observer’s original error bars in the CDS ages (as advocated by SVB), then the reduced \(\chi^2\) is 0.57 for all ages in common. On average, such a small value of the reduced \(\chi^2\) only occurs 5\% of the time, suggesting that the fit is ‘too good’. This is caused by overestimating the errors. Of course, when comparing ages for individual GCs between CDS and SW, discrepancies may be found. However, CDS did not concern themselves with individual ages, but the properties of the sample as a whole. As such, the excellent correlation between the CDS ages and SW ages provides yet another piece of evidence which supports the errors adopted by CDS and the conclusion that a total age range of ∼ 5 Gyr exists among the bulk of the Galactic GCs.\(^5\)

6. Age and Metallicity

The question of whether an age-metallicity relation exists among the Galactic GCs is a long-standing one. As the relative ∆(B − V) ages are only reliable over a small metallicity range, ages derived using ∆V are usually used to examine the correlation between age and metallicity among GCs. It has been traditionally assumed that ∆V is independent of metallicity, leading to the conclusion that the existence of an age-metallicity relation depends critically on the slope of the \(M_v\)\((RR)\)-[Fe/H] relation (Sandage 1982; Buonanno 1989; Sarajedini & King 1989; Sandage & Cacciari 1990). However, as shown by CDS there are now a number of clusters whose ∆V values are smaller than the average (e.g. Rup 106, Pal 12, Ter 7). As none of the known ‘young’ GCs are metal poor ([Fe/H] \(\approx\) −1.8), CDS found that an age-metallicity relation exists even for relatively steep \(M_v\)\((RR)\)-[Fe/H] slopes of 0.30. However, this correlation is driven by the small number of young clusters, and CDS found that the a global age-metallicity relation only exists when the \(M_v\)\((RR)\)-[Fe/H] slope is less than 0.26. This study was based on individual ∆V measurements, and somewhat older stellar models.

\(^4\)In performing this comparison, we have used the CDS ages derived assuming \(M_v\)(RR) = 0.20[Fe/H] + 0.82. This is the slope preferred by CDS. The adopted zero-point leads to a rather young mean age (though still older than the mean age found by SW). CDS found that the derived age range did not depend critically on the adopted \(M_v\)(RR)-[Fe/H] relation.

\(^5\)At the time of this writing, the absolute age scale of globular clusters is in a state of flux, but if absolute ages are indeed reduced by some factor, age differences would be reduced by approximately the same factor.
The large majority of $\Delta V$ measurements compiled by CDS are for $[\text{Fe}/\text{H}] < -1.2$. If we restrict our attention to these relatively metal-poor clusters, and require that the clusters belong to the ‘Old Halo’ group of Zinn (1993), then an examination of the $\Delta V$ values reveals that they are indeed consistent with a single mean value, independent of $[\text{Fe}/\text{H}]$. The linear Pearson R correlation coefficient between $[\text{Fe}/\text{H}]$ and $\Delta V$ is 0.166 (22 points), with a probability of correlation of 53%. Thus, among these old, relatively metal-poor clusters the assumption that $\Delta V$ is independent of $[\text{Fe}/\text{H}]$ is valid.

Assuming that $\Delta V$ is independent of $[\text{Fe}/\text{H}]$ (in the range $-2.4 \leq [\text{Fe}/\text{H}] \leq -1.2$) allows for a simple analytical analysis of the age-metallicity relation. The basic assumptions are equation (1): $M_v(\text{RR}) = \alpha [\text{Fe}/\text{H}] + \beta$, and

$$M_v(\text{TO}) = ct + d [\text{Fe}/\text{H}] + e$$

where $\alpha, \beta, c, d$ and $e$ are constants. The constants in equation (2) can be derived from theoretical isochrones. Note that CDS found that a more complicated quadratic relation was required to accurately model the behavior of the theoretical isochrones. This was due to the relatively large $[\text{Fe}/\text{H}]$ ($-2.5$ to $-0.5$ dex) and age (8 - 22 Gyr) range used by CDS. For the restricted $[\text{Fe}/\text{H}]$ and age range (10 - 16 Gyr) considered here, the linear relation given in (2) is adequate. Combining equations (1) and (2) along with the definition of $\Delta V$ yields

$$t = \frac{\Delta V}{c} - \frac{d - \alpha}{c} [\text{Fe}/\text{H}] - \frac{e - \beta}{c}.$$  \hspace{1cm} (3)

