Effects of Merging Histories on Angular Momentum Distribution of Dark Matter Haloes

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ABSTRACT
Effects of merging histories of protoobjects on angular momentum distribution of present dark matter haloes are analysed. In a analysis of the angular momentum distributions by analytical methods (e.g., Heavens & Peacock 1989, Catelan & Theuns 1996), they have assumed that initially haloes are homogeneous ellipsoids and that the growth of the angular momentum halts at the maximum expansion time of the haloes. However, more realistically, the maximum expansion time of a halo cannot be determined uniquely because each protoobject of the halo has each maximum expansion time in the hierarchical clustering scenario. So merging histories of haloes may affect the angular momentum of the haloes. Using the merger tree model proposed by Rodrigues & Thomas (1996), which includes the information of spatial correlations of density fluctuations owing to a grid-based realisation of density fluctuations, we investigate effects of the merging histories on the angular momentum distribution of dark matter haloes. We find that the merger effect does not strongly affect the final angular momentum distribution. Moreover, we investigate contribution of the orbital angular momentum to the total angular momentum when two or more pre-existing haloes merge each other. We find that the contribution is important especially in the case that merger occurs many times.

Key words: galaxies: formation – large-scale structure of Universe

1 INTRODUCTION

Galaxy formation is one of the most important problems in astrophysics. However, it includes many physical processes such as evolution of cosmological fluctuations, star formation, heating processes of gases from supernovae, dynamical and chemical evolution of gases, etc. Hence it is difficult to attack the problem of formation and evolution of galaxies in a way that connects all of the above complicated physical processes.

One of the important key to understanding the galaxy formation is the angular momentum of dark matter haloes. For example, there is a strong correlation between the angular momentum and morphology of galaxies. Angular momentum of ellipticals is much smaller than that of spirals. So we expect to understand the origin of the morphological distinction from analyses of angular momentum. In the standard picture, it is considered that dark matter dominates in our universe and that galaxies and clusters of galaxies have formed by the gravitational growth of small initial density fluctuations. The dark matter has collapsed and virialised by self-gravitational instability into objects which are called 'dark matter haloes' or 'dark haloes'. The larger haloes are generally considered to have formed hierarchically by clustering of smaller haloes (so called 'hierarchical clustering'). During this process, each dark halo obtains angular momentum by tidal forces (Hoyle 1949; Peebles 1969; White 1984; Barnes & Efstathiou 1988).

Peebles (1969) concluded that the growth of angular momentum of initially spherical volume is proportional to $t^{5/3}$ ($t$ is the cosmic time) during the linear regime in an Einstein-de Sitter universe. Peebles considered a spherical volume as a protoobject, so the analysis is based on a second-order perturbation of initial gravitational field because the second-order term is the lowest one in this case. On the other hand, Doroshkevich (1970) and White (1984) found that angular momentum of an object evolves proportionally to the cosmic time $t$ during the linear regime, because they considered initially homogeneous density ellipsoid as an object. In
this case, the effect from first-order term becomes the lowest one. This prediction has been confirmed by the N-body simulations by White (1984) and Barnes & Efstathiou (1987). It should be noted that Peebles and White did not consider whether the matter in the region under consideration in fact would grow into the collapsed objects or not. This criterion is considered by coupled with peak formalism of Gaussian random fields (Peacock & Heavens 1985; Bardeen et al. 1986, hereafter BEKS).


It should be noted that in almost all of these works connected with derivation of angular momentum distributions, they assumed that a object is initially homogeneous density ellipsoid and that the acquisition of angular momentum halts at maximum expansion time of the object. The maximum expansion time is usually estimated from the averaged density contrast of the object by assuming the spherically symmetric collapse. In the spherical collapse, the averaged linear density contrast of the object reaches to 1.05 at the maximum expansion time (e.g., Peebles 1993). However, in the hierarchical clustering, smaller objects collapse firstly. Then their objects merge each other into the present halo. In some cases, the dark matter accretes gradually or two or more haloes merge later than the maximum expansion time estimated by using the spherical collapse model of the homogeneous object. Hence we believe that in the merging history of protoobjects we should take into account the halting time of each protoobject, the contribution of the orbital angular momentum of merging protoobject, and the angular momentum of the accreting matter.

