Light Hadron Spectroscopy∗
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Recent developments in calculations of the light hadron spectrum are reviewed. Particular emphasis is placed on discussion of to what extent the quenched spectrum agrees with experiment. Recent progress, both for quenched and full QCD, in reducing scaling violation with the use of improved actions is presented.

1. Introduction

Deriving the light hadron spectrum from the first principles of QCD has been a major subject of lattice QCD simulations[1]. A precise determination of the known hadron spectrum would lead us to a fundamental verification of QCD. We should also clarify the nature of observed hadrons, provide predictions for hadrons not in the quark model, and give informations for quantities of phenomenological importance.

In order to achieve these goals, understanding and control of various systematic errors are required. One of major sources of systematic errors is that of a finite lattice spacing. Recent progress in reducing this systematic error has been made in two ways. For the quenched QCD spectrum, development of computer power has enabled to push simulations toward smaller lattice spacings on physically larger lattices with higher statistics than the previous attempts. As a result we are now in a status to discuss the problem of how well quenched QCD describes the experimental spectrum. Another progress in reducing scaling violation is brought with the use of improved quark actions. Tests of improvement, previously made mainly in quenched QCD, have been extended this year to full QCD.

Finite size effects and chiral extrapolations have been studied extensively in the past. Several studies to investigate these systematic errors were also reported at the Symposium.

In this review we attempt to describe the present status of spectroscopic studies. Progress in quenched QCD spectrum is summarized in sec. 2, emphasizing results in the continuum limit. Discussions on several issues in spectroscopic studies follow in sec. 3, which include study of finite size effects, chiral extrapolations, and quenching error in meson decay constants. After discussions on improvement of quark actions in sec. 4, attempts toward a realistic calculation in full QCD are presented in sec. 5. Sec. 6 is devoted to results for masses of glueballs and exotics. Our conclusions are given in sec. 7.

2. Progress in Quenched QCD Spectrum

2.1. major simulations

Recent quenched simulations made with the plaquette gauge action are compiled in Table 1. See sec. 4 for those with improved gauge actions.

Deriving precise quenched results in the continuum limit is a first step toward understanding the light hadron spectrum. The GF11 collaboration[14] carried out the first systematic effort to achieve this goal with the Wilson quark action using three lattices with \( a^{-1} = 1.4 - 2.8 \) GeV and the spatial size \( L = 2.3 \) fm. This year the CP-PACS collaboration reported further effort in this direction[4]. They made high statistics simulations on four lattices with \( a^{-1} = 2.0 - 4.2 \) GeV and \( L = 3 \) fm. Hadron masses are calculated for five quark masses corresponding to \( m_\pi/m_\rho = 0.75, 0.7, 0.6, 0.5 \) and 0.4, the last point being closer to the chiral limit than ever attempted for the Wilson action. They reported continuum values of hadron masses with a statistical error of 0.5 % for mesons and 1–3 % for baryons.

Another trend in this year’s simulations is a pursuit of reduction of scaling violation with the
use of the Sheikholeslami-Wohlert[15] or clover action. Efforts in this direction were made by the UKQCD[5–7] and JLQCD[12] collaborations for the tadpole-improved[16] clover action and by the UKqCD, QCDSF[8] and APETOV[11] collaborations for the non-perturbatively $O(a)$-improved[17] clover action (see also Ref.[18] on this subject). These studies have not yet reached the level of simulations with the Wilson action, being restricted to the parameter range $m_\pi/m_\rho > 0.5$, $a^{-1} < 3$ GeV, and $L a < 2.0$ fm.

For the Kogut-Susskind (KS) quark action, the MILC collaboration[19] last year reported a result of nucleon mass in the continuum limit based on simulations on four lattices with $a^{-1} = 0.6 - 2.4$ GeV and $L a \approx 2.7$ fm. Not much progress has been made this year[2,13].

2.2. quenched spectrum in the continuum limit

In Fig. 1 we plot the result for the quenched light hadron spectrum reported by the CP-PACS collaboration as compared to the GF11 result and experiment. The quenched spectrum depends on the choice of hadron masses to set the lattice scale and light quark masses. Results for two choices are shown in Fig. 1, one employing $m_\pi, m_\rho$ and $m_K$ and the other replacing $m_K$ with $m_\phi$. The disagreement of about 5–10% observed for strange hadrons between the two choices represent a manifestation of quenching error.

The GF11 result, albeit not covering the entire spectrum, showed agreement with experiment within the quoted error of 2% for mesons and 4–8% for baryons. Comparing their result with the CP-PACS result obtained with the same input (filled circles), one finds a sizable difference for $K^*, \phi, \Xi^*$ and $\Omega$. In fact the CP-PACS result with significantly reduced errors exhibits a clear systematic deviation from experiment both for mesons and baryons.

