Dirac magnetic monopole and the discrete symmetries

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Abstract

We examine several issues related to the processes of Dirac monopole-antimonopole production in high-energy collisions such as $e^+e^-$ annihilation. Perturbative calculations for such processes are known to be inherently ambiguous due to the arbitrariness of direction of the monopole string; this requires use of some prescription to obtain physical results. We argue that different prescriptions lead to drastically different physical results which suggests that at present we do not have an entirely satisfactory procedure for the elimination of string arbitrariness (this problem is quite separate from the problems caused by the large coupling constant). We then analyze the consequences of discrete symmetries (P and C) for the monopole production processes and for the monopole-antimonopole states. The P and C selection rules for the monopole-antimonopole states turn out to be different from those for the ordinary fermion-antifermion or boson-antiboson system. In particular, the spin $1/2$ monopole and antimonopole should have the same helicities if they are produced through the one-photon annihilation of an electron and positron. A stronger selection rule holds for spinless monopoles: CP symmetry absolutely forbids the monopole-antimonopole production through the one-photon annihilation of an electron and positron. Single-photon $e^+e^-\rightarrow g^+g^-$ amplitude
has been a key input in calculating the contribution of virtual $g^+g^-$ pairs to various physical processes such as the decay $Z \rightarrow 3\gamma$ and the anomalous magnetic moment of the electron. Applying our conclusions to these cases can lead to significant modifications of the results obtained in previous works. 14.80.Hv
I. INTRODUCTION

Despite the long history of magnetic monopole, the interplay between the discrete symmetries and the magnetic charge has not been completely elucidated in the literature. Some of the important issues are: What is the relative parity of the monopole and antimonopole? Is it negative, as in the case of ordinary fermion-antifermion, or not? What happens to the monopole string under the action of C,P,T? Are there any non-trivial selection rules that are based on discrete symmetries and would affect the processes of monopole creation in particle collisions? One example is the electron-positron annihilation into monopole-antimonopole pair. Our paper is an attempt to answer these and related questions.

Motivation, other than theoretical completeness, comes from the fact that the quantum field theory of magnetic monopole is a strong coupling theory. Consequently, conventional perturbative calculations are of little help and one has to rely rather on general principles such as various symmetries, unitarity and so on.

At present the electromagnetic duality is at the focus of the elementary particle physics. Various aspects of this concept have been explored recently leading to significant breakthroughs for instance in better understanding the structure of the string theory. Another area of considerable current interest is the study of magnetic monopole and string solutions in the context of the standard electroweak model. Also, much interest is generated in investigating the magnetic monopoles appearing in theories which combine the general relativity with the Yang-Mills-Higgs systems. All this indicates at the necessity of a more thorough analysis of the whole complex of problems associated with the idea of magnetic monopole both from a theoretical and experimental perspectives (for reviews on Dirac monopoles [?] see, e.g., [?,?,?]).

Despite several unconfirmed candidate events, the conventional verdict is that the magnetic monopole has never been observed at the laboratory. However the experimental efforts devoted to the monopole searches show no signs of subsiding. Work is being done aimed at introducing new methods of looking for monopoles, as well as improving sensitivity of more
traditional types of experiments.

There can be different starting points for conducting the magnetic monopole searches. One can think of producing monopoles directly at accelerators or observing monopoles bound to the nuclei of the ferromagnetic materials or, alternatively, look for the bound states of monopole and antimonopole. In all these cases we speak of production and detection of monopoles as real particles (although they may be confined). Another search strategy is to look for the effects of virtual monopoles in high-energy reaction such as $e^+e^-$ annihilation and Z-boson decay. Also of interest are similar effects related to the contribution of virtual magnetic monopoles to high-precision quantities, for instance, the anomalous magnetic moment of the electron or muon.

Although all of the experiments for their interpretation have to rely on the theory governing the behaviour of monopoles, some of the experiments are in fact less sensitive to the theoretical nuances than the others. For example, the traditional methods of monopole search employ only the basic fact that the monopole is the source of the strong magnetic field around itself. The strength of the magnetic field is assumed to be dictated by the Dirac quantization condition [?] that connects the electric and magnetic coupling constants:

$$\frac{eg}{4\pi} = \frac{1}{2^n} \quad (1)$$

Thus, a large value of the magnetic charge and the associated strong magnetic field is the key signature of the usual experimental methods.

