Perturbative renormalization parameters for heavy quarks

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We study the heavy quark mass dependence of the perturbative renormalization factors for the heavy-light currents involving Wilson, Clover, and NRQCD heavy quarks. We find that the one-loop $Z$-factor for the axial-vector current depends significantly on the heavy quark mass commonly for all actions we study, while that for the vector current has smaller dependence.

1. Introduction

Formulation of the heavy quark on the lattice, such as NRQCD\textsuperscript{[1]} or the Fermilab approach with the Wilson-like relativistic quarks\textsuperscript{[2]}, requires perturbative matching with the continuum theory. Since the quark mass in the lattice unit cannot be neglected for these theories, the perturbative calculation is much more involved and have not developed as that for the massless case. We report the current status of our effort to calculate the one-loop coefficients of the perturbative $Z$-factor $Z_V$ and $Z_A$. It is particularly important to calculate the mass dependence of the axial-current $Z$-factor $Z_A$, which is required for the calculation of the $B$ meson decay constant $f_B$, since the one-loop coefficient in the static limit is known to be very large\textsuperscript{[3]}.

The combinations of the heavy and light quark actions we investigate in this report are listed in Table 1. The Wilson and Clover heavy quarks are used with the Fermilab group's non-relativistic interpretation\textsuperscript{[2]}, with which the relativistic actions can be treated in parallel with NRQCD for heavy quarks, since the both methods are based on the small spatial momentum expansion, or equivalently the $\Lambda_{QCD}/M$ expansion. And in particular the both methods have the identical static limit.

<table>
<thead>
<tr>
<th>Heavy quark</th>
<th>Light quark</th>
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<tbody>
<tr>
<td>NRQCD</td>
<td>Wilson</td>
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<td>Wilson</td>
<td>Wilson</td>
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<td>Clover</td>
<td>Clover</td>
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The earlier works for the lattice perturbative calculation for NRQCD can be found in Refs.\textsuperscript{[4,5]}. Advanced studies using NRQCD-Clover actions are also reported by J. Shigemitsu and A. Ali-Khan\textsuperscript{[6]}.

2. Operator Matching

Let us present the operator matching problem taking the axial-vector current as an example. The matrix element of the time component of the axial-vector current $A_0$ for the free heavy and light quark external fields having momenta $p$ and $k$ have the following form:

\[
\langle k | J_{N/W/C} | p \rangle = \bar{u}_l(k) \left( C^{(0)}_1 + \alpha_s C^{(1)}_1 \right) \gamma_5 \gamma_4 + a \left( C^{(0)}_2 + \alpha_s C^{(1)}_2 \right) \gamma_5 \gamma_4 \frac{i \gamma \cdot p}{1} + a \alpha_s C^{(1)}_3 \gamma \cdot k \gamma_5 \gamma_4 + \text{higher orders} \right| h
\]

where $u_l$ is a light quark 4-component spinor, $h$ is a heavy quark 2-component spinor, and the coeffi-
coefficients $C_1^{(i)}, C_2^{(i)}, \ldots$ are functions of the bare quark mass $aM_0$. The first term gives the leading contribution surviving in the static limit, and the second and third terms contribute to the $\Lambda_{QCD}/m_Q$ and $a\Lambda_{QCD}$ corrections.

We have to match the continuum ($\overline{\text{MS}}$) and the lattice theories to give the same matrix elements. The matching of the tree level coefficients $C_i^{(0)}$ and $C_2^{(0)}$ is almost trivial, and we have calculated in this study the one-loop correction $C_1^{(1)}$, which gives the multiplicative $Z$-factor $Z_A$, for all combinations of the heavy and light quark actions. The remaining coefficients $C_2^{(1)}$ and $C_3^{(1)}$ must be calculated to achieve higher accuracy.

3. NRQCD-Wilson

We use two types of the NRQCD action including terms through $O(\Lambda_{QCD}/M)$ for the heavy quark. The actions are given by

$$S = \sum_{t,x} Q(t, x) \left[ Q(t, x) - \left(1 - \frac{aH_0}{2n}\right)^n \right] \times \left(1 - \frac{a\delta H}{2}\right) U_4^{-1} \left(1 - \frac{a\delta H}{2}\right) \times \left(1 - \frac{aH_0}{2n}\right)^n Q(t-1, x), \text{ type } A, \quad (2)$$

and

$$S = \sum_{t,x} \left[ Q(t+1, x) \left(1 - \frac{aH_0}{2n}\right)^{-n} U_4^{-1} \left(1 - \frac{aH_0}{2n}\right)^n Q(t, x) - Q(t, x)(1 - a\delta H)Q(t, x) \right], \text{ type } B, \quad (3)$$

where $Q$ is the effective 2-component spinor field. $H_0 = -\Delta^{(2)}/[2M_0]$ and $\delta H = -g\sigma \cdot B/[2M_0]$ are defined as usual. The original 4-component spinor field $\psi$ is related via Foldy-Wouthuysen-Tani transformation through $O(\Lambda_{QCD}/M)$:

$$\psi = \left(1 - \frac{\gamma \cdot \Delta}{2M_0}\right) \left[ \begin{array}{c} Q \\ 0 \end{array} \right] \equiv R_h. \quad (4)$$