2.3. meson spectrum

The CP-PACS result in the continuum shows that the value of $m_{K^*}$ is 3%(6σ) smaller than experiment and $m_\phi$ by 5% (7σ) if $m_K$ is used as input. Alternatively, with $m_\phi$ as input, they find that $m_{K^*}$ agrees with experiment to 0.6%, but $m_K$ is larger by 9%(7σ). This means that a small value of hyperfine splitting, previously observed at finite lattice spacings[14,20], remains in the continuum limit, which is different from the conclusion of the GF11 collaboration after the continuum extrapolation.

The origin of the discrepancy is clearly seen in Fig.2 where the continuum extrapolations of $m_{K^*}$ and $m_\phi$ are plotted. The CP-PACS data
Table 1
Recent spectrum runs in quenched QCD with the standard gauge action. New results since Lattice 96 are marked by double asterisks and those with increased statistics by asterisks. Quark actions are denoted in parentheses by W: Wilson, C: clover, and KS: Kogut-Susskind. Clover coefficients are denoted by 1: tree level, TP: tadpole improved, TP1: one-loop tadpole improved, and NP: non-perturbatively improved.

<table>
<thead>
<tr>
<th>$\beta$ size (fm)</th>
<th>$m_\pi/m_\rho$</th>
<th>#conf.</th>
<th>$m_\pi/m_\rho$</th>
<th>#m</th>
<th>ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>MILC (W)**</td>
<td>5.70 (12 – 24)$^3 \times 48$</td>
<td>1.7–3.4</td>
<td>404-170</td>
<td>0.90-0.50</td>
<td>6 [2,3]</td>
</tr>
<tr>
<td>CP-PACS (W)**</td>
<td>5.90 $3^3 \times 56$</td>
<td>3.21</td>
<td>800</td>
<td>0.75-0.40</td>
<td>5 [4]</td>
</tr>
<tr>
<td>CP-PACS (W)**</td>
<td>6.10 $4^3 \times 70$</td>
<td>3.04</td>
<td>600</td>
<td>0.75-0.40</td>
<td>5 [4]</td>
</tr>
<tr>
<td>CP-PACS (W)**</td>
<td>6.25 $48^3 \times 84$</td>
<td>3.03</td>
<td>420</td>
<td>0.75-0.40</td>
<td>5 [4]</td>
</tr>
<tr>
<td>CP-PACS (W)**</td>
<td>6.47 $64^3 \times 112$</td>
<td>3.03</td>
<td>91</td>
<td>0.75-0.40</td>
<td>5 [4]</td>
</tr>
<tr>
<td>UKQCD (C=TP)</td>
<td>5.70 (12,16)$^3 \times 24$</td>
<td>(2.1,2.8)</td>
<td>(482,145)</td>
<td>0.78,0.65</td>
<td>2 [5,6]</td>
</tr>
<tr>
<td>UKQCD (C=TP)</td>
<td>6.00 $16^3 \times 48$</td>
<td>1.6</td>
<td>499</td>
<td>0.76-0.62</td>
<td>3 [5,7]</td>
</tr>
<tr>
<td>UKQCD (C=TP)*</td>
<td>6.20 $24^3 \times 48$</td>
<td>1.8</td>
<td>218</td>
<td>0.75-0.49</td>
<td>3 [5,7]</td>
</tr>
<tr>
<td>UKQCD (C=NP)**</td>
<td>6.00 (16,32)$^3 \times 48$</td>
<td>(1.7,3.3)</td>
<td>(497,70)</td>
<td>0.77-0.50</td>
<td>3 [7]</td>
</tr>
<tr>
<td>UKQCD (C=NP)**</td>
<td>6.20 $24^3 \times 48$</td>
<td>1.7</td>
<td>251</td>
<td>0.71-0.54</td>
<td>3 [7]</td>
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<tr>
<td>QCDSF (C=1)**</td>
<td>5.70 $16^3 \times 32$</td>
<td>2.4</td>
<td>0.66-0.44</td>
<td>3 [8]</td>
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<tr>
<td>QCDSF (C=NP)**</td>
<td>5.70 $16^3 \times 32$</td>
<td>3.3</td>
<td>0.77-0.56</td>
<td>6 [8]</td>
<td></td>
</tr>
<tr>
<td>QCDSF (W)*</td>
<td>6.00 (16,24)$^3 \times 32$</td>
<td>(1.4,2.0)</td>
<td>(5000,100)</td>
<td>0.93-0.50</td>
<td>4 (3) [8,9]</td>
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<tr>
<td>QCDSF (C=NP)**</td>
<td>6.00 (16,24)$^3 \times 32$</td>
<td>(1.7,2.6)</td>
<td>(1000,200)</td>
<td>0.90-0.41</td>
<td>6 (3) [8,9]</td>
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<tr>
<td>QCDSF (W)**</td>
<td>6.20 $24^3 \times 48$</td>
<td>1.6</td>
<td>O(100)</td>
<td>0.94-0.61</td>
<td>5 [8]</td>
</tr>
<tr>
<td>QCDSF (C=NP)**</td>
<td>6.20 $24^3 \times 48$</td>
<td>1.8</td>
<td>O(300)</td>
<td>0.90-0.59</td>
<td>5 [8]</td>
</tr>
<tr>
<td>QCDSF (C=NP)**</td>
<td>6.20 $32^3 \times 64$</td>
<td>2.4</td>
<td>O(40)</td>
<td>0.55-0.39</td>
<td>3 [10]</td>
</tr>
<tr>
<td>APETOV (W)**</td>
<td>6.20 $24^3 \times 48$</td>
<td>1.7</td>
<td>50</td>
<td>0.98-0.56</td>
<td>7 [11]</td>
</tr>
<tr>
<td>APETOV (C=NP)**</td>
<td>6.20 $24^3 \times 48$</td>
<td>1.9</td>
<td>50</td>
<td>0.98-0.56</td>
<td>7 [11]</td>
</tr>
<tr>
<td>JLQCD (C=TP1)**</td>
<td>5.90 $16^3 \times 40$</td>
<td>2.0</td>
<td>400</td>
<td>0.76-0.56</td>
<td>4 [12]</td>
</tr>
<tr>
<td>JLQCD (C=TP1)**</td>
<td>6.10 $24^3 \times 64$</td>
<td>2.1</td>
<td>200</td>
<td>0.77-0.50</td>
<td>4 [12]</td>
</tr>
<tr>
<td>JLQCD (C=TP1)**</td>
<td>6.30 $32^3 \times 80$</td>
<td>2.2</td>
<td>100</td>
<td>0.81-0.52</td>
<td>4 [12]</td>
</tr>
<tr>
<td>Kim-Ohta (KS)*</td>
<td>6.50 $48^3 \times 64$</td>
<td>2.6</td>
<td>350</td>
<td>0.65-0.28</td>
<td>4 [13]</td>
</tr>
</tbody>
</table>