The age-metallicity relation is simply defined by the derivative of the above equation

$$\frac{dt}{d[\text{Fe}/\text{H}]} = \frac{\alpha - d}{c}$$ \hspace{1cm} (4)

which assumes that $\Delta V$ is independent of $[\text{Fe}/\text{H}]$. From this, it is clear that the existence of an age-metallicity relation depends both on the theoretical isochrones (through $c$ and $d$) and on the slope of the $M_v(\text{RR})$-$[\text{Fe}/\text{H}]$ relationship (the coefficient $\alpha$). Previous work has focused on the later point. Here, we address both points, by considering a variety of isochrones and $M_v(\text{RR})$-$[\text{Fe}/\text{H}]$ slopes.

The Monte Carlo isochrones of Chaboyer et al. (1996a,1996b) provide a large set (1000) of isochrones which have been calculated under a wide variety of physical assumptions (different oxygen abundances, mixing lengths, opacities, etc). For each set of isochrones, the coefficients $c$ and $d$ were calculated and combined with a variety of $M_v(\text{RR})$-$[\text{Fe}/\text{H}]$ slopes to derive an age-metallicity relation. The results for 4 slopes are presented in Table 1. For an $M_v(\text{RR})$-$[\text{Fe}/\text{H}]$ slope of 0.2, Table 1 shows that the most likely value for the age-$[\text{Fe}/\text{H}]$ slope is $-1.6$ Gyr/dex, and the 95% confidence limits on this bound are $-1.0$ to $-2.0$. Thus, if the $M_v(\text{RR})$-$[\text{Fe}/\text{H}]$ slope is 0.20, then GCs at $[\text{Fe}/\text{H}] = -1.2$ are 1 - 2 Gyr younger than $[\text{Fe}/\text{H}] = -2.2$ GCs.

Alternatively, one may determine the critical $M_v(\text{RR})$-$[\text{Fe}/\text{H}]$ slope, above which no age-metallicity relation exists. Based upon the typical errors in determining $c$ and $d$, a minimum
### TABLE 1

AGE-METALLICITY RELATION

<table>
<thead>
<tr>
<th>( dM_v(RR) ) ( d[\text{Fe/H}] )</th>
<th>( dt ) ( d[\text{Fe/H}] )</th>
<th>95% confidence limits Upper</th>
<th>Lower</th>
</tr>
</thead>
<tbody>
<tr>
<td>(mag/dex)</td>
<td>(Gyr/dex)</td>
<td>(Gyr/dex)</td>
<td>(Gyr/dex)</td>
</tr>
<tr>
<td>0.15</td>
<td>-2.2</td>
<td>-2.8</td>
<td>-1.8</td>
</tr>
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<td>-2.0</td>
<td>-1.0</td>
</tr>
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<td>0.25</td>
<td>-0.6</td>
<td>-1.4</td>
<td>-0.1</td>
</tr>
<tr>
<td>0.30</td>
<td>0.0</td>
<td>+0.5</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

The slope of 0.50 Gyr/dex is required to conclusively demonstrate the existence of an age-metallicity slope. Using the Monte Carlo isochrones, the mean critical value of the \( M_v(RR)-[\text{Fe/H}] \) slope is 0.26, above which no age-metallicity relation exists (very similar to that found by CDS). The 95% confidence limits on this slope are 0.23 and 0.30. Thus, even \( M_v(RR)-[\text{Fe/H}] \) slopes as shallow as 0.23 may not lead to an age-metallicity relation, depending on which isochrones are used.

The exact value of the \( M_v(RR)-[\text{Fe/H}] \) slope (denoted by \( \alpha \)) is still open to debate (see §2.2). It would appear that the evidence favors somewhat shallow slopes (\( \alpha \leq 0.25 \)). However, one cannot rule out slopes around \( \alpha = 0.23 \). Thus, based upon the Monte Carlo analysis presented in the previous paragraph, one cannot conclusively state whether or not an age-metallicity relationship exists among the old, metal-poor Galactic GCs.