A different class of experiments is based on attempts of producing monopoles at accelerators or studying the effects of virtual monopoles in high-energy collisions or in static elementary particle properties. The latter class of methods for monopole searches attracted significant attention recently (see [??,?] and references therein).

The main problem with this class of experiment lies in theoretical interpretation. It is sometimes not recognized clear enough that these experimental results (usually reported in terms of a specific bound on the monopole mass) depend much more on the theoretical formulation of how the monopoles interact than do “classical” search methods. The difficult
question is how to obtain *unambiguous* predictions from the monopole theory. There are two sources of difficulties. The first is a well-known fact that the coupling constant (that is, the magnetic charge of the pole) should be very large if the Dirac quantization condition is to be true. That makes impossible the use of perturbation theory for practical calculations (although it can be used within an effective field theory approach).

The second difficulty (which has not been as much popularized) is, perhaps, more fundamental. It has nothing to do with the magnitude of the coupling constant at all; rather, it is related to the existence of Dirac string – infinitely thin line of magnetic flux stretching from the magnetic pole to infinity. It has been often repeated in the literature that the Dirac quantization condition makes the string invisible. However, in reality the situation is far from being so simple and clear. This is especially true in the context of quantum field theory where monopoles are allowed to be created and annihilated (recall that the Dirac quantization condition was initially derived for a simple quantum mechanical system “electron plus monopole”). It is generally believed that the full quantum field theory does not depend on how we choose the position of the string which can be arbitrary. However, the peculiarity of the monopole theory is that the formulation of the theory cannot be made without recourse to the string concept in one or another form. In other words, the quantum theory of monopoles is not *manifestly* string-independent. Since the string fixes a specific direction, the theory is not *manifestly* Lorentz invariant either. Perhaps, it is a unique example of a physical theory possessing implicit Lorentz invariance which nevertheless cannot be formulated in a *manifestly* invariant way.

What are the practical implications of this fundamental theoretical feature? One consequence is this. Imagine that we forget for a moment about the large coupling constant and attempt to calculate some physical quantity in the first order of perturbation theory (such as the monopole-antimonopole production in $e^+e^-$ annihilation). The result will be discouraging because it will be ambiguous. More exactly, the result will depend explicitly on the string direction which is clearly unacceptable. Obviously, a serious question is how to deal with this type of situation.
Consider, for example, the process of $e^+e^-$ annihilation into monopole-antimonopole pair (assumed to be fermions). It has a virtue of being physically interesting and simple enough at the same time. This process has been previously considered and a prescription has been given for elimination of string dependence [?] which has been subsequently adopted in [?,?]. The resulting cross-section is not very different from the cross-section for the creation of a pair of usual fermion-antifermion.

However, we believe that the prescription is not entirely satisfactory. One reason for concern is that it only gives the value of the squared modulus of the amplitude, but not the amplitude itself.

Therefore, it would be difficult to generalize it for the cases when an interference of two amplitudes is involved (for instance, if we want to calculate the interference between electromagnetic and Z-boson contributions to the monopole-antimonopole production in $e^+e^-$ annihilation).

One purpose of this paper is to consider an alternative procedure and see if the physical results would be the same. More specifically, we propose an alternative prescription based on the averaging of the amplitude over all possible directions of the string. This procedure has a clear physical meaning since the string is supposed to be unobservable. However, it leads to a drastically different answer: according to this prescription, the amplitude of $e^+e^-$ annihilation into the monopole-antimonopole pair should be zero to the lowest order of perturbation theory.

This result suggests that the task of extracting the physically meaningful results from the inherently ambiguous perturbative calculations should be considered as an open problem requiring further investigation.

In this paper we try to circumvent this problem by using only general principles of quantum field theory whose validity does not rely on the use of perturbation theory. It is natural to start with the consideration of the role of the discrete symmetries such as C, P and T transformations and to see what constraints are provided by these symmetries.

We show that the behaviour of the monopole-antimonopole system under discrete sym-
metries is rather different from that of standard fermion-antifermion or boson-antiboson system (standard means not carrying magnetic charge). In particular, there arise selection rules for the process of the monopole-antimonopole production through one-photon annihilation of an electron and positron. For spin 1/2 monopole the P and C symmetries require that the monopole and antimonopole have the same helicities. For spinless monopoles CP symmetry absolutely forbids the monopole-antimonopole production through the one-photon annihilation of an electron and positron.

