Neutrino Oscillations: a source of Goldstone fields

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Abstract

It is proved that true Goldstone bosons develop coherent fields whenever the associated charges of the matter particles are not conserved in a macroscopic scale. The sources of the Goldstone fields are the time rates of quantum number violation. The case of neutrino flavour oscillations is studied with application to Supernovae. It is shown that if the Lepton numbers break at the Fermi scale, the $\nu$ potentials and oscillation patterns change in the periods of largest $\nu$ fluxes. In this way, $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ oscillations may occur in the first instants of $\nu$ emission.

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I. INTRODUCTION

In theories with spontaneous breaking of global symmetries the Goldstone theorem asserts the existence of massless particles - the Goldstone bosons - one per each non-zero quantum number of the vacuum. In principle such massless scalar bosons could mediate long range forces and lead to observable coherence effects just like the gravitational force. However, the very structure imposed by exact global symmetries at the Lagrangian level implies that these goldstons have nothing but derivative couplings with matter fields. It was shown that if the symmetry constraints are relaxed either through anomalies or with soft symmetry breaking terms then, scalar couplings of pseudo-goldstons with fermions naturally arise. Consequently, macroscopic forces are developed by matter aggregates but one price is paid: the intermediate bosons get non-zero masses which means that the range of those forces is finite. In this paper it is shown that under special conditions a true Nambu-Goldstone boson may develop a macroscopic field, the source being the rate of non-conservation of the matter quantum number associated with that field. That may be significant in astrophysical compact objects and the case of supernovae is analysed.

To be definite I consider an abelian global symmetry $U(1)_\Lambda$ where $\Lambda$ denotes generically any linear combination of flavour lepton numbers such as $L_e - L_\mu$ or the total lepton number $L$. After symmetry breaking non-invariant terms are generated in masses and/or couplings but the symmetry is still realized non-linearly and strongly constrains the couplings to the Goldstone boson $\phi$. Under $U(1)_\Lambda$, the matter fields and $\phi$ transform as

$$
\psi^a \to e^{-i \alpha \Lambda \phi/V_\Lambda} \psi^a , \tag{1a}
$$

$$
\phi \to \phi + \alpha V_\Lambda , \tag{1b}
$$

respectively, where $V_\Lambda$ is the scale of symmetry breaking.

Consider for instance the lepton masses. Let $\nu^L_a$ denote all left-handed neutrino fields including the standard weak interacting partners $\nu^e_L, \nu^\mu_L, \nu^\tau_L$ and any extra neutral singlets. The mass terms generated either radiatively or at tree-level are necessarily accompanied by couplings to $\phi$ completely determined by the lepton charges $\Lambda_a$:

$$
\mathcal{L}_m = -m_{ab} \nu^a_L C \nu^b_L e^{i (\Lambda_a + \Lambda_b) \phi/V_\Lambda} + \text{h.c.} - m_\ell \ell^\dagger \ell , \tag{2}
$$

where a sum over $a, b$ and $\ell = e, \mu, \tau$ is assumed. Moreover, in any other term of the Lagrangian each field $\psi^a$ always comes with a factor $\exp(i \Lambda_a \phi/V_\Lambda)$. Consequently, a spatially uniform $\phi$ would be absorbed in the mass eigenstate fields leaving no couplings with $\phi$. Matter only cares about the gradient of $\phi$ and that, as shown in [1], is better expressed in what I term as weak-unitary basis:

$$
\psi^a \ (\text{old}) = e^{-i \Lambda_a \phi/V_\Lambda} \psi^a \ (\text{uni}) . \tag{3}
$$

The new fields preserve the original quantum number assignments but are invariant under the global symmetry transformations (1). They reduce now to a translational invariance $\phi \to \phi + \beta$ and therefore, the goldston interactions can only depend on the $\phi$ derivatives.
In the unitary basis the effective Lagrangian writes as

\[ L = \sum \bar{\psi}_i \gamma_\mu \psi - \sum_{e,\mu,\tau} m_e \bar{\ell}_e \ell - \sum_{a,b} \nu_L^a C m_{ab} \nu_L^b + \text{h.c.} \]
\[ + \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + J_\Lambda^\mu \partial_\mu \phi/V_\Lambda + \cdots \] (4)

All of the goldston interactions are condensed in the current

\[ J_\Lambda^\mu = \sum_{e,\mu,\tau} \bar{\ell}_e \gamma_\mu \Lambda_{\mu} \ell + \sum_a \bar{\nu}_L^a \gamma_\mu \Lambda_a \nu_L^a + \cdots, \] (5)

which includes the matter current derived from the kinetic Lagrangian of \( \psi \) (old) but also in general model dependent higher order corrections. The \( \phi \) equation of motion reads

\[ \partial_\mu \partial_\mu \phi = -\partial_\mu J_\Lambda^\mu / V_\Lambda \] (6)

and clearly the source term is not some sort of matter density.