The merging history models have been used for semi-analytical method of galaxy formation (Cole & Kaiser 1988; Kauffmann & White 1993; Rodrigues & Thomas 1996; Rodighera et al. 1997). Cole & Kaiser developed the Block model. This model is very simple and easy to use, while taking into account only one-point distribution function of density fluctuations. So spatial correlations of density field cannot be analysed. Kauffmann & White extended the Press-Schechter formalism and constructed the merging histories by coupled the extended formalism with Monte Carlo method. This extension is based on Bower (1991). However this formalism also includes only the information of one-point distribution function. On the other hand, Yano, Nagashima & Gouda (1996) claimed that the spatial correlation is important when calculating mass functions of dark haloes, by using Jedamzik formalism (Jedamzik 1995). The mass function derived analytically by using their formula, which includes the information of two-point correlation function explicitly, is changed significantly in comparison with Press-Schechter mass function. So we believe that merging history should also include spatial information. Rodrigues & Thomas constructed the merging history model (we call ‘the merging cell model’ for convenience) which includes the information of the spatial correlations. In the merging cell model, the random Gaussian density fluctuation field is realised on spatial grids as in the construction of initial conditions of N-body simulations, so it is expected that this model naturally includes information of the spatial correlation. Then, by finding the region of the collapsed cells or blocks with each mass scale whose linear density contrast \( \delta_c = 1.69 \) (\( \delta_c \) is a critical density contrast for collapse, see, e.g., Peebles 1993), we can construct a merger tree. We expect that this model is more realistic and useful for galaxy formation although spherical collapse of each block is assumed. This merging cell model is analysed by comparing the mass functions given by the merging cell model and those of Yano, Nagashima & Gouda (Nagashima & Gouda 1997). The former ones are in agreement with those of Yano, Nagashima & Gouda (1996).

In this paper, using the merging cell model, we calculate not only density contrasts but also velocity fields in order to obtain angular momentum of dark haloes. According to a picture of previous analytical works, initial angular momentum of each halo is calculated by investigating the velocity field firstly, then the angular momentum is evolved proportionally to the cosmic time. Since the overdensity of the halo is known, we can estimate the maximum expansion time of the halo. By using this time, we obtain the angular momentum of the halo. When merger between haloes occurs, we investigate the spin of each halo, the orbital angular momentum of each halo around a center-of-mass of the new common halo, and the angular momentum of accreting matter into the new halo. It should be noted that the block model and the extension of the PS formalism cannot treat the angular momentum because these models do not have the information about velocity field.

In order to compare with previous works, we also calculate the angular momentum of homogeneous density haloes by smoothing density contrasts in a region of each halo at present epoch. In this way we investigate effects of merging histories by showing angular momentum distributions of dark haloes.

An advantage of our method is that it is easy to understand the processes of the acquisition of angular momentum qualitatively. Of course, we must perform N-body simulations in order to understand quantitatively (Efstathiou & Jones 1979; Barnes & Efstathiou 1987; Warren et al. 1992). It is, however, hard to distinguish the effect from the each process, as the contribution from orbital angular momentum in N-body simulations. Moreover, we do not spend a time for calculations so much in our semi-analytical method, so we can investigate many model parameters in a short time. We believe that semi-analytical methods are complementary to N-body simulations.

In Section 2, previous analytical works, especially the growth rate of the angular momentum, are reviewed briefly. In Section 3, the Merging Cell model is reviewed briefly. In Section 4, the method of calculating the angular momentum in the Merging Cell model is shown. In Section 5, we show angular momentum distributions. We show the merger effects and also the role of the orbital angular momentum of
merging objects in their common halo. Section 6 is devoted to conclusions and discussions.

2 TIME DEPENDENCE OF ANGULAR MOMENTUM

In this section, we review the evolution of the angular momentum according to White’s method. The notations are almost the same as CT.

The angular momentum $L$ of the protoobject is

$$L(t) = \int_V \rho(r)(r - r_0) \times (v - v_0) \, dr,$$

where $\rho$ is the density, $r$ is the Eulerian coordinate, $v$ is the velocity, and subscript $0$ denotes the center-of-mass of the volume $V$. This is transformed to the Lagrangian description,

$$L = \int \eta_i a^2(q_i - q_0 + D\nabla(\phi - \phi_0)) \times D\nabla(\phi - \phi_0) \, dq,$$