The plan of the work is as follows. Section 2 summarizes the Feynman rules for the monopole field theory and the structure of the amplitude $e^+e^-\text{annihilation into monopole-antimonopole pair.}$ The appearance of string dependence is emphasized. In Section 3 we suggest an alternative prescription for eliminating the string dependence of the amplitude. The averaging of the amplitude over the string directions is carried out which results in vanishing of the one-photon exchange amplitude for the monopole-antimonopole production in $e^+e^-\text{annihilation.}$ Section 4 is devoted to the analysis of the discrete symmetries in the quantum field theory of magnetic monopoles. The results are applied to the derivation of selection rules for the monopole-antimonopole system in Section 5. Finally, we present our conclusions in Section 6.

II. THE FEYNMAN RULES AND THE ELECTRON-POSITRON ANNIHILATION INTO MONOPOLE-ANTIMONOPOLE

The Feynman rules [?] describing the interactions of photons and monopoles have the following form (Fig. ??):

$$-ig\frac{\epsilon^{\mu\nu\lambda\rho}\gamma^\nu n^\lambda q^\rho}{qn + i\epsilon}.$$  \hspace{1cm} (2)

The photon and fermion propagators, as well as the photon-electron vertex, remain the same as in the standard QED. Note that in other formulations of the monopole quantum field theory the Feynman rules would be different (for details, see [?]). The most notable feature
of these Feynman rules is the fact that they depend on the vector $n$ which corresponds to the direction of the string. Thus, these Feynman rules are not manifestly invariant. However, it is believed that the full theory is nevertheless Lorentz-invariant, that is physical predictions should not depend on the specific direction of the vector $n$.

Now, let us write down the amplitude of the process of the electron-positron annihilation into the monopole-antimonopole pair. The amplitude has the following form (Fig. ??):\[A = iegK^\beta\epsilon^{\mu\beta\gamma\delta}n^\gamma q^\delta \frac{1}{qn} q^2 J^\mu.\] (3)

where \[J^\mu = \bar{v}_e(p_2)\gamma^\mu u_e(p_1), K^\beta = \bar{u}_g(p_3)\gamma^\beta v_g(p_4).\] (4)

The dependence on $n$ remains even after the squaring of the amplitude is made. An obvious question is how to make sense out of the $n$-dependent quantity. It has been suggested in Ref. [?] that one should drop the terms which have no pole in $q^2$ and thus to arrive at the following result: \[|A|^2 = \frac{e^2 g^2}{q^4} [(KJ^\dagger)(JK^\dagger) - (JJ^\dagger)(KK^\dagger)].\] (5)

III. A DIFFERENT PRESCRIPTION

However, the consistency of such a prescription can be questioned on the grounds that it gives the corrected value of the squared matrix element but not of the amplitude itself. Therefore, it would be difficult to generalize it for the cases when an interference of two amplitudes is involved (for instance, if we want to calculate the interference between electromagnetic and Z-boson contributions to the monopole-antimonopole production in $e^+e^-$annihilation). Another concern is whether Eq. (??) is positively definite or not. There exist a different approach to the problem of dealing with the $n$ dependence. The idea is to average over all possible directions of $n$. Since there are no physically preferred directions of $n$, all the directions should be taken with the same weight. Because all these directions
are physically indistinguishable, we have to perform averaging of the amplitude rather than of the squared matrix element. Therefore, we need to find the average value:

\[ \langle \frac{n^\gamma}{qn} \rangle. \]  

(6)

By Lorentz invariance, it is sufficient to find this average value in a system where \( n^0 = 0 \) and, consequently, \( n^2 = 1 \):

\[ \langle \frac{n}{-qn} \rangle = \frac{1}{4\pi} \int \frac{n}{-qn} d\Omega. \]  

(7)

In evaluating this integral one should be careful about a possible singularity arising when the vector \( n \) becomes orthogonal to \( q \). Let us choose the \( z \)-axis of the spherical coordinate system such as to be parallel to \( q \), and calculate the \( x, y, z \) components of the average:

\[ \frac{1}{4\pi} \int \frac{n_x}{-qn} d\Omega = -\frac{1}{4\pi|q|} \int_{-1}^{1} \frac{\sqrt{1-t^2}}{t} dt \int_{0}^{2\pi} \cos \phi d\phi, \]  