(filled circles) show very small scaling violation, in contrast to an increase exhibited by the GF11 results. The continuum extrapolation of GF11 strongly depends on the small values of results at $\beta = 5.7$ obtained on a lattice of size $La \approx 2.3$ fm ($L = 16$). Their additional results for a larger lattice with $La \approx 3.4$ fm ($L = 24$), also shown in Fig. 2, are higher by 2–3%, and are more compatible with the CP-PACS results. Whether one can attribute the difference of the GF11 results between $L = 16$ and 24 to finite-size effects is not clear since values of the two groups for smaller lattice spacings are consistent.

In Fig. 3 we plot the meson hyperfine splitting as a function of the pseudo-scalar meson mass squared where $m_K$ is used as input. The CP-PACS data at four values of $\beta$ (filled symbols) scale well and do not reproduce the experimental value of $K^*-K^*$ mass splitting.

In Figs. 2 and 3, the clover results have also been plotted with open symbols. We observe that they lie slightly above the Wilson results. This agrees with the expectation that the clover term should increase the hyperfine splitting compared to that of the Wilson action. However, there is a problematical feature that the difference of results for the two actions increases toward the continuum limit rather than decreasing as $O(a)$. In fact the UKQCD collaboration[7] concluded this year that $m_{K^*}$ linearly extrapolated to the continuum limit is consistent with experiment using either $m_K$ or $m_\phi$ as input.
Figure 3. Meson hyperfine splitting obtained with $m_K$ as input.

We should emphasize that the difference of meson masses for the two actions is tiny (1–2%) and no more than a $3\sigma$ effect at finite $\beta$. Lattice sizes of $La \lesssim 2$ fm employed in the clover studies may be too small to avoid finite-size errors at this level of precision. Statistical errors of the clover results, which are larger by a factor 2–3 compared to those of the Wilson action, also need to be reduced to resolve the discrepancy.

We compile results for the $J$ parameter[21] in Fig. 4. As has been known, results for the Wilson action and its improved ones consistently lie below the experimental value for a wide range of lattice spacing. Results for the KS action[23] also converge to a similar value from above.

A small value of $J$ is equivalent to a small hyperfine splitting if the latter is a linear function of quark mass. This correspondence is satisfied for the Wilson results, while it is apparently not for the clover case. This represents another problem which needs to be understood in the quenched meson spectrum.