$$= \int \eta_i a^2\partial_t(q_i - q_0) \cdot \nabla(\phi - \phi_0) \, dq,$$  

where $\eta_i = \bar{\rho} a^3$ (the comoving mean density of the universe), $a$ is the cosmic scale factor, $q_i$ is the Lagrangian coordinate, $\phi$ is proportional to the gravitational potential in the Einstein-de Sitter universe, $D$ is the linear growth factor $(D = a$ in the Einstein-de Sitter universe), and $\Gamma$ denotes the Lagrangian volume corresponding to $V$ in the Eulerian space. The $\Gamma$ is defined as an ellipsoidal region surrounded with an isodensity surface with $\delta = 0$, which is estimated from the peak shape parameters in a smoothed density fluctuation field in CT. The dot represents time derivative, $\partial_t$. In the above transformation, we use the Zel’dovich approximation (Zel’dovich 1970)

$$\mathbf{r}(t, \mathbf{q}) = a(t)(\mathbf{q} + D(t)\nabla\phi(\mathbf{q})).$$

Thus we find the growth rate of the angular momentum as

$$L \propto a^2 D \propto t$$

in the Einstein-de Sitter universe.

Note that the determinations of the potential $\phi$ and the Lagrangian volume $\Gamma$ depend on the smoothing scale of density fluctuation field. In CT, the scale is fixed.

3 MERGING CELL MODEL

We briefly review the merging cell model (hereafter MCM) according to the procedure and the notations shown in Rodrigues & Thomas (1996) (see also Nagashima & Gouda 1997).

First, the random Gaussian density field is realised in a periodic cubical box of side $L$. In the random Gaussian distribution, the Fourier mode of density contrast $\delta = (\rho - \bar{\rho})/\bar{\rho}$; $\rho$ is density and $\bar{\rho}$ is the mean density of the universe obeys the following probability for its amplitude and phase (BBKS),

$$p(\delta_k, \phi_k) \, d\delta_k \, d\phi_k = \frac{2|\delta_k|}{P(k)} \exp \left( -\frac{|\delta_k|^2}{P(k)} \right) \, d|\delta_k| \, d\phi_k / 2\pi,$$

where $\phi_k$ is the random phase of $\delta_k$, $\delta_k = |\delta_k| \exp(i\phi_k)$ and $P(k)$ is the power spectrum $\langle |\delta_k|^2 \rangle$, where the angle brackets mean the ensemble average of the universe. Then, the density contrast at each grid (‘cell’) is given by the Fourier transform,

$$\delta(x) = \frac{V}{(2\pi)^3} \int_0^{k_c} \delta_k e^{ik \cdot x} \, d^3k,$$

where $k_c$ is a cut-off wavenumber, which is corresponding to the cell size.

Next, averaging the density fluctuations within cubical blocks of side 2, 4, $L_{\text{box}}$; the fluctuations of the various smoothing levels are constructed. At each smoothing level, displacing the smoothing grids by half a blocklength in each direction of each axis, eight sets of overlapping grids are constructed in order that a density peak will be approximately centred within one of them.

Then, the density fluctuations within blocks and cells are combined into a single list and ordered in decreasing density. The fluctuations are investigated from the top of the list. It is decided by the following rules whether each block or cell can collapse. Note terminology that halo is a block or cell which has already collapsed, and an investigating region is a block or cell whose linear density contrast is just equal to $\delta_i$ at the reference time.

(a) If an investigating region includes no haloes, the investigating region (block or cell) can collapse and can be identified as a new halo.

(b) When an investigating region includes a part of a halo, if the overlapping region has at least half the minimum of the masses of the halo and the investigating region, then the investigating region can collapse. This is the criterion of collapse of the investigating region and merging for the overlapped haloes. We call this criterion the overlapping criterion in this paper.

(c) If the investigating region has half or more of its mass contained in exactly one pre-existing halo and the overlapping criterion (b) is satisfied for any overlapped haloes then they can be merged together as part of the new structure. This is the criterion for linking of haloes.

These criteria are those chosen by Rodrigues & Thomas. Moreover, we introduce the overlapping parameters $x$ and $y$, according to Nagashima & Gouda (1997). The $x$ means the ratio of the mass of the overlapping region to the lesser of the masses of the halo and the investigating region. Then, $\text{RT’s criterion}$ corresponds to $x = 1/2$. The parameter $y$ quantifies the linking criterion. Using this parameter $y$, we change the criterion (c) as follows: If the investigating region has $y$ times ($x \leq y \leq 1/2$) or more of its mass contained in exactly one pre-existing halo and the overlapping criterion (b) is satisfied for any overlapped haloes then they can be merged together.

In the case of $x = y = 1/2$, it is shown that the mass function given by the merging cell model is in agreement with those of Yano, Nagashima and Gouda (1996) well by Nagashima and Gouda (1997).
4 METHOD OF CALCULATIONS OF ANGULAR MOMENTUM

In this section, we mention on the method of calculations of the angular momentum of each halo.