(8)

where \( t = \cos \theta \). Although the integral over \( \phi \) vanishes, we need to prove that the integral over \( t \) is not singular. For this purpose we have to invoke the \( qn + i\epsilon \) rule (or, in 3-dimensional terms, the \( qn - i\epsilon \) rule):

\[ \int_{-1}^{1} \frac{\sqrt{1-t^2}}{t} dt \rightarrow \int_{-1}^{1} \frac{\sqrt{1-t^2}}{t-i\epsilon} dt = \varphi \int_{-1}^{1} \frac{\sqrt{1-t^2}}{t} dt + i\pi \int_{-1}^{1} \delta(t) \sqrt{1-t^2} dt = i\pi \]  

(9)

Thus, indeed, the \( t \)-integral is finite and, therefore,

\[ \frac{1}{4\pi} \int \frac{n_x}{-qn} d\Omega = 0. \]  

(10)

Furthermore, a similar argument shows that the \( y \)-component of the average value also vanishes:

\[ \frac{1}{4\pi} \int \frac{n_y}{-qn} d\Omega = 0. \]  

(11)

Now, the \( z \)-component is;

\[ \frac{1}{4\pi} \int \frac{n_x}{-qn} d\Omega = -\frac{1}{4\pi|q|} \int d\Omega = -\frac{1}{|q|}. \]  

(12)
Thus, finally, we obtain:

\[
\frac{1}{4\pi} \int \frac{n}{-qn} d\Omega = -\frac{q}{q^2}.
\]

(13)

Consequently,

\[
\langle \frac{n^\gamma}{qn} \rangle = \frac{q^\gamma}{q^2}.
\]

(14)

Now, if we insert this value into the \( e^+e^- \) annihilation amplitude, we obtain

\[
A = iegK^\beta \epsilon^{\alpha\beta\gamma\delta} q^\gamma q^\delta J^\mu = 0.
\]

(15)

Thus, we arrive to the same conclusion: if one uses the averaging procedure to eliminate the string dependence of the amplitude, than the one-photon amplitude of the \( e^+e^- \) annihilation into monopole-antimonopole pair turns out to be zero. To summarize, we have shown that two different prescriptions used to eliminate the string dependence of the amplitude lead to drastically different physical results. Therefore, we suggest to try to circumvent this problem by using only general principles of quantum field theory whose validity does not rely on the use of perturbation theory. It is natural to start with the consideration of the role of the discrete symmetries such as C, P and T transformations and to see what constraints are provided by these symmetries.

**IV. DISCRETE SYMMETRIES IN THE QUANTUM FIELD THEORY OF MAGNETIC MONOPOLES**

There exist several formulations of the quantum field theory with electric and magnetic charges. However, all of the formulations have been shown [?] to be equivalent (except the formulation due to Cabibbo and Ferrari). Therefore, we will not need to specify exactly in which theoretical context are going to work. Rather, we will focus on the properties of the quantum field theory under the action of the discrete symmetries such as space reflection and charge conjugation. It can be shown [?,?],? that the quantum field theory of the electric and
magnetic charges is invariant under the following discrete transformations (we use Majorana representation, and denote the magnetically charged fields by the subscript $g$):

$$ C : \mathbf{E}, \mathbf{H}, \psi, \psi_g \rightarrow -\mathbf{E}, -\mathbf{H}, \psi^\dagger, \psi_g^\dagger. \quad (16) $$

$$ P : \mathbf{E}(x), \mathbf{H}(x), \psi(x), \psi_g(x) \rightarrow -\mathbf{E}(-x), -\mathbf{H}(-x), \gamma^0\psi(-x), \gamma^0\psi_g^\dagger(-x) \quad (17) $$

$$ T : \mathbf{E}(t), \mathbf{H}(t), \psi(t), \psi_g(t) \rightarrow \mathbf{E}(-t), -\mathbf{H}(-t), \gamma^0\gamma^5\psi(-t), \gamma^0\gamma^5\psi_g^\dagger(-t). \quad (18) $$

Here, a comment on terminology is in order. There is some confusion in the literature as to whether we should retain the names “P reflection” and “T inversion” for the above operations or we should call them “PM” and “TM” transformations, where M stands for the inversion of the magnetic charge. However, this difference is of semantical rather than of physical character; switching from one terminology to the other does not entail any physical consequences. In this paper we adopt the the first point of view, i.e. we keep the names parity and T inversion for the operations we have just introduced without making any further qualifications (the same view is adopted in [?]).