2.4. baryon spectrum

In Fig. 5 we plot the continuum extrapolation of representative baryon masses reported by the GF11 and CP-PACS collaborations. The quenched value of nucleon mass has been a long debated issue. Previous high statistics results[27, 28, 20] (see also Ref.[7]) at $\beta \approx 5.7 - 6.2$ obtained by a chiral extrapolation from $m_\pi/m_\rho \gtrsim 0.5$ yielded a value higher than experiment. The GF11 results also shared this feature, and agreement with experiment in the continuum limit was obtained only after a finite-size correction.

The CP-PACS data down to $m_\pi/m_\rho \gtrsim 0.4$ show that the nucleon and $\Lambda$ masses have a negative curvature in terms of $1/K$ toward the chiral limit. The bending significantly lowers the nucleon mass even at finite $\beta$ as shown in Fig. 5, and a linear continuum extrapolation leads to a value 2.3% lower than experiment, albeit consistent within a 3% statistical error. The nucleon mass for the KS action from the MILC collaboration[2, 19] is also consistent with experiment. See Sec. 3.2 for further discussion on the chiral extrapolation.

For $\Delta$ and $\Omega$ masses, the GF11 and CP-PACS results are reasonably consistent at similar lattice spacings. The continuum extrapolation is different, especially for $\Omega$, with the GF11 case strongly affected by the results at $\beta = 5.7$ on an $L = 16$ lattice.

In the continuum limit, the CP-PACS results show a systematic deviation from experiment.
For the octet, the non-strange nucleon mass is consistent with experiment, while strange baryon masses are lower by 5–8% (3–5%) with \( m_K \) \((m_\phi)\) as input. However, the Gell-Mann-Okubo (GMO) relation is well satisfied at a 1% level.

The GMO relation is also well satisfied for the decuplet, where it takes the form of an equal spacing rule, with at most 10% deviations. However, the average spacing is too small by 30% (20%) with \( m_K \) \((m_\phi)\) as input.

Baryon mass splittings were extensively studied at \( \beta = 6.0 \) on a \( 32^3 \times 64 \) lattice in Ref. [20], which reported the validity of the GMO relations and the smallness of the decuplet mass splitting. The CP-PACS data confirm these results and extend them as the property of the quenched baryon spectrum in the continuum.

2.5. quark mass for the Wilson action

The Wilson action explicitly breaks chiral symmetry at finite lattice spacing. One of its manifestations is that quark mass \( m_q^{WI} \) defined by the Ward identity [29–31] does not agree with quark mass \( m_q^P \) defined perturbatively at finite lattice spacings [30,20].

This problem was examined by four groups this year. The CP-PACS collaboration compared the two definitions for the Wilson action, and reported that they linearly extrapolate to a consistent value in the continuum limit [4]. The

JLQCD collaboration employed an extended current and found indications that scaling violation for \( m_q^{WI} \) becomes smaller than that for the local current [32]. The QCDSF collaboration [8] reported that the two definitions give consistent results in the continuum limit also for the non-perturbatively \( O(a) \) improved clover action.

The Ape collaboration [33] reported that \( m_q^{WI} \) are compatible with \( m_q^P \) at each \( \beta \) when renormalization factors determined non-perturbatively are used.

We summarize results for the strange quark mass in Fig. 6. The agreement of \( m_q^{WI} \) with \( m_q^P \) in the continuum limit supports our expectation that chiral symmetry of the Wilson and clover actions is recovered in the continuum limit. The disagreement of the values \( m_s \approx 135 \text{ MeV} \) obtained with \( m_\phi \) as input and \( m_s \approx 110 \text{ MeV} \) found with \( m_K \) as input originates from the small meson hyperfine splitting, and hence represents a quenching uncertainty. Further results on quark masses are reviewed in Ref. [34].
3. Issues in Spectroscopic Studies

3.1. finite size effects in quenched QCD

In quenched QCD finite-size effects of hadron masses are expected to be smaller than in full QCD due to $Z(3)$ symmetry. For the nucleon mass with the KS action, the magnitude has been estimated to be less than 2% at $m_\pi/m_\rho \approx 0.5$ for $La \geq 2$ fm[35,36]. On the other hand, the GF11 result[14] for the Wilson action at $\beta = 5.7$ showed a larger effect of 5% between the sizes $L = 16$ (2.3 fm) to 24 (3.4 fm).

The MILC collaboration carried out extensive runs at $\beta = 5.7$ with the Wilson action for the sizes $L = 12$−24, and we reproduce their results for the nucleon mass[2,3] together with those of GF11 in Fig. 7.

The GF11 result for $L = 16$ significantly depends on the source/sink size, with the value for the size 4 consistent with those for $L = 24$. The MILC results for $L = 16$ do not show a source size dependence. Their values for the sizes $L = 12$ − 24 mutually agree within the statistical error of about 2%, and are also consistent with the GF11 results for $L = 24$.