Then the lattice current operators at tree level can be written as

$$J_n = \bar{q}_i \Gamma h = \frac{1}{2M_0} \bar{q}_i \Gamma (\gamma \cdot \Delta) h$$

![Figure 1. One-loop coefficients for the vector and axial-vector current renormalization factors for the type A action. $\circ$: time component of the axial vector current; $\square$: time component of the vector current; $\diamond$: spatial component of the vector current. Open symbols are obtained with the leading term $J_n^{(0)}$ only and filled symbols are with $J_n = J_n^{(0)} + J_n^{(1)}$.](image)

We calculate the multiplicative part of current renormalization factors

$$Z = 1 + g^2(C^{\text{cont}} - C^{\text{latt}}), \quad (6)$$

where $C^{\text{cont}}$ and $C^{\text{latt}}$ are the one-loop corrections for the continuum and lattice (axial-)vector currents. Figure 1 shows the $1/M_0$ dependence of $C^{\text{cont}} - C^{\text{latt}} - \log(aM_0)/4\pi^2$. We have applied the tadpole improvement using the average plaquette for the mean link variable. The axial-vector current in the continuum ($\overline{\text{MS}}$ scheme) is defined with the totally anti-commuting $\gamma_5$.

We observe that the time-component of the axial-vector current and the spatial component of the vector current have a large $1/M_0$ dependence, and in the static limit we reproduce the value $-0.1346$ for the axial-vector current[3]. This large dependence has an impact on the calculation of $f_B$. For a typical value of the lattice spacing the heavy quark mass becomes $aM_0 = 1.8 - 2.5$, and the difference of $Z_A$ from the static value could be large as $\sim 10\%$. The mass dependence of the time component of the vector current is rather mild, on the other hand. A similar behavior is observed for the type $B$ action.
4. Wilson/Clover

The renormalization factors for the Wilson-Wilson and Clover-Clover heavy-light current are calculated in a similar manner as the NRQCD-Wilson case. The lattice current operator is written at tree level as

\[
J_{W/C} = \bar{q}_l \Gamma q_h + d_1 \bar{q}_l \Gamma (\gamma \cdot \Delta) q_h
\]

\[
= J_{W/C}^{(0)} + J_{W/C}^{(1)}, \quad (7)
\]

where \(d_1 = am_0/[2(2 + am_0)(1 + am_0)]\) with \(am_0\) the bare heavy quark mass appearing in the action. In the non-relativistic interpretation approach the heavy quark kinetic mass \(am_2\) is given by \(am_2 = e^{am_1} \sinh am_1/(1 + \sinh am_1)\) and \(am_1 = \log (1 + am_0)\). The coefficient \(d_1\) is small (~0.1 or smaller) around the b-quark mass region and we neglect in this study.

The one-loop coefficient for the time component of the vector and axial-vector currents is shown in Figure 2 for Wilson-Wilson and Clover-Clover as well as NRQCD-Wilson. We apply the tadpole improvement by using the critical hopping parameter to define the mean link variable for the relativistic actions. Overall tendencies of the mass dependence are similar for all actions, except in the lighter heavy quark mass region, where the results for the relativistic actions tend to have smaller mass dependence while the NRQCD-Wilson result still has large slope. The mass dependences for the vector current are weak compared with that for the axial-vector current.

5. Summary

We have investigated the heavy quark mass dependence of multiplicative part of the renormalization factors for the heavy-light currents systematically using NRQCD, Wilson, and Clover actions. We found the large heavy quark mass dependence for \(Z_A\)

The calculation of the heavy-light decay constant is presented in Ref.[7] for NRQCD and in Ref.[8] for the relativistic actions.

This work is supported in part by Grants-in-Aid of the Ministry of Education under the contract numbers 08044089 and 09740226. H.M. is supported by the JSPS for Young Scientists for a research fellowship.

REFERENCES

5. J. Shigemitsu, hep-lat/9705017.
6. J. Shigemitsu, A. Ali Khan, these proceedings.
7. N. Yamada, S. Hashimoto, K.-I. Ishikawa, H. Matsufuru and T. Onogi, these proceedings.
8. JLQCD Collaboration (S. Aoki et al.), presented by S. Hashimoto, in these proceedings.