Sometimes one can find in the literature the statements to the effect that the theory of monopoles is not invariant under P and T symmetries. These statements refer to the situation when the discrete symmetries are assumed to act on the magnetically charged particles in exactly the same way as they act on the electrically charged particles, that is their action on the magnetically charged states does not include the sign inversion of the magnetic charge. It is easy to see that if the discrete transformations are defined in that way, then the theory is indeed P and T non-invariant. However, the possibility to define P and T symmetries in such a way that they are conserved makes the “non-conserving” definition irrelevant.

Note also that we assume that there are no particles carrying simultaneously both the electric and magnetic charge; in other words, there are no dyons in the theory; in this case the conserving P and T operations do not exist [?].
Now, we need to write these transformations in terms of creation (or annihilation ) operators rather than in terms of local fields. Let us first recall the standard formulas for spinor fields in the Majorana representation (we follow the Bjorken-Drell notation [?]):

\[
\psi(x, t) = \frac{1}{(2\pi)^{3/2}} \int d^3 p \sum_s \sqrt{\frac{m}{E}} a(p, s) u(p, s) \exp(-iEt + i px) \\
+ b^\dagger(p, s) v(p, s) \exp(iEt - i px).
\]

(19)

We are working in the Majorana representation which is connected with the standard one (i.e., with diagonal \(\gamma^0\)) via the following relationships:

\[
\gamma^\mu = U \gamma^\mu_S U^\dagger \quad \text{(20)}
\]

\[
u(p, s) = U u_S(p, s) \quad \text{(21)}
\]

\[
v(p, s) = U v_S(p, s), \quad \text{(22)}
\]

where the subscript \(S\) marks the standard representation and the transition matrix \(U\) is

\[
U = \begin{pmatrix}
I & \sigma_2 \\
\sigma_2 & -I
\end{pmatrix}.
\]

(23)

The spinors \(u_S\) and \(v_S\) are defined according to:

\[
u_S(p, s) = \sqrt{\frac{E + m}{2m}} \begin{pmatrix}
\chi_1(s_0) \\
\sigma_p \frac{E + m}{E + m} \chi_1(s_0)
\end{pmatrix} \quad \text{(24)}
\]

\[
v_S(p, s) = \sqrt{\frac{E + m}{2m}} \begin{pmatrix}
\sigma_p \frac{E + m}{E + m} \chi_2(s_0) \\
\chi_2(s_0)
\end{pmatrix}. \quad \text{(25)}
\]

Here, \(s_0\) is the spatial part of the spin 4-vector \(s_\mu\) taken in the rest system of the 4-vector \(p_\mu\); the 2-column spinors \(\chi_1(s_0)\) and \(\chi_2(s_0)\) correspond to the spin parallel and antiparallel to the direction \(s_0\):

\[
\sigma s_0 \chi_1(s_0) = \chi_1(s_0) \quad \text{(26)}
\]

\[
\sigma s_0 \chi_2(s_0) = -\chi_2(s_0). \quad \text{(27)}
\]
Thus, explicitly, we have in the Majorana representation:

\[
\gamma^0 = \begin{pmatrix}
0 & \sigma_2 \\
\sigma_2 & 0
\end{pmatrix}, \quad \gamma^1 = \begin{pmatrix}
i\sigma_3 & 0 \\
0 & i\sigma_3
\end{pmatrix}
\]
\[
\gamma^2 = \begin{pmatrix}
0 & -\sigma_2 \\
\sigma_2 & 0
\end{pmatrix}, \quad \gamma^3 = \begin{pmatrix}
-i\sigma_1 & 0 \\
0 & -i\sigma_1
\end{pmatrix}.
\]

(28)

(29)

All the \(\gamma\)-matrices are purely imaginary which is the characteristic of the Majorana representation. Therefore, the spinors \(u(p, s)\) and \(v(p, s)\) are:

\[
u(p, s) = U u_S(p, s) = \sqrt{\frac{E + m}{2m}} \begin{pmatrix}
(1 + \frac{\sigma_2 p}{E + m}) \chi_1(s_0) \\
(\sigma_2 - \frac{\sigma_2 p}{E + m}) \chi_1(s_0)
\end{pmatrix}.
\]

(30)

\[
v(p, s) = U v_S(p, s) = \sqrt{\frac{E + m}{2m}} \begin{pmatrix}
(\sigma_2 + \frac{\sigma_2 p}{E + m}) \chi_2(s_0) \\
(-1 + \frac{\sigma_2 p}{E + m}) \chi_2(s_0)
\end{pmatrix}.
\]

(31)