These comparisons strongly suggest that finite-size effect at $La \approx 2$ fm is already 2% or less also for the Wilson action, rather than 5% estimated by GF11. This implies that finite-size effects are negligible for $La \approx 3$ fm as employed by the CP-PACS collaboration.

3.2. chiral extrapolation of nucleon mass

Last year the MILC collaboration[2,19] emphasized the difficulties in reliable chiral extrapolation for the nucleon mass using their high precision data with the KS action. The results obtained for light quarks down to $m_\pi/m_\rho \approx 0.3 − 0.4$ exhibit a negative curvature, and the mass in the chiral limit is sensitive to the choice of fitting functions.

The CP-PACS data for the nucleon mass for the Wilson action measured down to $m_\pi/m_\rho \approx 0.4$ also show a negative curvature. They tried to fit their data using four fitting functions; a cubic function in quark mass, a form predicted by chiral perturbation theory ($\chi$PT) in full QCD[37] given by $m_N = c_0 + c_1 m_\pi^2 + c_2 m_\pi^3$, and two forms in quenched QCD ($Q\chi$PT)[38–40] given by $m_N = c_0 + c_1 m_\pi + c_2 m_\pi^2$ and $m_N = c_0 - 0.53 m_\pi + c_1 m_\pi^2 + c_2 m_\pi^3$ where in the latter the coefficient of the linear term is fixed to a value estimated from experiment. As shown in Fig. 8(a), the four fitting functions describe data equally well, but deviate significantly toward the chiral limit.
In Fig. 8(b) we show how the choice of chiral extrapolations affects the nucleon mass in the continuum limit. Having precision results down to $m_\pi/m_\rho = 0.4$ at each $\beta$ helped to constrain the uncertainty in the continuum limit almost within the statistical error of 3%.

A major difficulty in exploring the chiral limit in quenched QCD simulations is the presence of exceptional configurations. A method has recently been proposed to avoid this difficulty[41]. It would be very interesting to see if the method allows to obtain reliable results near the chiral limit as close as $m_\pi/m_\rho \approx 0.2$, which would be needed to control the chiral extrapolation at a few % precision level.

The APETOV collaboration[11] studied quark mass dependence of octet baryon masses for the non-perturbatively $O(a)$ improved action for the range of $m_\pi/m_\rho = 0.98 - 0.56$. They found that linearity is better if one includes the $O(m_\rho a)$ improvement term in the definition of quark mass.

3.3. decay constants and quenching error

It has been observed for the Wilson action that $f_K/f_\pi - 1$ in quenched QCD is much smaller than experiment, which is considered to be a quenching error (see Ref. [42] for a recent review). In Fig. 9 we compile recent results for the ratio. Small values in the range 0.1–0.15 are also obtained for the clover and KS actions. A discrepancy of 30–40% with experiment roughly agrees with estimates based on quenched chiral perturbation theory[39].

In Fig. 10 we summarize the status with the determination of the pion decay constant. Continuum values for the Wilson action reported by various groups are consistent with each other, and are slightly smaller than experiment, while the situation with the clover results is very unsatisfactory, suffering from a large discrepancy among groups.

4. Improvement of Quark Actions

Several groups have been testing improved quark actions with improved gauge actions. In this section we discuss quenched results in this category. New simulations since Lattice 96 are listed in Table 2.
Table 2
Tests of improved quark actions with improved gauge actions. Abbreviations for gauge actions in brackets are TILW: tadpole-improved Lüscher-Weisz[46,16,47], TISY: tadpole-improved Symanzik[48,16], SY: Symanzik[48].

<table>
<thead>
<tr>
<th>Action</th>
<th>( \beta_{pl} )</th>
<th>size (fm)</th>
<th>#conf.</th>
<th>( m_\pi/m_\rho )</th>
<th>#m</th>
<th>ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCRI (C=NP)[TILW]</td>
<td>7.75-12</td>
<td>8 × 15</td>
<td>O(1000)</td>
<td>1</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>Alford et al. (D234c,C)[TISY]</td>
<td>1.157</td>
<td>5 × 18</td>
<td>2.0</td>
<td>0.76-0.70</td>
<td>2</td>
<td>50</td>
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<td>8 × 20</td>
<td>2.0</td>
<td>0.76-0.70</td>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>DeGrand Fixed point actions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[51]</td>
</tr>
<tr>
<td>MILC (KS,Naik)[TILW]</td>
<td>7.60</td>
<td>16 × 32</td>
<td>100</td>
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<td>3</td>
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<tr>
<td>MILC (KS,Naik)[TILW]</td>
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<td>16 × 32</td>
<td>200</td>
<td>0.76-0.33</td>
<td>5</td>
<td>3</td>
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<tr>
<td>MILC (KS,Naik)[TILW]</td>
<td>7.90</td>
<td>16 × 32</td>
<td>200</td>
<td>0.80-0.27</td>
<td>6</td>
<td>3</td>
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<tr>
<td>Bielefeld (lat)[SY]</td>
<td>4.1</td>
<td>16 × 30</td>
<td>57</td>
<td>( \approx 0.65 )</td>
<td></td>
<td>52</td>
</tr>
</tbody>
</table>

Figure 11. Comparison of \( m_N/m_\rho \) at \( m_\pi/m_\rho = 0.7 \) for various quark actions. C-ML and D234c-ML employ mean link for the tadpole factor. Lattice spacings are set with the string tension (\( \sqrt{\sigma} = 427 \) MeV) except for results with TISY gauge action which use the charmonium spectrum.