From the Fourier component $\delta_k$ obtained by the Monte Carlo method using eq.(5) and the Poisson equation
\[
\nabla^2 \phi = -\frac{\delta}{D_0},
\]
we can estimate the velocity $v$ and potential $\phi$ at each cell,
\[
\phi(q) = \frac{V}{(2\pi)^3 D_0} \int \frac{\delta_k}{k^3} e^{i k \cdot q} \, dk,
\]
\[
v(q) = \frac{i}{(2\pi)^3 D_0} \int \frac{k \delta_k}{k^3} e^{i k \cdot q} \, dk,
\]
where $a_0$ and $D_0$ are the scale factor and growing mode at present epoch.

When a halo collapses, we divide the angular momentum into three components, that is, (1) spin angular momentum of pre-existing halos $L_{spin}$, (2) orbital angular momentum of pre-existing halos $L_{orb}$, and (3) angular momentum of accreting matter into the new halo, $L_{acc}$. In order to calculate the above quantities, we need the mass of the pre-existing halos, and the position and the velocity of the center-of-mass of both the pre-existing halos and the new common halo. Moreover we need the density contrast of the new common halo in order to estimate the maximum expansion time. We estimate the relation between the linear density contrast at the present epoch ($a = 1$) and the maximum expansion time $t_M (= a_M^{3/2})$ using the spherical collapse model,
\[
\delta = 1.05 \frac{a}{a_M}.
\]
However, we use the growth rate of angular momentum in the nonspherical model, i.e., eq.(4). Thus these components are expressed as
\[
L_{spin} = \sum_i L_i,
\]
\[
L_{orb} = \sum_i M_i (q_i - q_{C M}) \times (v_i - v_{C M}) \left( \frac{1.05}{\delta} \right)^3
\]
\[
L_{acc} = \sum_i M_{cell} \sum_j (q_j - q_{C M}) \times (v_j - v_{C M}) \left( \frac{1.05}{\delta} \right)^3
\]
where subscripts $i$ and $j$ stand for pre-existing halos and cells of accreting matter respectively, subscripts $C M$ denotes the center-of-mass, $M_{cell}$ is a mass of one cell, and $M_i$ is a mass of $i$-th pre-existing halo. We normalize $M_{cell} = 1$. Finally, we obtain the total angular momentum $L$ of the new collapsed halo by summing the above quantities,
\[
L = L_{spin} + L_{orb} + L_{acc}.
\]

When considering in the homogeneous case as in the previous analytical works, firstly we calculate the regions of collapsed halos following the Section 3. Then we average the density contrasts in each halo. This averaged density contrast is used for estimation of maximum expansion time. Finally, using only eq.(13), we obtain the angular momentum of each halo.

5 RESULT

In this section we show distributions of the angular momentum $|L|$ and the mass-weighted angular momentum $|L|/M^{5/3}$, according to HP and CT. We assume that linear density fluctuations obey a random Gaussian distribution with a power-law power spectrum $P(k) \propto k^p$, where $n = 0$ and $-2$. We consider only the Einstein-de Sitter background universe ($\Omega = 1$, $\Lambda = 0$) in this paper. The calculating box size is $L_{box} = 128$. The normalization of the power spectrum is given that the variance of density contrasts is unity at a block with eight cells. The overlapping parameter $x$ is set to $1/2$ and $1/8$.

According to CT, a final angular momentum $|L_f|$ is approximated as follows:
\[
|L_f| \sim a_M^3 D_M \nabla^2 \phi M R^2 = a_M^3 \frac{D_M}{D_M} M R^2 \times \frac{D_M}{D_M} \bar{p}^{3/2} M^{5/3}
\]
\[
\times \frac{D_M}{D_M} (\frac{a_M}{a_M})^{-1/3} M^{5/3},
\]
where $M$ and $R$ are the mass and radius of the object. It should be noted that in CT it is assumed that $\phi$ is adequately represented in the first three terms of the Taylor expansion about the center of the volume, $q_0$, so we obtain the second-derivative term of the same order as $\nabla^2 \phi(q_0)$. In the Einstein-de Sitter universe, the growing mode $D$ equal to the scale factor $a$, so we obtain
\[
|L_f| \propto M^{5/3} a_M^{1/3}.
\]
Thus the value $|L|/M^{5/3}$ depends only on the maximum expansion time $t_M$. It should also be noted that $t_M$ depends on $M$.