The relative phase of the spinors \(u\) and \(v\) is chosen in such a way that the following equality holds:

\[
u^*(p, s) = v(p, s).
\]

(32)

Now, the action of the parity operator on the magnetically charged field \(\psi_g\) reads:

\[
P\psi_g(x, t)P^{-1} = \frac{1}{(2\pi)^{3/2}} \int d^3p \sum_s \sqrt{\frac{E}{|E|}} \left[ P a_g(p, s) P^{-1} u(p, s) \exp(-iEt + ipx) + P b_g(p, s) P^{-1} v(p, s) \exp(iEt - ipx) \right].
\]

(33)

On the other hand, using the rule (??) we can write:

\[
P\psi_g(x, t)P^{-1} = \frac{1}{(2\pi)^{3/2}} \int d^3p \sum_s a_g^\dagger(p, s) \gamma^0 u^*(p, s) \exp(iEt + ipx) + b_g(p, s) \gamma^0 v^*(p, s) \exp(-iEt - ipx).
\]

(34)

Now, using Eq. (??) and (??) one can show that

\[
\gamma^0 u^*(p, s) = -v(-p, s), \quad \gamma^0 v^*(p, s) = u(-p, s).
\]

(35)
Therefore, we obtain the following law of transformation of the creation and annihilation operators of a magnetically charged fermion:

$$P a_g(p, s) P^{-1} = b_g(-p, s), \quad P b_g^\dagger(p, s) P^{-1} = -a_g^\dagger(-p, s).$$

(36)

For a magnetically uncharged fermion $\psi$, the transformation law is:

$$P a(p, s) P^{-1} = a(-p, s), \quad P b(p, s) P^{-1} = -b(-p, s).$$

(37)

In a similar fashion we can derive the laws of C transformation of a magnetically charged fermion:

$$C a_g(p, s) C^{-1} = b_g(p, s), \quad C b_g(p, s) C^{-1} = a_g(p, s).$$

(38)

For a fermion without magnetic charge, the C transformation has the same form. In a similar way one can obtain the formulas for the T reversal but we will not need to use them in the present paper. Hence, we see a clear difference between the behavior of the states with the electric charge and the magnetically charged states. The parity and time inversion acting on the electrically charged states do not change the electric charge of these states, that is under P transformation the electron is carried into an electron with opposite momentum and, likewise, positron is transformed into positron state with the opposite momentum. Similarly, under time inversion the electron state is transformed into the electron state with opposite momentum and spin; the positron is turned into the positron with opposite momentum and spin. So, the P and T transformation do not change the electric charge at all. On the contrary, for magnetically charged particles the situation is exactly opposite: the P and T reflections necessarily include the change of sign of the magnetic charge. For instance, P transformation acting on the magnetic monopole takes it into antimonopole with the opposite momentum; likewise, under P parity the antimonopole is transformed into monopole with the opposite momentum. The same is true for T reversal: the T transformation changes the monopole into antimonopole with opposite momentum and spin; the antimonopole is changed into monopole with inverse momentum and spin.
V. DISCRETE SYMMETRIES AND THE MONOPOLE STRING

So far we have completely ignored the existence of a string (that is, the infinitely thin line of infinitely strong magnetic field) attached to the magnetic monopole. Note that it is possible to formulate quantum mechanics of the magnetic charge in such a way as to avoid introduction of the string [?]. However, at the level of quantum field theory all known formulations do introduce the string under different guises [?]. It is therefore of obvious importance to know how the discrete symmetries act on the string, if at all.

We would like to stress that our considerations in this section are of heuristic rather than of rigourous character. We try to pinpoint those aspects of the problem that would be common to all specific theories of quantum electromagnetodynamics, rather than making a theory-by-theory analysis.

We should, however, make one very important distinction from the beginning. Namely, we have to distinguish between two types of strings: first, the semi-infinite string and, second, the string that is infinite in both directions. For brevity, we shall call them “short strings” and “long strings”, respectively. Let us consider the short string first. Denote by \( n \) the unit vector in the direction of the string and by \( H \) the magnetic field of the string. Under charge conjugation (which, by definition, includes both electric and magnetic charge reversal) we have (see Fig.3a):

\[
H \rightarrow -H \quad g \rightarrow -g \quad n \rightarrow n.
\]

(39)

Next, the parity transformation \( P \) acts as follows (Fig.3b):

\[
H(x) \rightarrow H(-x) \quad g \rightarrow -g \quad n \rightarrow -n.
\]