4.1. improvement of the Wilson action

Improvement of the Wilson quark action by adding the clover term has been extensively investigated both with the standard gauge action[5–12] and with improved gauge actions[53,24,26,50]. We plot in Fig. 11 the mass ratio \( m_N/m_\rho \) at \( m_\pi/m_\rho = 0.7 \). We clearly observe that the clover term significantly reduces scaling violation so that the ratio agrees with the phenomenological value[54] within 5% already at \( a \approx 0.4 \) fm.

The D234 action[55] is designed to achieve improvement beyond the clover action. Results[25, 26,50,53] for a class of D234 actions, however, do not show clear improvement for the mass ratio compared with those for the clover action.

Scaling test of hadron masses themselves at a fixed \( m_\pi/m_\rho \) is useful to examine the functional dependence of scaling violation on the lattice spacing. Using the tadpole-improved Lüscher-Weisz (TILW) gauge action for which we expect only small scaling violation, the SCRI group[24] showed last year that mass results for the tadpole-improved clover action are consistent with an \( O(a^2) \) scaling behavior, while Wilson data need both \( O(a) \) and \( O(a^2) \) terms.

In Fig.12 we reproduce their figure for the vector meson mass at \( m_\pi/m_\rho = 0.7 \), adding new results for the Wilson[4](open circles) and clover[7]
(filled circles) actions on the standard plaquette gauge action. The results for the two actions lie on the respective extrapolation curves of the SCRI results, showing a reduction of scaling violation with the clover action also for the plaquette gauge action.

The Cornell group[50] tested improvement using mean value of link in the Landau gauge rather than plaquette for the tadpole factor(right triangles). They reported that the mean link is superior in reducing scaling violation effects over plaquette.

Let us also mention that non-perturbative determinations of the clover coefficient with improved gauge actions have been attempted[49,56]. Spectrum calculations are in progress.

4.2. improvement of the KS action

The MILC collaboration[19] studied the KS and Naik[57] three-link actions using the TILW gauge action, and compared them with those for the KS action on the standard gauge action. They found that $m_N/m_\rho$ is improved by the use of the improved gauge action, but the Naik improvement has a relatively small effect on the mass ratio. Pushing the calculation toward higher $\beta$[3], they found little difference between the Naik and KS actions.

Another direction of improvement tested by the MILC collaboration[58] is the use of fat link, in which one replaces a link variable with a weighted sum of the link and staples. This is expected to improve flavor symmetry, and indeed they found a substantial reduction in the mass difference between the Goldstone and non-Goldstone pions.

The Bielefeld group[52] studied the fat link improvement with the Symanzik gauge action. They also observed improvement of flavor symmetry for this quark action, while $O(p^2)$ and $O(p^4)$ improved actions which include many link paths do not show any significant improvement of flavor symmetry.

5. Toward Full QCD Spectrum

With progress of our understanding of the quenched spectrum, increasingly larger efforts are beginning to be spent in simulations of full QCD.

Figure 13. Edinburgh plot for $N_f=2$ KS quarks reported by MILC[64] together with those from HEMCGC[67], Columbia[66,68], and Kyoto-Tsukuba[69]. In parentheses are lattice sizes.

Here we summarize recent work listed in Table 3.

5.1. progress with the KS action

The MILC collaboration[19] continued their study of the $N_f=2$ KS spectrum for $\beta = 5.3 - 5.6$ employing large lattices of a size $La \geq 2.6$ fm. In Fig. 13 we show their results in the Edinburgh plot together with those of previous studies[66–69].

The ratio $m_N/m_\rho$ decreases toward weak coupling. Taking advantage of improved precision of their results as is clear from Fig. 13, the MILC collaboration attempted a continuum extrapolation of $m_N/m_\rho$ for a fixed value of $m_\pi/m_\rho$. They find $m_N/m_\rho = 1.252(37)$ at the physical point in the continuum limit.

Also of interest is the problem of how the KS spectrum depends on the number of dynamical quark flavors. Columbia group[66] showed that the four flavor hadron spectrum is nearly parity doubled on a $16^3 \times 32$ lattice at $\beta = 5.4$. Chiral symmetry breaking effects are smaller for four flavors than for two or zero flavors.