In the following, we consider three methods in estimating angular momentum. One is the MRG, in which is given by eq.(14). Second is the ORB, in which we remove the component of orbital angular momentum, $L_{orb}$. Third is the HOM (homogeneous estimation). In the HOM, firstly estimating a region occupied by each halo, and averaging density contrasts in the region, then we estimate the maximum expansion time from the averaged density contrast. So we assume that each cell of a halo has the same maximum expansion time, that is, the halo is homogeneous in the HOM case. This procedure is almost the same as that in the previous works.

In all cases, we consider only halos with mass larger than nine cells, owing to the numerical resolution. Moreover, we consider only halos with the averaged density contrast larger than $1.69$ in the HOM case because each block must satisfy the criterion of the spherical collapse.

5.1 Mass dependence of angular momentum

First of all, we show the mean angular momentum against the mass of the halos in the case of $x = 1/2$ (Fig.1) and $x = 1/8$ (Fig.2). The horizontal axis denotes the mass, that is, the number of cells of each halo. The vertical axis denotes the angular momentum averaged over each mass bin. The solid lines corresponds to the MRG case. The dotted lines and the dashed lines correspond to the ORB case and the HOM case, respectively. The short thick solid line denotes $L \propto M^{5/3}$. The minimum mass in the figures is corresponding to the halo with eight cells ($\log 8 \sim 9.1$), the values of
Effects of Merging Histories on Angular Momentum Distribution

the mean angular momentum in the minimum mass are unphysically small owing to the numerical resolution. In the HOM case, there are no data in large masses because the averaged density contrasts in large haloes are smaller than 1.69.

In all cases, the angular momentum \( L \) is proportional to \( M^{7/3} \), because \( L \propto M^{7/3} \delta^{1/3} \) and moreover the distribution of \( \delta \) is not so broad.

In the case of \( x = 1/2 \) (Fig.1), the difference among the lines is only slight, but we find the tendencies that the angular momentum in the MRG case is a little smaller than in the HOM case, and that the angular momentum becomes smaller slightly when the orbital angular momentum component is removed. We find that the orbital angular momentum does not play important role in the case of \( x = 1/2 \). In the case of \( x = 1/2 \), haloes grow almost only through accretion events; large blocks with \( 8^3 \) or more cells should collapse in order to merge two or more pre-existing haloes. However it does not occur that such large blocks would collapse, including two or more haloes and satisfying the criterion (c) in Section 3. Thus the merger events between two or more pre-existing haloes occur rarely. This is the reason that the orbital angular momentum do not play important role in the case of \( x = 1/2 \).

On the other hand, in the case of \( x = 1/8 \) we can see that the difference between the MRG case and the ORB case is large, about one order of magnitude of \( L \) at larger mass scales, while the difference between the MRG case and the HOM case is only slightly (Fig.2). In this case, the merger events occur easily. Since the orbital angular momentum can grow until relatively later time, the role of the orbital angular momentum becomes large.

In both cases, we find that the behavior of the HOM case is almost the same as that of the MRG case, while the importance of the orbital angular momentum in the total angular momentum depends on the overlapping parameter \( x \). These properties are almost independent of the spectral index \( n \). It seems to be strange that the MRG case coincides with the HOM case independent of the value of \( x \). First of all, we will investigate this problem by considering the distribution in detail in the next subsection.

5.2 Angular momentum distributions

In order to investigate the distribution in detail, we show the comparisons of the distributions of angular momentum \( L \) in Fig.3a (\( x = 1/2 \)) and in Fig.4a (\( x = 1/8 \)), and mass-weighted angular momentum \( L/M^{7/3} \) in Fig.3b (\( x = 1/2 \)) and in Fig.4b (\( x = 1/8 \)). Each figure includes the MRG case and the HOM case. The solid lines denote the distributions of the MRG cases, and the dashed lines the HOM cases. The thick lines denote the spectral index \( n = 0 \), and the thin lines \( n = -2 \).

In the case of \( x = 1/2 \), we can see that the distributions of the HOM case are almost the same as those of the MRG case. The degrees of the differences between them are similar to those of the mean values (Fig.1). These results suggest
that the final distribution of angular momentum does not change so much when considering the merger effect.