(40)

Performing the time reversal \( T \), we obtain (Fig.3a):

\[
H(t) \rightarrow -H(-t) \quad g \rightarrow -g \quad n \rightarrow n.
\]

(41)

Under CP parity the transformation law is (Fig.3c):

\[
H(t) \rightarrow -H(-t) \quad g \rightarrow -g \quad n \rightarrow n.
\]
\[ H(x) \rightarrow -H(-x) \quad g \rightarrow g \quad n \rightarrow -n. \] (42)

Finally, under CPT we have (Fig.3b):

\[ H(x, t) \rightarrow H(-x, -t) \quad g \rightarrow -g \quad n \rightarrow -n. \] (43)

For the case of the long string (i.e., infinite in both directions) the discrete transformations look as follows (Fig.4):

\[ C : \quad H \rightarrow -H \quad g \rightarrow -g \] (44)

\[ P : \quad H(x) \rightarrow -H(-x) \quad g \rightarrow -g \] (45)

\[ T : \quad H(t) \rightarrow -H(-t) \quad g \rightarrow -g \] (46)

\[ CP : \quad H(x) \rightarrow -H(-x) \quad g \rightarrow g. \] (47)

\[ CPT : \quad H(x, t) \rightarrow H(-x, -t) \quad g \rightarrow -g. \] (48)

Thus we are led to the following conjecture about the behavior with respect to discrete symmetries of the quantum field theories describing magnetic monopoles with short (i.e. semi-infinite) strings. These theories either conserve CPT, P and CP symmetries or they violate all of them, CPT, P and CP, simultaneously. Thus, according to this conjecture it would be hard to conceive a CPT invariant magnetic monopole theory that would violate parity. Another interesting question is whether one can construct an example of CPT non-invariant theory along these lines. Of course, that would not contradict the famous CPT theorem because one of the requirements for this theorem to be true is the condition of locality whereas the monopole string appears to be a non-local object.
VI. DISCRETE SYMMETRIES AND PHYSICAL PROCESSES

Now we are in a position to apply the discrete symmetries to consideration of specific physical processes in order to establish whether any selection rules can be obtained or not. Since the monopoles are expected to be relativistic, let us use the helicity basis for their consideration. In this basis the pair of monopole-antimonopole is described by a wave function $\psi_{JM\lambda_1\lambda_2}$ where $J$ is the total angular momentum of the pair, $M$ is the projection of $J$ and $\lambda_1$ and $\lambda_2$ are the helicities of the monopole and antimonopole. The action of discrete symmetries is given by:

$$P\psi_{JM\lambda_1\lambda_2} = \psi_{JM-\lambda_2-\lambda_1},$$

$$C\psi_{JM\lambda_1\lambda_2} = (-1)^J\psi_{JM\lambda_2\lambda_1}. \quad (50)$$

Using these rules, we can construct the wave function that has the photon quantum numbers, i.e. $J = 1$, $P = -1$ and $C = -1$:

$$\psi_{1M} = \frac{1}{\sqrt{2}}(\psi_{1M\frac{1}{2}\frac{1}{2}} - \psi_{1M-\frac{1}{2}-\frac{1}{2}}). \quad (51)$$

Thus we see that in order to couple to the photon, the monopole and antimonopole should have the same helicities. To further understand the physical meaning of this condition, let us consider the non-relativistic limit, in which the monopole-antimonopole pair is described by the wave function $\psi_{JLSM}$ where $L$ and $S$ are the total orbital momentum and spin, respectively. The connection between the wave functions $\psi_{JLSM}$ and $\psi_{JM\lambda_1\lambda_2}$ is given by [?]:

$$\psi_{JLSM} = \sum_{\lambda_1\lambda_2} \psi_{JM\lambda_1\lambda_2} \langle JM\lambda_1\lambda_2|JLSM\rangle, \quad (52)$$

where the coefficients are expressed through the $3j$ symbols as follows:

$$\langle JM\lambda_1\lambda_2|JLSM\rangle = (-i)^L(-1)^S(2L+1)(2S+1)\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & S \\ \lambda_1 & -\lambda_2 & -\Lambda \end{pmatrix} \begin{pmatrix} L & S & J \\ 0 & \Lambda & -\Lambda \end{pmatrix}. \quad (53)$$
It can be shown that the wave function Eq. (??) corresponds to the state $S = 0, L = 1$ in the non-relativistic limit, i.e.:

$$\psi_{110M} = \frac{1}{\sqrt{2}}(\psi_{1M \frac{1}{2} \frac{1}{2}} - \psi_{1M \frac{1}{2} - \frac{1}{2}}),$$  \hspace{1cm} (54)$$

$$\Lambda = \lambda_1 - \lambda_2$$  \hspace{1cm} (55)