Finally, there is some evidence that \( \alpha \) is not the same in all metallicity ranges, and that even within a given metallicity range, \( \alpha \) changes with time as the HB type evolves from red to blue. This last non-linearity is most pronounced for the most metal-poor clusters, for which \( M_v(RR) \) varies by a larger amount during evolution (Lee 1991; Demarque et al. 1997b). This additional complexity makes the existence of an simple age-metallicity relationship somewhat problematic.

### 7. How DID the Milky Way Form?

Given all of the points in the above discussion, it seems to us that the most viable formation model for the Milky Way is one that combines the properties of the ELS and SZ scenarios as suggested by several previous authors (e.g. Norris 1994). Recall that the ELS model was based on the kinematics of nearby stars and is therefore only applicable to the region inside of \( \sim 10 \) kpc from the Galactic plane. In this region, ELS maintained that the age range among the globular clusters is very small. This assertion is actually in agreement with one of the basic tenets of the SZ model, namely that inside of 8 kpc from the Galactic center, no second parameter effect exists. In other words, the age range among the halo globular clusters inside 8 kpc is very small. Furthermore,
inside of \( \sim 8 \) kpc, there is a relation between [\( \text{Fe/H} \)] and Galactocentric distance for the halo globular clusters (Zinn 1986). All of these characteristics point to a monotonic collapse \textit{a la} ELS inside of \( \sim 8 \) kpc of the Galactic center.

In contrast, outside of 8 kpc, there appears to be a significant age range among the Galactic globulars of as much as 5 Gyr. Additionally, the second parameter effect becomes stronger as Galactocentric distance increases indicating the need for a larger age spread to explain the wide range of HB morphologies at a given metallicity. Furthermore, there is no relation between [\( \text{Fe/H} \)] and Galactocentric distance. All of these observations point to a prolonged chaotic collapse highlighted by the fragmentation of the proto-galactic gas cloud. In this manner, we reconcile the two competing paradigms for the formation of the Galactic halo.

8. Summary

The relative ages of the Galactic globular clusters provide an insightful look into the earliest formation epochs of the Galaxy’s halo. In particular, studying these ages can allow us to distinguish between a rapid collapse of the halo characterized by the scenario of ELS or a more gradual collapse similar to the one described by SZ.

To begin with, we need measurement techniques that will yield globular cluster ages to the required level of precision. These methods can be classified into four general groups: 1) Direct measures of the cluster distance, hence age, via main sequence fitting, 2) Vertical measures which utilize the magnitude difference between various features in the color-magnitude diagram, 3) Horizontal measures which use the color difference between CMD features, and 4) measures which rely on counting stars in various stages of evolution, i.e. using the luminosity function.

Through the use of these age-determination methods, we have examined various questions concerning the Galactic globular clusters. First, we investigate the second parameter effect in detail. Weighing all of the evidence presented in numerous previous works as well as new analyses described herein, we conclude that the age of a globular cluster is the most likely candidate to be the global second parameter, which along with metal abundance, controls the morphology of the horizontal branch.

Next, we address the question of an age range among the globular clusters using the \( \Delta V \) age diagnostic. The answer to this question is greatly dependent on obtaining a robust estimate for the observational errors in the measured \( \Delta V \) values of clusters. We present arguments in favor of the assertion that the quoted errors in \( \Delta V \) represent 90% confidence intervals, and that Gaussian 1-\( \sigma \) errors are approximately 60% of the quoted errors. When the measured \( \Delta V \) values for 43 globular clusters are combined with these updated errors, we find a total age range of \( \sim 5 \) Gyr among the bulk of these clusters.

Finally, we use theoretical isochrones to derive an algebraic expression which describes the age
- metallicity relation. The slope of this equation determines whether there is a relation between age and metallicity. Monte Carlo simulations of theoretical isochrones indicate that there is a significant relation between age and metallicity among the Galactic globular clusters if the slope of the $M_v(RR)$-[Fe/H] relation is less than $\sim 0.20$.