The divergences of \( \bar{\psi} \gamma_\mu \psi \) and \( \bar{\psi} \gamma_\mu \gamma_5 \psi \) only yield pseudo-scalar \( \bar{\psi} \gamma_5 \psi \) charges that do not generate long range '1/r' fields, but rather spin dependent '1/r^3' potentials [2]. In the intent of escaping the derivative couplings law, models were made with explicit symmetry breaking terms. These may be anomaly terms [3], as in the axion case [4], or soft symmetry breaking terms [5]. In this way scalar couplings with the goldston field are produced but at the cost of getting a mass for the so-called pseudo Nambu-Goldstone boson which ultimately means a finite range for the long range forces.

Here, I will explore the fact that the source term is different from zero whenever the \( \Lambda \) number of the matter particles is not conserved. To be accurate, there is a Noëther current associated to the symmetry \( \phi \rightarrow \phi + \alpha \) namely \( J_\Lambda^\mu + V_\Lambda \partial_\mu \phi \), that is exactly conserved, but the non-standard interactions do not conserve the fermions \( \Lambda \) number as a result of the very spontaneous symmetry breaking.

Non-conservation of \( J_\Lambda^\mu \) may occur in macroscopic proportions in a stationary basis in stars, or in Supernovae for a short period of time. For instance, the total lepton number is violated at a constant rate in the process of neutrino emission of a star if \( \nu_e \) has a Majorana mass: the non-conservation takes place in the neutrino production reaction where a fraction of the neutrino wave function is an anti-neutrino \( \bar{\nu}_e \). This fraction is of course suppressed by the ratio \( m_\nu^2/E_\nu^2 \). That is not the case if a significant portion of the \( \nu_e \) spectrum is converted into \( \bar{\nu}_\mu \) anti-neutrinos under appropriate magnetic field and transition magnetic moments. Another example is a Goldstone boson associated with the breaking of \( L_e - L_\mu \). If there is resonant conversion of \( \nu_e \)s into \( \nu_\mu \)s then the goldston source has macroscopic dimensions and a long-range scalar field is generated.

II. GOLDSTONE FIELDS

The solution of the \( \phi \) equation of motion is found using standard Green function methods. In the source term, \( \partial_\mu J_\Lambda^\mu = \dot{\rho}_\Lambda^{cr} \) represents the rate of creation of \( \Lambda \)-charge per unity of time and volume. In a stationary regime where this rate is constant or varies slowly enough one gets a Coulombian field.
\[
\phi(t, \vec{r}) = \frac{-1}{4\pi} \int d^3x \frac{\langle \partial_\mu J_\mu^\nu(t, \vec{x}) \rangle}{V_\Lambda |\vec{r} - \vec{x}|},
\]
where the bracket means expectation value of the operator \(J_\mu^\nu\) over the appropriate particle state density. The Lagrangian of Eq. (4) tells however that the fermions couple to the gradient \(A_\mu^\nu = \partial_\mu \phi/V_\Lambda\). In the stationary limit \(A_\mu^\nu\) is negligible and the vector \(\vec{A}_\Lambda = -\vec{\nabla} \phi/V_\Lambda\) obeys a Gauss law:
\[
\vec{\nabla} \cdot \vec{A}_\Lambda = -(\dot{\rho}_\Lambda + \vec{\nabla} \cdot \vec{j}_\Lambda)/V_\Lambda^2.
\]
It implies that the flux of \(\vec{A}_\Lambda\) over a closed surface is proportional to the time rate \(\dot{Q}_\Lambda^\tau\) of creation of \(\Lambda\)-charge inside the surface. In turn \(\dot{Q}_\Lambda^\tau\) is equal to the difference between the flux of \(\langle \vec{j}_\Lambda \rangle\) and the flux of the current \(\vec{j}_\Lambda^0\) that would exist if there was no \(\Lambda\) violation inside the surface. Hence,
\[
\vec{A}_\Lambda = -\left(\langle \vec{j}_\Lambda \rangle - \vec{j}_\Lambda^0\right)/V_\Lambda^2.
\]
The value of \(\vec{j}_\Lambda^0\) is determined for a given \(\dot{\rho}_\Lambda\) and obeys the identity \(\dot{\rho}_\Lambda + \vec{\nabla} \cdot \vec{j}_\Lambda^0 = 0\). One reaches the same solution by noting that the Noether current \(j_\mu^\nu + V_\Lambda \partial_\mu \phi\) is conserved.