In the case of $x=1/8$, it seems that the distributions of $L$ of the MRG case become wide especially in the case of $n=-2$ (Fig.4a). It should be noted that haloes with averaged density contrast smaller than 1.69 are removed in the HOM case, so the differences between the HOM and the MRG cases are large apparently at larger angular momentum. In Fig.4b, we do not need to take care of this removal effect in the HOM case, because statistically $L$ is nearly proportional to $M^{2/3}$ (the thick solid line in Fig.2). The distribution of the HOM case is also almost the same as that of the MRG case also in the case of $x=1/8$. We find that the merger effect does not affect the final angular momentum distribution, independent of $x$.

Anyway, the difference between the MRG case and the HOM case is very small. We consider its reason in Section 5.3.

5.3 Distributions of maximum expansion time

In this subsection, firstly we investigate the distribution of the product of mass and maximum expansion time of each halo, because absolute value of angular momentum depends strongly on the maximum expansion time. This quantity is represented as follows:

$$M\langle t_{m,i}\rangle \equiv \sum_i t_{m,i},$$

$$\langle t_{m,i}\rangle /M^{2/3} \equiv \frac{\sum_i t_{m,i}}{M^{2/3}},$$

where $t_{m,i}$ is the first maximum expansion time of the cell labelled by $i$. The sum is carried out in each halo’s region. We distinguish the MRG case from the HOM case by superscripts ‘MRG’ and ‘HOM’, e.g., $\langle t_{m,i}^{\text{MRG}}\rangle$ and $\langle t_{m,i}^{\text{HOM}}\rangle$, because values of $t_{m,i}$ are different in the case of the MRG and the HOM. In the HOM case $t_{m,i}^{\text{HOM}}$ of a cell is estimated from the density contrast $\delta_i$ of the cell. On the other hand, in the MRG case $t_{m,i}^{\text{MRG}}$ is estimated from the density contrast of the block to which the cell belongs.

We show the distributions of $M\langle t_{m}\rangle$ in Fig.5a ($x=1/2$) and in Fig.6a ($x=1/8$), and $\langle t_{m}\rangle /M^{2/3}$ in Fig.5b ($x=1/2$) and in Fig.6b ($x=1/8$), in the MRG and the HOM cases. The solid lines denote the distributions of the MRG case, and the dashed lines the HOM case. The thick lines and the thin lines denote the spectral index $n=0$ and $n=-2$, respectively.

In the case of $x=1/2$, the differences between the MRG case and the HOM case are about 0.3 order of magnitude at larger side in Fig.5a and at smaller side in Fig.5b. They are large relative to the distribution of the angular momentum (Fig.3). This is because there are many cells which halt the growth of the angular momentum at the earlier time than in the HOM case especially for the large haloes. The results in the case of $x=1/8$ are the same as in the case of $x=1/2$.

If we considered only this effect of merging, the final
angular momentum in the MRG were much different from that in the HOM.

We should consider the effect of the direction of the angular momentum. If angular momentum components with a direction grow in a halo, the total angular momentum of the halo can become large in spite of the averaged maximum expansion time \( t_{\max}^{\text{HOM}} \) is small, and the final angular momentum has the special direction. So, we investigate the maximum expansion time weighted by the direction which will be defined below.

Defining the time-independent vector part of angular momentum of a cell labelled by \( i \) as follows:

\[
L_i^{\text{vec}} = (\mathbf{q}_i - \mathbf{q}_{\text{CM}}) \times (\mathbf{v}_i - \mathbf{v}_{\text{CM}}).
\]

(19)

So the absolute values of angular momentum of the MRG case and the HOM case are

\[
|L^{\text{MRG}}_i| = \left| \sum_{k} L^{\text{vec}}_k |t_{\max}^{\text{MRG}}| \right|
\]

(20)

\[
|L^{\text{HOM}}_i| = \left| \sum_{k} L^{\text{vec}}_k \frac{t_{\max}^{\text{HOM}}}{M} \right| = \left| \sum_{k} L^{\text{vec}}_k (t_{\max}^{\text{HOM}}) \right|
\]

(21)

respectively. Considering the MRG case, defining \( \theta_f \) as the angle of the direction of the final angular momentum, we can transform eqs. (20) and (21) into

\[
|L^{\text{MRG}}_i| = \sum_{k} |L^{\text{vec}}_k| \cos(\theta_i - \theta_f)t_{\max}^{\text{MRG}}
\]

(22)

\[
|L^{\text{HOM}}_i| = \sum_{k} |L^{\text{vec}}_k| \cos(\theta_i - \theta_f)t_{\max}^{\text{HOM}}
\]

(23)