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Fig. 1.— Definitions of the $\Delta V$ and $\Delta (B-V)$ age diagnostics discussed in the text. Note that the magnitude level of the horizontal branch is essentially constant with age.

Fig. 2.— (a) The fiducial sequences of NGC 362 (x’s), NGC 1851 (filled circles), and NGC 288 (solid line) are offset to match the horizontal portion of the subgiant branch. (b) Same as in (a) except that the fiducials have been shifted to match the unevolved main sequences.

Fig. 3.— Utilizing the magnitude and color offsets that match the cluster subgiant branches (see Fig. 2a), the top panel shows a comparison of the NGC 362 photometry with the NGC 1851 fiducial sequence. This same fiducial is compared with the NGC 288 photometry from Bolte (1992) in the middle panel and Bergbusch (1993) in the lower panel.

Fig. 4.— The top panel illustrates the metal abundance of Milky Way globular clusters as a function of their horizontal branch (HB) morphology as quantified by $(B-R)/(B+V+R)$. This represents the difference between the number of stars on the blue side of the RR Lyrae instability strip (B) and on the red side (R), divided by the total number of blue side, red side, and variable (V) stars on the HB. Blue HB clusters have an index of +1 while red HB clusters have an index of −1. Filled circles are clusters with Galactocentric distances inside 8 kpc, while open circles are those outside 8 kpc. The solid lines are the results of synthetic HB calculations (LDZ94) and indicate the expected locations of clusters for different relative ages. The lower panel shows the behavior of observed $\Delta V$ values for clusters in two narrow ranges of metallicity. The solid and dashed lines indicate the expected behavior based on the synthetic models shown in the top panel.

Fig. 5.— (a) The observed values of metallicity and mean dereddened horizontal branch (HB) color for globular clusters with purely red HB morphologies. The open circles represent the clusters Kron 3, NGC 121, and Pyxis. The first two are populous clusters in the Small Magellanic Cloud and the latter one is a newly discovered Milky Way globular cluster. The dashed lines represent the mean intrinsic colors of zero age horizontal branches for scaled solar abundances and masses of 0.90$M_\odot$ (left line) and 0.66$M_\odot$ (right line). (b) Same as in (a) except that the difference in color between the HB and red giant branch is plotted.

Fig. 6.— Color-magnitude diagram comparisons between Kron 3, NGC 121, Pyxis, and NGC 362. Each panel shows the $B-R$ photometry for each of the first three clusters compared with the fiducial sequence of NGC 362. Note that there is a correlation between the relative main sequence turnoff locations and the relative colors of the red horizontal branch. Relative to NGC 362, younger clusters (i.e. brighter turnoffs) have redder horizontal branch colors.

Fig. 7.— Same as Fig. 6 except that $B-V$ photometry is plotted for the globular clusters Palomar 4, NGC 121, Lindsay 1, ESO121-SC03, and Kron 3 again compared with the fiducial sequence of NGC 362.

Fig. 8.— The photometry for the globular cluster Arp 2 compared with the fiducial sequence of M68, where both have been matched at the magnitude of the horizontal branch (HB) and the color
of the red giant branch at the level of the HB. The vertical dotted lines represent the published value of $\Delta(B - V)$ for Arp 2.

Fig. 9.— (a) The horizontal branch (HB) morphology of Galactic globulars as quantified by $B2/(B + V + R)$ as a function of the peak dereddened color of the HB. The HB-type index is composed of the number of stars blueward of $(B - V)_o = -0.02$ (i.e. the blue HB tail), denoted by B2, divided by the sum of the number of stars blueward of the instability strip (B), redward of the instability strip (R), and the number of RR Lyrae variables (V). The filled circles are the clusters to which the lines have been fit. The solid line represents the direct linear fit and the dashed line shows the ‘inverse’ fit (i.e. independent and dependent variables are interchanged). Figures (b) and (c) show the residuals from these fits, respectively, as a function of cluster central concentration.
A. Estimating the Gaussian Error in $\Delta V$