A typical situation of potential interest is the flavour violation driven by neutrino oscillations in their way out of a star. The quantum numbers are the partial lepton quantum number \(L\), the interaction Lagrangian \(L^\tau\) gives for the standard neutrinos
\[
\delta \vec{j}_\ell = \bar{\ell} \gamma^\mu \ell + \bar{\nu}_L \gamma^\mu \nu_L + \sum_{a > \tau} \bar{\nu}_L^a \gamma^\mu (L_a) \nu_L^a,
\]
where \(\nu_L^a\), \(a > \tau\), denote any non-standard neutrinos. To be specific suppose that \(L_\tau\) is spontaneously broken. The Goldstone boson \(\phi_\tau\) associated with it couples to a current \(J_\mu^\nu\) that is essentially \(j_\mu^\nu\) but contains in general higher order terms arising from radiative corrections. Their existence implies that even if \(L_\tau\) is conserved in a certain process, the very reaction rate \(i.e.,\) of particle production or absorption, is enough to yield a goldston field. However those terms are of higher order and I will limit myself to the bare tree-level currents.

For definiteness I assume that all non-standard neutrinos are extra-heavy singlet neutrinos that cannot be produced at energies of interest and in addition, no significant neutrino \(\leftrightarrow\) anti-neutrino transitions occur either because the light neutrino masses are too small or because transition magnetic moments do not come into play.

Suppose that in the way out of a star \(\nu_e s\) convert into \(\nu_\tau s\) and the flux of converted neutrinos is \(\vec{j}(\nu_e \rightarrow \nu_\tau)\). Then, a macroscopic goldston field \(\phi_\tau\) exists with non-zero gradient outside the sphere where the conversion takes place. In the Supernovae case one also gets a flux \(\vec{j}(\nu_\tau \rightarrow \nu_e)\). Eq. (9) gives \(\vec{A}_\tau = -\delta \vec{j}_\tau/V_\tau^2\) with
\[
\delta \vec{j}_\tau = \vec{j}(\nu_e \rightarrow \nu_\tau) - \vec{j}(\nu_\tau \rightarrow \nu_e).
\]
If the conversion involves anti-neutrinos one has instead \(\delta \vec{j}_\tau = -\vec{j}(\bar{\nu}_e \rightarrow \bar{\nu}_\tau) + \vec{j}(\bar{\nu}_\tau \rightarrow \bar{\nu}_e)\), because the antineutrinos have negative Lepton numbers.

The field \(\phi_\tau\) yields a potential energy for the \(\tau\) neutrinos. Generalising now to any quantum number \(L_\ell\) and goldston field \(\phi_\ell\), the interaction Lagrangian \(L_{\text{int}} = J_\ell \mu A_\ell^\mu\) in Eq. (4) gives for the standard neutrinos \(\nu_\ell\),
The potential energy of a neutrino with velocity $\vec{v}_\nu$ is therefore

$$V_{\nu_i} = -A^0_{\nu_i} + \vec{A}_{\nu_i} \cdot \vec{v}_\nu = -V_{\bar{\nu}_i}. \quad (13)$$

In the stationary limit $A^0_{\nu_i} = 0$ and for radially moving neutrinos $V$ is just the radial component $A_r$. The interesting point is that such kind of potential is not universal in flavour and in addition the sign of the potential is not solely determined by the matter content of the medium where neutrinos propagate as it also depends on the flavour structure of the neutrino fluxes. This allows in principle other than the standard MSW type of oscillations. The most interesting effect is probably the occurrence of $\bar{\nu}_e \leftrightarrow \bar{\nu}_x$ in a situation where it would not normally occur i.e., with a mass hierarchy $m_{\nu_e} < m_{\nu_\mu}, m_{\nu_\tau}$. In the following I model such kind of phenomenon with application to Supernovae neutrinos.