5.2. progress with the Wilson action

Simulations of full QCD with the Wilson quark action for $N_f=2$ have been pushed forward by the SESAM[59,61] and TχL[60,61] col-
Table 3
Recent spectrum runs in full QCD for $N_f=2$. New results since Lattice 96 are marked by double asterisks and those with increased statistics by asterisks.

<table>
<thead>
<tr>
<th>Collaboration (Action)</th>
<th>$\beta$</th>
<th>size (fm)</th>
<th>traj.</th>
<th>$m_\pi/m_\rho$</th>
<th>#$m$</th>
<th>ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SESAM (W)*</td>
<td>5.6</td>
<td>$16^4 \times 32$</td>
<td>1.4</td>
<td>200 $\times$ 25</td>
<td>0.84-0.7</td>
<td>3 [59,61]</td>
</tr>
<tr>
<td>TχL (W)*</td>
<td>5.6</td>
<td>$24^3 \times 40$</td>
<td>2.0</td>
<td>O(3000)</td>
<td>0.70.55</td>
<td>2 [60,61]</td>
</tr>
<tr>
<td>UKQC (C=1.76)**</td>
<td>5.2</td>
<td>$12^4 \times 24$</td>
<td>50 conf.</td>
<td>0.85-0.75</td>
<td>4 [62]</td>
<td></td>
</tr>
<tr>
<td>CP-PACS (W,C=1,TP)**</td>
<td></td>
<td>(12,16)$^4 \times 32$</td>
<td>study of action improvement</td>
<td>[63]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MILC (KS)</td>
<td>5.30</td>
<td>$12^3 \times 32$</td>
<td>3.7</td>
<td>1000-5000</td>
<td>0.8-0.3</td>
<td>8 [19]</td>
</tr>
<tr>
<td>MILC (KS)*</td>
<td>5.415</td>
<td>$16^3 \times 32$</td>
<td>3.2</td>
<td>1000-2000</td>
<td>0.77-0.44</td>
<td>6 [19,64]</td>
</tr>
<tr>
<td>MILC (KS)*</td>
<td>5.415</td>
<td>$12^3 \times 24$</td>
<td>2.4</td>
<td>2000</td>
<td>0.46</td>
<td>1 [19,64]</td>
</tr>
<tr>
<td>MILC (KS)</td>
<td>5.50</td>
<td>$24^3 \times 64$</td>
<td>3.6</td>
<td>1000-2000</td>
<td>0.69-0.63</td>
<td>2 [19]</td>
</tr>
<tr>
<td>MILC (KS)</td>
<td>5.50</td>
<td>$20^3 \times 48$</td>
<td>3.0</td>
<td>2000</td>
<td>0.56-0.48</td>
<td>2 [19]</td>
</tr>
<tr>
<td>MILC (KS)*</td>
<td>5.60</td>
<td>$24^3 \times 64$</td>
<td>2.6</td>
<td>1500-2000</td>
<td>0.75-0.53</td>
<td>4 [19,64]</td>
</tr>
<tr>
<td>Columbia(KS,$N_f=2$)*</td>
<td>5.70</td>
<td>$16^4 \times 32(40)$</td>
<td>1.5</td>
<td>1400-4900</td>
<td>0.70-0.57</td>
<td>4 [65,66]</td>
</tr>
<tr>
<td>Columbia(KS,$N_f=4$)*</td>
<td>5.40</td>
<td>$16^4 \times 32$</td>
<td>1.5</td>
<td>2700-4500</td>
<td>0.72-0.67</td>
<td>2 [65,66]</td>
</tr>
</tbody>
</table>

Laborations. Simulations were initially made at $\beta = 5.6$ on a $16^4$ spatial lattice ($La \approx 1.4$ fm) for $m_\pi/m_\rho = 0.85 - 0.7$ (SESAM), which have been extended to those on a larger lattice $24^4$ ($La \approx 2.0$ fm) and closer to the chiral limit with $m_\pi/m_\rho \approx 0.7$. The magnitude is comparable to that for the KS action[69,71]. They estimated strange hadron masses, treating the strange quark as a valence quark in the presence of two light dynamical quarks. The $K-K^*$ mass splitting is smaller than experiment by 15%, contrary to the expectation that dynamical sea quark effects alleviate the small hyperfine splitting of quenched QCD. It is possible that dynamical quarks employed is still too heavy to improve the splitting significantly.

SESAM and TχL also studied the static potential and several hadron matrix elements to explore effects of sea quarks. See Ref. [72] for a review.

5.3. full QCD with improved actions
Till last year there were only sporadic attempts toward full QCD simulations of the light hadron spectrum with improved actions[73]. This year the CP-PACS collaboration[63] and the UKQC collaboration[62] presented preliminary results of a systematic attempt in this direction.