In other words, the $J^{PC} = 1^{--}$ of the monopole-antimonopole pair corresponds to the $^1P_1$ state in the non-relativistic limit. This should be contrasted with the case of the standard fermion-antifermion pair (such as positronium or quarkonium) for which the $1^{--}$ state is $^3S_1$ (or $^3D_1$).

Now, let us consider spin 0 monopoles for we do not have any evidence concerning the possible value of the monopole spin. From the similar considerations as the above, it can be shown that the spinless monopole-antimonopole system has the following quantum numbers:

$$P = 1, \quad C = (-1)^J,$$  \hspace{1cm} (56)

where $J$ is the total angular momentum of the system. Thus, the spinless monopole-antimonopole production through the one-photon $e^+e^-$ annihilation is absolutely forbidden. Next, it follows from Eq. (??) that in the state with the total angular momentum $J = 1$ the monopole-antimonopole pair has always $CP = -1$. Therefore, CP symmetry absolutely forbids the $1^{--}$ and $1^{++}$ states of the monopole-antimonopole system. Note that this conclusion holds true even if P and C parities do not conserve separately, but CP does. This means that the the decay of Z-boson into spin 0 monopole-antimonopole pair would be absolutely forbidden in a CP invariant theory.

Thus we have shown that C and P invariance imposes exact selection rules on the monopole-antimonopole state produced through the one-photon channel of $e^+e^-$ annihilation.

It remains to be investigated whether these selection rules can help us to understand why the monopoles have not been observed experimentally.
Recently the contribution of virtual monopoles to various physical processes has been examined in several papers. One of them was the contribution of virtual monopole-antimonopole pairs to the anomalous magnetic moment of the electron [?] (see Fig. 5). Another process is the monopole loop contribution to the decay of Z boson into 3 photons [?]. Single-photon $e^+e^- \rightarrow g^+g^-$ amplitude has been a key input in calculating the contribution of virtual $g^+g^-$ pairs to these processes. Applying our conclusions to these cases can lead to significant modifications of the results obtained in previous works [?].

VII. CONCLUSION

We have examined several issues related to the processes of Dirac monopole-antimonopole production in high-energy collisions such as $e^+e^-$ annihilation. Perturbative calculations for such processes are known to be inherently ambiguous due to the arbitrariness of direction of the monopole string; this requires use of some prescription to obtain physical results. We argue that different prescriptions lead to drastically different physical results which suggests that at present we do not have an entirely satisfactory procedure for the elimination of string arbitrariness (this problem is quite separate from the problems caused by the large coupling constant). We then analyze the consequences of discrete symmetries (P and C) for the monopole production processes and for the monopole-antimonopole states. The P and C selection rules for the monopole-antimonopole states turn out to be different from those for the ordinary fermion-antifermion or boson-antiboson systems. In particular, the spin 1/2 monopole and antimonopole should have the same helicities if they are produced through one-photon annihilation of an electron and positron. In the case of spinless monopoles CP symmetry absolutely forbids the monopole-antimonopole production through the one-photon annihilation of an electron and positron. Single-photon $e^+e^- \rightarrow g^+g^-$ amplitude has been a key input in calculating the contribution of virtual $g^+g^-$ pairs to various physical processes such as the decay $Z \rightarrow 3\gamma$ and the anomalous magnetic moment of the electron. Applying our conclusions to these cases can lead to significant modifications of the results obtained
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FIGURES

FIG. 1. Feynman rules for the photon-monopole interaction

FIG. 2. Electron-positron annihilation into monopole-antimonopole

FIG. 3. Action of the discrete symmetries on the semi-infinite string

FIG. 4. Action of the discrete symmetries on the infinite string

FIG. 5. Contribution of virtual monopole-antimonopole pair to the anomalous magnetic moment of the electron
Fig. 3
Fig. 4

\[ g \quad \text{C, P, T, CPT} \quad \bar{g} \]

\[ H \quad H \quad H \quad H \]

\[ g \quad \text{CP} \quad g \]

\[ H \quad H \quad H \quad H \]
Fig. 5