If a partial Lepton number is spontaneously broken it is natural that the same happens for all the three flavours. Let $\sigma_i$ be the set of scalar fields, singlets of $SU(2) \times U(1)$, with quantum numbers $\Lambda_i = (L_e, L_\mu, L_\tau)$ under the symmetry group $U(1)_e \times U(1)_\mu \times U(1)_\tau$. I assume that the vacuum expectation values $\langle \sigma_i \rangle$ break all the Lepton numbers producing 3 Goldstone bosons. By expanding around the vacuum as $\sigma_i = \exp \{ -i \sum_\Lambda \xi_\Lambda \Lambda_i \} (\langle \sigma_i \rangle + \cdots)$, where the dots stand for the massive degrees of freedom, one obtains for the Lagrangian of the Goldstone fields

$$\mathcal{L} = \frac{1}{2} V^2_{\Lambda M} \partial_\mu \xi^\Lambda \partial^\mu \xi^M + J^\mu_\Lambda \partial_\mu \xi^\Lambda, \quad (14)$$

a sum over the quantum numbers $\Lambda, M$ is implicit. The matrix

$$V^2_{\Lambda M} = 2 \sum_i \Lambda_i M_i |\langle \sigma_i \rangle|^2 \quad (15)$$

is in general non-diagonal, the sole condition being that some of the fields $\sigma_i$ have simultaneously more than one quantum number different from zero. Such mixing implies that the Goldstone bosons can mediate an action at a distance between two distinct currents as the equations of motion show:

$$\partial_\mu \partial^\mu \xi^\Lambda = -\sum_M G_{\Lambda M} \partial_\mu J^\mu_M, \quad (16)$$

where $G_{\Lambda M}$ is the inverse matrix of $V^2_{\Lambda M}$. Hence, in a stationary regime the vector field solutions are now

$$\vec{A}_\Lambda = -\sum_M G_{\Lambda M} (\langle \vec{J}_M \rangle - J^0_M) \quad (17)$$

instead of Eq. (9). The expression of the one-particle potential remains the same as in Eq. (13) because the interaction Lagrangian is the same as before namely, $J^\mu_\Lambda A_{\Lambda \mu} = J^\mu_\Lambda \partial_\mu \xi^\Lambda$.

It is not the aim of this paper to construct specific models but as a proof of existence I present here a suitable adaptation of the abelian familon [6] and seesaw Majoron [2] models. Three neutral singlets $N^e_R, N^\mu_R, N^\tau_R$ join the standard lepton iso-doublets $L^e, L^\mu,$
$L^\tau$, the charged singlets $e_R, \mu_R, \tau_R$ and the Higgs doublet $\Phi$. The fields $N^\ell_R$ only have Yukawa interactions namely, flavour conserving Dirac couplings

$$\mathcal{L}_D = -\left(m_\ell \bar{L}^\ell \Phi \ell_R + \mu_\ell \bar{\ell}^\ell \Phi N^\ell_R \right)/\langle \Phi^0 \rangle + \text{h.c.},$$

plus Majorana couplings with a set of scalar singlets $\sigma_{\ell m}$:

$$\mathcal{L}_M = -\frac{1}{2} \sum_{\ell, m} \left(N^\ell_R C N^m_R \right) M_{\ell m} \sigma_{\ell m}/\langle \sigma_{\ell m} \rangle.$$ 

The invariance under $U(1)_e \times U(1)_\mu \times U(1)_\tau$ determines the $(L_e, L_\mu, L_\tau)$ quantum numbers of the scalar fields namely, $\sigma_{ee}(-2, 0, 0)$, $\sigma_{e\mu}(-1, -1, 0)$, $\sigma_{\mu\tau}(0, -1, -1)$, etc. The expectation values of these fields break all the three Lepton numbers which gives three Goldstone bosons. The matrix $G_{\ell m}$ that enters in the goldston equations of motion, Eq. (16), has full generality with respect to its flavour structure. For Majorana masses $M$ much higher than the Dirac masses $\mu_\ell$, the neutrinos separate in heavy $N^\ell$ with mass matrix $M$ and very small mixing with the three light neutrinos, $\nu = \nu_1, \nu_2, \nu_3$, that acquire a Majorana mass matrix $m_{\ell m} = -\mu_\ell \mu_\ell M_{\ell m}^{-1}$. The magnitudes relevant for the subsequent application lie around $m \sim 10^{-2} \text{eV}$, and $M = 0.1 - 1 \text{TeV}, \mu \sim 30 \text{KeV}$, very far from the original [7] seesaw GUT scales but not excluded from first principles.