So we investigate the direction-weighted maximum expansion time instead of \( t_{\max} \)

\[
\langle t_{\max} \rangle_{\cos \theta} \equiv \frac{\sum_i \cos(\theta_i - \theta_f) t_{\max}^{\text{MRG}}}{\sum_i \cos(\theta_i - \theta_f)}
\]

(24)

Then

\[
|L^{\text{MRG}}_i| \approx \sum_{i} |L^{\text{vec}}_i| \cos(\theta_i - \theta_f) \langle t_{\max} \rangle_{\cos \theta}
\]

(25)

Here we used the assumption that the absolute value of \( L^{\text{vec}}_i \) is statistically independent of the direction of \( L^{\text{vec}}_i \). Thus if \( \langle t_{\max}^{\text{HOM}} \rangle \approx \langle t_{\max} \rangle_{\cos \theta} \) it is expected that \( |L^{\text{HOM}}| \approx |L^{\text{MRG}}| \).

We show the relation of the direction-weighted maximum expansion time \( \langle t_{\max} \rangle_{\cos \theta} \), no-weighted maximum expansion time \( \langle t_{\max}^{\text{HOM}} \rangle \), and the average maximum expansion time \( \langle t_{\max}^{\text{HOM}} \rangle \) in Fig. 7 (\( x = 1/2 \)) and in Fig. 8 (\( x = 1/8 \)). The left sides denote the spectral index \( n = 0 \) and the right sides \( n = -2 \). We show the comparisons of \( \langle t_{\max}^{\text{HOM}} \rangle \) with \( \langle t_{\max} \rangle_{\cos \theta} \) in the upper panels, and with \( \langle t_{\max}^{\text{MRG}} \rangle \) in the lower panels. The solid lines denote lines whose slope is unity.

The difference of the extents of the distributions of points in horizontal axis between the case of \( n = 0 \) and \( n = -2 \) is due to the difference of the power spectrum of density fluctuation field. The mass dependence of maximum expansion time is

\[
t_M \propto \alpha_M^{3/2} = \left( \frac{1.05}{\delta} \right)^{3/2} \sim M^{3+\alpha}/
\]

(26)
so points in the case of $n = 0$ spread out wider than those in $n = -2$.

In lower panels, there are most of points below the solid lines, as we have already found in Figs. 5 and 6. However, in the upper panels, the points distribute in wider range in vertical axis than those in the lower panels. This result shows that in some merging histories, the growth of the angular momentum with the opposite direction to the final one happens to halt earlier than that with the same direction of the final one, and so some haloes satisfy the condition \( t_{m} \cos \theta \geq \langle t_{m}^{MRG} \rangle \). Then \( t_{m} \cos \theta \sim \langle t_{m}^{HOM} \rangle \) while the dispersion of \( t_{m} \cos \theta \) is large.

This is the reason that the distribution of the angular momentum in the MRG case is almost the same as those in the HOM case. Although \( t_{m}^{MRG} \) is smaller than \( t_{m}^{HOM} \), protobodies having the angular momentum with the same direction to the final one have large maximum expansion time \( t_{m, \text{max}}^{MRG} \), and those with the opposite direction have small maximum expansion time. So the final angular momentum grows almost the same as that of the HOM case. Therefore both distributions become almost the same.

5.4 Orbital angular momentum

In order to investigate the contribution of the orbital angular momentum to the total angular momentum, we show the case of removing the component of the orbital angular momentum (the ORB case) in the case of $x = 1/2$ in Fig. 9. The solid lines show the total angular momentum \( (L_{\text{spin}} + L_{\text{acc}} + L_{\text{orb}}) \) in Fig. 9a and those divided by $M^{1/3}$ in Fig. 9b of the MRG cases, and the dashed lines show those without the orbital angular momentum component, that is, \( L_{\text{spin}} + L_{\text{acc}} \) in Fig. 9a and those divided by $M^{1/3}$ in Fig. 9b. The thick lines denote the spectral index $n = 0$, and the thin lines $n = -2$. In the case of $x = 1/2$, reflecting the small differences in Fig. 1, the differences between the MRG case and the ORB case are very small. As already mentioned in Section 5.1, in the case of $x = 1/2$ haloes grow almost only through accretion events, and the merger events between two or more pre-existing haloes occur rarely. Thus the contribution of the orbital angular momentum is small.