In order to estimate the Gaussian error associated with the $\Delta V$ measurements, CDS analyzed repeated observations. However, as pointed out by SVB, a few of the observations that CDS thought to be independent could be traced back to the same original photometry. For this reason, we have repeated the original analysis of CDS, carefully re-checking the literature to ensure that only independent $\Delta V$ measurements are included. We have discovered a few more errors in our original work, and have searched the literature for new, independent observations of $\Delta V$ which have appeared since the work of CDS. The results of our literature search are presented in Table 1, which gives the cluster name, $\Delta V$ value and the references. Using the $\Delta V$ values and errors, we have calculated and tabulated the quantity $\delta \equiv (\Delta V_a - \Delta V_b)/(\epsilon_a^2 + \epsilon_b^2)^{1/2}$, where $\Delta V_a$ is the measured $\Delta V$ with its error ($\epsilon_a$) as reported by observer $a$, and $\Delta V_b$ and $\epsilon_b$ are the same quantities reported by observer $b$. Essentially, $\delta$ is simply the difference in the $\Delta V$ observations, normalized by the quoted errors. If the observers are quoting Gaussian 1-σ error bars, then $\delta$ should have a Gaussian distribution, with $\sigma = 1$. There are 16 measurements of $\delta$ in Table 1; for this sample size one would expect 5 values of $\delta$ in excess of 1 for Gaussian errors. However, this occurs only once. This suggests that the reported errors are an overestimate of the Gaussian 1-σ error bars. Indeed, the F-test (Press et al. 1992) finds that there is only a 1% chance that $\delta$ has a standard deviation of 1.0. The quantity $\delta$ has an actual standard deviation of 0.55. If one choses the hypothesis that $\delta$ has a standard deviation of 0.61 (as advocated by CDS in the original analysis), this has a 69% probability of being true from the F-test. Thus, multiplying the quoted errors by 0.61 yields a good estimate of the Gaussian 1-σ error in $\Delta V$. This implies that the original errors are actually 1.64 σ error bars, which corresponds to 90% confidence limits.
<table>
<thead>
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<th>Cluster</th>
<th>ΔV</th>
<th>δ</th>
<th>References (TO,HB)</th>
</tr>
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<tbody>
<tr>
<td>NGC 104</td>
<td>3.61 ± 0.10</td>
<td></td>
<td>Hesser et al. 1987</td>
</tr>
<tr>
<td></td>
<td>3.81 ± 0.18</td>
<td>−0.971</td>
<td>Buonanno et al. 1989</td>
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<tr>
<td></td>
<td>3.62 ± 0.06</td>
<td>−0.086</td>
<td>Grundahl 1996</td>
</tr>
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<td>NGC 288</td>
<td>3.73 ± 0.12</td>
<td></td>
<td>Buonanno et al. 1989</td>
</tr>
<tr>
<td></td>
<td>3.70 ± 0.14</td>
<td>0.163</td>
<td>Pound et al. 1987; Olszewski et al. 1984</td>
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<td>NGC 1261</td>
<td>3.57 ± 0.11</td>
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<td>Ferraro et al. 1993</td>
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<td></td>
<td>3.50 ± 0.14</td>
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<td>3.34 ± 0.10</td>
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<td>3.45 ± 0.21</td>
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<td>3.44 ± 0.12</td>
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<td>Samus et al. 1996</td>
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<td>3.51 ± 0.10</td>
<td>−0.639</td>
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Figure 1 - Sarajedini, Chaboyer, & Demarque
Figure 2 - Sarajedini, Chaboyer, & Demarque

(a)

(b)

NGC 1851
NGC 288
NGC 362
Figure 3 - Sarajedini, Chaboyer, & Demarque
Figure 4 - Sarajedini, Chaboyer, & Demarque
Figure 5 - Sarajedini, Chaboyer, & Demarque
Figure 6 - Sarajedini, Chaboyer, & Demarque
Figure 7 - Sarajedini, Chaboyer, & Demarque
Figure 8 - Sarajedini, Chaboyer, & Demarque
Figure 9 - Sarajedini, Chaboyer, & Demarque