It is known that this type of singlet Majoron or familons easily escape present astrophysical bounds on the couplings of Nambu-Goldstone bosons. Here, they couple primarily to neutrinos with pseudo-scalar couplings $g \sim m_\nu/V_\Lambda$ which, for the specified range of neutrino masses, are far below $10^{-4}$ to play a role in Supernova collapse dynamics [8], and even below the limit of $\sim 10^{-4.5}$ to produce Supernova cooling through singlet Majoron emission [9]. On the other hand, pseudo-scalar couplings to electrons, that could be responsible for energy loss in stars [10], only arise through radiative corrections and are so further suppressed.

### III. SUPERNova NEutrinos

If the cause of the well known solar neutrino anomalies lies in neutrino flavour oscillations then one expects that the same phenomenon occurs with Supernovae neutrinos. In the accepted understanding of Supernova type-II all neutrino species including their anti-particles are emitted with similar importance after a first electron neutrino burst. As a working frame I adopt here the common prejudice that the neutrino mass eigenstates are in first approximation - but not exactly - $\nu_1 \approx \nu_e, \nu_2 \approx \nu_\mu, \nu_3 \approx \nu_\tau$, with the following mass hierarchy: $m_1 < m_2 < m_3$. In their way out from the star the electron neutrinos have a potential energy that exceeds by $V_W$ the equal potentials of $\nu_\mu$ and $\nu_\tau$. $V_W = \sqrt{2} G_F n_e$ [11,12] is positive and proportional to the electron density $n_e$ and therefore the neutrinos propagate from the region of emission where $V_W$ is much larger than $m_3$ (so I assume) to the outer space where $V_W$ goes to zero. It is then in principle possible to have MSW resonant conversions from $\nu_e$ into $\nu_\tau$ and vice-versa [13,14]. The anti-particle $\bar{\nu}_e$ on the contrary, receives a negative potential $-V_W$ and cannot be resonantly converted into the other flavours, given the assumed mass hierarchy.
In a Supernova the $\nu_e$ flux is larger than the $\nu_\tau$ flux and consequently more $\nu_e$ transform into $\nu_\tau$ than the reverse so making a net $L_e - L_\tau$ violation. As a result, the goldston fields $\phi_e$, $\phi_\mu$, $\phi_\tau$ develop as described above. Applying Eq. (17) with flux transfers
\[
\delta j_\tau = -\delta j_e = j(\nu_e \rightarrow \nu_\tau) - j(\nu_\tau \rightarrow \nu_e),
\]
one derives the vector potentials
\[
\vec{A}_\ell = (G_{e\ell} - G_{\ell\tau}) \delta j_\tau,
\]
\[\ell = e, \mu, \tau.\] 
They vanish of course inside the sphere where $\nu_e \leftrightarrow \nu_\tau$ takes place. The magnitude of the constants $G_{AM}$ is directly given by the inverse square of the Lepton symmetry breaking scale, but the flavour structure is quite arbitrary and is not directly related to the neutrino masses as they also depend on the Yukawa couplings. The matrix $G_{AM}$ is only constrained to be symmetric and have positive eigenvalues as demands the goldston kinetic Lagrangian.

An interesting case is the following hierarchy: $|G_{e\tau}|, |G_{e\mu}|, |G_{e\tau}| < |G_{\mu\tau}|, G_{\tau\tau}$. Then, the vector $\vec{A}_e$ is much smaller in magnitude than $\vec{A}_\mu$ and $\vec{A}_\tau$. Far enough from the neutrinosphere and assuming spherical symmetry, the $\nu$ potentials are well approximated by
\[
V_{\nu_e} = V_W + A_e, \quad |A_e| < < |V_{\nu_e}|,
V_{\nu_\mu} = A_\mu \simeq -G_{\mu\tau} \delta j_\tau,
V_{\nu_\tau} = A_\tau \simeq -G_{\tau\tau} \delta j_\tau < 0,
\]
ignoring the universal neutral current and gravitational potentials. The anti-neutrino potentials are the symmetric of these.