In Fig. 10, we show the same figures as Fig. 9 but for the case of $x = 1/8$. In the upper panel, in contrast to the Fig. 9a, the differences at higher angular momentum are larger than those at lower one. In the case of $x = 1/8$, two or more pre-existing haloes can merge each other many times. We find that the number of times of the merger event is nearly proportional to $0.01M^{1.25}$ in $n = 0$ and $0.02M$ in $n = -2$ for larger haloes than $\sim 100$ cells. Then the haloes with large mass generally experience many merging. Moreover the haloes with large mass have the large angular momentum. Hence the contribution of the orbital angular momentum is greater for the larger angular momentum.
Effects of Merging Histories on Angular Momentum Distribution

6 CONCLUSIONS & DISCUSSIONS

In this paper we analysed the acquisition and distribution of angular momentum of protoobjects with inhomogeneous density distribution by using the MCM. The assumptions in our calculations are spherically symmetric collapse of each block, some overlapping conditions, the growth of angular momentum proportional to the cosmic time $t$, and that the halting time of the acquisition of angular momentum is determined by the maximum expansion time.

In almost all of the previous works, it is assumed that protoobjects are homogeneous density ellipsoid, and estimate the maximum expansion time from the smoothed density contrasts. However, if we consider hierarchical clustering scenarios on which most of previous analyses of angular momentum are based, we should consider the merging effect. If we consider the hierarchical clustering, it is natural that each protoobject has different halting time of the acquisition of angular momentum. Hence it is very important to consider merging histories of dark haloes. However, we cannot find any significant differences in distribution of angular momentum in our analyses by using the MCM, which includes the effect of the merging histories of dark matter haloes. We find this reason by dividing the angular momentum into two components, that is, the time-independent vector part and the maximum expansion time (see Section 5.3). The distribution of the maximum expansion time, which is directly influenced by the merging history, change to lower values on average. However angular momentum components with a direction which is the same as that of the final angular momentum grow later than those with the other directions. This is shown by comparing simply averaged maximum expansion time in each halo ($t_{m}^{MRG}$) with direction-weighted maximum expansion time ($\langle t_{m}^{MRG} \rangle_\alpha$). We find that many haloes satisfy $\langle t_{m}^{MRG} \rangle_\alpha \geq \langle t_{m} \rangle$. Thus these two effects are cancelled out each other, then the difference between the MRG case and the HOM case becomes negligible. Note that this conclusion is independent of the overlapping parameter $x$.

Moreover, we show the contributions of orbital angular momentum of pre-existing haloes when mergers occur. This analysis is enabled by using semi-analytical models like the MCM. The difference between the MRG case and the ORB case which does not include the orbital angular momentum component depends on the overlapping parameter $x$ strongly. In the case of $x = 1/2$, in which there are not so many merger events between two or more pre-existing haloes, the contribution of the orbital angular momentum to the total angular momentum is small. On the other hand, in the case of $x = 1/8$, the orbital angular momentum plays an important role. The contribution of the orbital angular momentum is about a order of magnitude larger than other components. Note that it is easy for blocks to collapse in this case $x = 1/8$ and that there are many merger events. We cannot decide the value of the overlapping parameter $x$ without a detailed study on the overlapping condition, so we can only mention our conclusions qualitatively. However, even under this limitation, we find that the orbital angular momentum plays a significant role in the total angular mo-

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**Figure 9.** (a) Angular momentum distributions. The thick lines denote the case of $n = 0$, and the thin lines $n = -2$. The solid lines are the same as Fig.3 (the MRG case), and the dashed lines denote the ORB case (angular momentum without the orbital angular momentum). (b) Mass-weighted angular momentum ($L/M^{5/3}$) distributions. Types of the lines are the same as (a).

**Figure 10.** The same as Fig.6 but for $x = 1/8$. 
momentum if merger event occurs many times. When a gaseous component is considered, the process of merger of galaxies as gaseous and stellar components will involve a release of angular momentum to dark matter and running away gases. So the distribution of the angular momentum of the gaseous component may follow the distributions without the orbital angular momentum. If elliptical galaxies are formed through such a merger process, this result is very suggestive, because the difference between the angular momentum of ellipticals and spirals is about one order of magnitude by observation (Fall 1983 or Fig.9 in CT). That figure resembles Fig.2 in our paper. We need hydro-dynamical simulations in order to decide whether the distribution of angular momentum of galaxies are followed by the distribution including orbital angular momentum or not, since our model considers only dark matter component.

As mentioned above, we assume spherically symmetric collapse of each block in the MCM. For this assumption, we need the overlapping parameter $x$. So we believe that we need a new model which includes nonspherically collapse in order to discuss the merger effect quantitatively. We will develop such a new model and discuss quantitatively in the near future.

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