The CP-PACS collaboration made a comparative study of improvement at a coarse lattice $a^{-1} \approx 0.9 - 1.5$ GeV employing the plaquette and an RG-improved action[74] for gluons and the Wilson and tadpole-improved clover action for quarks. For one action combination, they also explored the chiral limit down to $m_\rho/m_\rho \approx 0.4$ with simulations on a $16^4 \times 32$ lattice. The UKQC collaboration employed the plaquette action at $\beta = 5.2$ and the clover action with a clover coefficient of 1.76. Simulations were made for four values of sea quark masses and the spectrum is calculated for four values of valence quark masses on each dynamical quark ensemble.

In Fig.14 we compile full QCD results for $m_N/m_\rho$ as a function of $m_\rho$, both calculated at $m_\pi/m_\rho = 0.7$. Results for the Wilson quark action have large scaling violation and approximately lie on a single curve, irrespective of the choice of gauge actions. In contrast the lattice spacing dependence is much weaker for the clover actions, again irrespective of the gauge action,
Figure 14. $m_N/m_\rho$ in full QCD with $N_f = 2$ as a function of $m_\rho$, both calculated at $m_\pi/m_\rho = 0.7$. Abbreviations for gauge actions are P: plaquette and R: RG-improved[74], and for quark actions W: Wilson and C: clover. Data are taken from CP-PACS[63,4], SCRI[75], and SESAM[61].

and the value of the ratio is close to a phenomenological estimate even on a very coarse lattice of $a^{-1} \approx 1.0$ GeV. These results show that a significant improvement of $m_N/m_\rho$ due to the clover term observed for the quenched case also holds in full QCD.

Another interesting question in full QCD is to what extent the lattice scale obtained from the hadron spectrum agrees with that from the static potential. The clover term is important also in this regard. A mismatch of the scale determined from $m_\rho$ in the chiral limit and that with the string tension observed for the Wilson action at $a^{-1} \approx 1.0$ GeV is much reduced by the use of the clover action[63]. The UKQCD collaboration reported that the scale determined from $m_K$, approximately agrees with that from $m_0$ for each value of dynamical quark.

For an effect of improvement of gauge actions, rotational symmetry of the potential is improved to a great extent also in full QCD[63].

The effects of improvement summarized here are parallel to those observed in quenched QCD, and come mainly from valence quarks rather than dynamical sea quarks. Nevertheless, they are important since they show that realistic full QCD simulations are possible without having to reduce the lattice spacing below $a^{-1} \approx 2$ GeV which is needed with the standard action.

6. Other Topics

Calculation of glueball masses in quenched QCD has reached a stage to pinpoint the mass ranges at least for the scalar glueball. The GF11 collaboration[76] reported $m_{0^{++}} = 1710(63)$ MeV as the infinite volume value in the continuum from a reanalysis of their data[77]. This value is consistent or slightly higher than the previous results by other groups[78–80].

The central effort of the GF11 collaboration has been a calculation of the mass of the $s\bar{s}$ scalar meson[76,81], for which they found values below $m_{s\bar{s}} < 1500$ MeV. They conclude that the observed meson $f_J(1710)$ is mainly a scalar glueball, while $f_0(1500)$ is mainly an $s\bar{s}$ quarkonium.

The SESAM collaboration[82] made a glueball mass measurement with their full QCD runs. No clear dynamical quark effects are seen in the glueball masses. Instead, they observed strong finite size effects in the scalar glueball mass, which may be an indication of the presence of mixing between the glueball and the $s\bar{s}$ scalar meson.

Two groups have contributions for spin exotic meson masses. The UKQCD collaboration[83] increased statistics since last year. Calculating masses at one combination of $\beta$ and the quark mass and employing a model to estimate masses at the strange quark, they obtained $m_{1^{--}}(s\bar{s}) = 2000(200)$ MeV. The MILC collaboration[84] made simulations at $\beta = 5.85$ and 6.15. Extrapolation to the strange quark mass was made to obtain $m_{1^{--}}(s\bar{s}) = 2170(80)$ MeV. The two results are consistent within 10%.

7. Conclusions

A number of interesting studies have been made this year, making a step forward toward a precise determination of the light hadron spectrum.

For the quenched spectrum, a systematic deviation from experiment has been uncovered both in the meson and baryon sectors. Quantitative
results have been accumulated with improved actions both for quenched and full QCD, clarifying to what extent improving actions reduce scaling violations in the light hadron spectrum. Quenched clover simulations are moving toward high precision determination of physical quantities exploiting the improved scaling behavior, and similar effort should be pursued with other improved actions.

And finally, attempts toward a realistic simulation in full QCD have begun. In my opinion, there is real hope that such a calculation could be achieved with the current generation of computers through application of improved actions.

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