In the following the numbers for the neutrino fluxes are extrapolated from the Supernovae neutrino luminosities obtained in [15] with a $25 M_\odot C$ model and in the generic model of [16]. The energy fluxes follow this pattern: $\nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau$ have all the same luminosity say $L_{\nu_\nu}$, and $\bar{\nu}_e$ gets rapidly close to the $\nu_e$ luminosity after the initial $\nu_e$ burst (instant $t = 0$). From then to the supernova explosion ($t \approx 0.45$ sec), the average value of $L_{\nu_e}$ is around $(4 - 8) \times 10^{52}$ ergs/sec and $L_{\nu_e}$ is about 2/3 to 3/4 of $L_{\nu_e}$. After the explosion the spectrum is thermal and the luminosities $L_{\nu_\nu} \approx L_{\bar{\nu}e} \approx L_{\nu_e}$ decrease along the time. At $t = 1$ sec, $L_{\nu_e} = (4 - 10) \times 10^{51}$ ergs/sec. The fluxes are obtained from the number of emitted particles per second $F = j(4\pi r^2)$, and $F = L/\bar{E}$ by definition of average energy $\bar{E}$. I take $\bar{E} = 10$ MeV for $\nu_e$ and 20 MeV for $\nu_\mu, \nu_\tau$. If the resonance is adiabatic i.e., all $\nu_e$ transform into $\nu_\tau$ and viceversa, the fluence transfer is $\delta F_\tau = F_{\bar{\nu}_e} - F_{\nu_e}$. Then, in unities of $10^{51}$ ergs/MeV/sec, $\delta F_{51} \simeq 2 - 5$ before the explosion and $\delta F_{51} \simeq 0.2 - 0.5$ at 1 sec time. In these unities,
\[
V_{\nu_\mu} = -1.48 \frac{G_{\mu\tau}^F \delta F_{51}}{r_{10}^2} \times 10^{-12} \text{ eV},
\]
where $G_{\mu\tau}^F = G_{\mu\tau}/G_F$, $G_F = 11.66$ TeV$^{-2}$ is the Fermi constant and $r_{10} = r/10^{10}$ cm.

The standard charge current potential $V_W$ is proportional to the electron density $n_e$ given by $Y_e \simeq 1/2$ times the nucleon density $\simeq n_n$. At large distances the mass density
\( \rho \) behaves as \( \tilde{M}/r^3 \) with a constant \( \tilde{M} = \tilde{M}_{31} \times 10^{31} \text{g}, \ \tilde{M}_{31} = 1 - 15 \) depending on the star [17]. Then,

\[
V_W = \sqrt{2} G_F n_e = 7.6 Y_e \tilde{M}_{31} \frac{1}{r_{10}^3} \times 10^{-13} \text{eV} .
\]  

(24)

\( V_W \) decays necessarily faster than the goldston potentials which being proportional to the \( \nu \) fluxes inevitably overcome at large enough radii. That is a crucial point. The effective \( M^2 \) matrix [12] relevant for \( \bar{\nu}_e - \bar{\nu}_\mu \) oscillations is, up to a universal term, given by:

\[
\begin{pmatrix}
M^2_e = 2E(V_{\bar{\nu}_e} - V_{\bar{\nu}_\mu}) & M^2_{e\mu} = \Delta m^2 \sin 2\theta \\
M^2_{\mu e} = \Delta m^2 \sin 2\theta & M^2_\mu = \Delta m^2 \cos 2\theta
\end{pmatrix}
\]  

(25)

An electron anti-neutrino is emitted with a negative potential \( V_{\bar{\nu}_e} = -V_W \) and a resonance would not be possible because \( \Delta m^2 > 0, \ \theta < \pi/4 \), by assumption. But, if the constant \( G_{\mu \tau} \) is negative, \( \bar{\nu}_\mu \) gets a negative goldston potential \( V_{\bar{\nu}_\mu} = G_{\mu \tau} \delta j_\tau \) and the resonance condition \( M^2_\mu = M^2_\mu \) may be met. In the Fig. 1 it corresponds to the interception of the "e" and "\( \mu \)" curves.

\[
\text{FIG. 1. } 2E(V_{\bar{\nu}_e} - V_{\bar{\nu}_\mu}) \text{ is plotted in the solid and dotted curves with } \delta F_{51} = 1 \text{ and } 3 \text{ respectively. The dashed line stands for a particular value of } M^2_\mu = \Delta m^2 \cos 2\theta \text{ and the squares curve is the adiabatic boundary for } M^2_\mu \tan^2 2\theta \text{ in resonance 2.}
\]

The positive part of the 'potential' \( M^2_\mu \) is plotted in Fig. 1 with values \( G_{\mu \tau}^E = -1, \ \delta F_{51} = 1 \) or 3, \( Y_e = 1/2, \ \tilde{M}_{31} = 4 \), and a neutrino energy of 20 MeV, the same for all curves. Take \( M^2_\mu = 6 \times 10^{-7} \text{eV}^2 \) for example. Its dashed line crosses the relative potential curve at two points: the first, \( r_1 \), lies in the remarkably sharp transition from the \( -V_W < 0 \)
dominated region to the goldston dominated one and very likely a non-adiabatic level crossing occurs there i.e., \( \nu_e \) continues \( \nu_e \) and \( \nu_\mu \) continues \( \nu_\mu \). In the second point, \( r_2 \), the potential is smooth enough (\( \propto r^{-2} \)) for adiabatic transitions. In general, the conversion probability is enhanced \([12,18]\) if \( M_\mu^2 \tan^2 2\theta \) is larger than the rate \( f(r) = |E\ dV/Vdr| \) at the resonance position. This function is plotted with squares in the figure for \( V \propto r^{-2} \). One can see that small mixing angle adiabatic conversions are possible for a wide range of neutrino masses. Using the resonance condition (neglecting \( V_W \)) one gets a lower bound on the mixing angle,

\[
M_\mu^2 \tan^4(2\theta) > \frac{0.52 \ E_{10}}{G_{\mu\tau}^F \delta F_{51}} \times 10^{-10} \text{eV}^2 ,
\]

while the maximum value of the \( \nu \) potential sets an upper limit to the square mass difference:

\[
M_\mu^2 < 1.66 \ E_{10} \frac{|G_{\mu\tau}^F \delta F_{51}|^3}{(Y_e M_{31})^2} \times 10^{-5} \text{eV}^2 ,
\]

where \( E_{10} = E/10 \text{MeV} \).

The requirement of non-adiabatic crossing in the first sharp resonance puts an upper limit on \( M_\mu^2 \tan^2 2\theta \). The slope of the potential on that point is equal (up to a numeric factor between 1 and 1/2) to the slope on the second resonance times the ratio \( (r_2/r_1)^3 \) and therefore that is the size of the window of interesting \( \tan^2 2\theta \) values for a given \( M_\mu^2 \). The net result is that for a wide range of parameters \( \nu_e, \nu_\mu \) undergo one level crossing with maximum probability \( P_c = 1 \) \([19,20]\) plus one adiabatic resonance. Their survival probabilities \([12,20]\) are therefore \( \sin^2 \theta \) and not \( \cos^2 \theta \) (mixing with \( \nu_\tau \) neglected).

As far as \( \nu_e, \nu_\mu \) is concerned, the potentials and \( M_\mu^2 \) are symmetric to the anti-neutrino case. There is still one resonance position as in the standard model but now in the abrupt part of the potential. It results in a non-adiabatic crossing and the oscillation pattern is \( \nu_e \to \nu_\tau \) and \( \nu_\tau \to \nu_e \), \( \nu_\mu \to \nu_\mu \) in contrast to the ‘standard’ one \([13,14]\), \( \nu_\tau \to \nu_e \to \nu_\mu \), \( \nu_\mu \to \nu_e \), for the same mass mixing pattern. This does not change the observation predictions because \( \nu_\mu \), \( \nu_\tau \) have the same energy spectrum. Finally, the late \( \nu_e \leftrightarrow \nu_\mu \) oscillations alter the net flavour violation and modify the goldston fields out of the sphere where they take place but that does not create new resonances.

This all picture may relate to solar neutrinos in two different ways: 1) \( \nu_e \to \nu_\tau \) in the Sun with \( 10^{-4} \text{eV}^2 \geq \Delta m^2_{\tau\tau} > \Delta m^2_{\mu\mu} \). Then, \( \nu_e \to \nu_\tau \) (\( \nu_\tau \to \nu_e \)) triggers the appearance of goldston fields in the supernova (only out of the sphere where \( \nu_e \to \nu_\tau \) takes place in particular, if \( \Delta m^2_{\tau\tau} \approx 10^{-5} \text{eV}^2 \), \( r \gtrsim 2 \times 10^{10} \text{cm} \)) and eventually \( \nu_e \leftrightarrow \nu_\mu \) occurs. 2) \( \Delta m^2_{\tau\tau} \) lies above \( 10^{-4} \text{eV}^2 \) and it is \( \nu_e \to \nu_\mu \) that takes place in the Sun. Then, resonant \( \nu_e \leftrightarrow \nu_\mu \) transitions occur in the supernova with the kinematic parameters proper of solar neutrinos. The solar non-adiabatic solution \([19–21]\) with parameters \( M^2_\mu \leq 10^{-5} \text{eV}^2 \) and \( M^2_\mu \tan^2 2\theta \approx 4 \times 10^{-8} \text{eV}^2 \) satisfies the desired condition if the neutrino fluxes are high enough (compare the dotted \( \delta F_{51} = 3 \) curve and the solid one, \( \delta F_{51} = 1 \)).

This points to a distinctive feature of that phenomenon: the strong dependence on the neutrino fluxes (third power in the bound of Eq. (27)) should manifest by the detection of \( \bar{\nu}_e \), s in the first moments of supernova observation with a energy spectrum proper of \( \bar{\nu}_\mu \) (25 MeV instead of 15 MeV) that is not maintained later when the fluxes drop down.
A crucial condition is of course that the scale of Lepton numbers symmetry breaking lies at or below the Fermi scale and that the goldston potentials have the right sign. The observation of the described correlation between oscillations and flux magnitudes would provide a measurement of the breaking scale.

The analyses of the SN1987A events [22] have either disfavoured the occurrence of $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ oscillations by comparing the best-fit energy spectrum with the current predictions for $\bar{\nu}_e$ and $\bar{\nu}_\mu$ spectra [23] or put a maximum limit of 35% for the $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ permutation probability [24]. However, the oscillations driven by goldston fields, if they happen at all, only occur in the periods of largest fluxes. If they occur in the first 1/2 second or so of emission, the resultant hotter spectrum may manifest as a short artificial cooling time scale on the top of the real neutrino spectrum evolution. The fits to the SN1987A time evolution indeed exhibit two time scales, one of about 4.0 s and another $\leq 1.0$ s [25]. This seems consistent with an intrinsic two phase cooling process [25] nevertheless, it remains as a provocative piece of evidence.

Another point that should be kept in mind is that $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ oscillations appear [26] to be consistent or even favoured by the data when the analysis is made model independent. This may call for a $\nu_\mu$ spectrum of energies below those predicted so far but, as has been remarked recently [27], this may be obtained if non-elastic neutrino scatterings in the transport sphere are taken in consideration.

A typical signature of the assumed $\nu$ mixing pattern is the non-observation of the initial $\nu_e$ burst. It turns more subtle however when one approaches the non-adiabatic frontier because the level crossing probability [19,20] depends exponentially on the inverse of the neutrino energy. Thus, almost-adiabatic transitions for 10 MeV neutrinos may be consistent with the survival of the highest energy neutrinos present in the electron capture $\nu_e$ burst. The survival of 32 MeV $\nu_e$s was studied in [28] and in particular, its consistency with the solar non-adiabatic solution.

IV. CONCLUSIONS

The main result of this paper is that despite the fact that true Goldstone bosons only have derivative couplings with matter particles, one can prove that coherent Goldstone fields develop whenever the matter charges they are associated with are not conserved in macroscopic scales. The equations of motion of the Goldstone bosons are equivalent to the conservation law of the Noether currents associated to nonlinear symmetries of the type $\phi \rightarrow \phi + \alpha$, these currents are a sum of a Goldstone boson piece and a matter current, and, whenever the quantum numbers of the matter particles are not conserved, the Goldstone fields develop compensating terms. Examples are where this may take place in a macroscopic proportion and as a result, the matter particles become subject to long range forces.

In part because the potential energies are inversely proportional to the square symmetry breaking scale and on the other hand, proportional to the time rates of particle reactions, the situations of interest are special, but not unique! Here I exercised a scenario where Lepton flavour violation is realized through neutrino oscillations in a Supernova with large enough intensity to alter the $\nu$ potentials and oscillation patterns, provided that the symmetry breaking scale is less than 1 TeV. The characteristic feature of those
kind of oscillations is their correlation with the periods of largest $\nu$ fluxes thus providing a challenge to better understand the time evolution of the neutrino signal and the flavour dependent energy spectra as well. It seems worth to explore other physical systems and symmetries.

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REFERENCES