Neutron star properties with relativistic equations of state

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Abstract

We study the properties of neutron stars adopting relativistic equations of state of neutron star matter, calculated in the framework of the relativistic Brueckner–Hartree–Fock approximation for electrically charge neutral neutron star matter in beta–equilibrium. For higher densities more baryons (hyperons etc.) are included by means of the relativistic Hartree– or Hartree–Fock approximation. The special features of the different approximations and compositions are discussed in detail. Besides standard neutron star properties special emphasis is put on the limiting periods of neutron stars, for which the Kepler criterion and gravitation–reaction instabilities are considered. Furthermore the cooling behaviour of neutron stars is investigated, too. For comparison we give also the outcome for some nonrelativistic equations of state.

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I Introduction

A necessary ingredient for solving the structure equations for (rotating) neutron stars (NS) is the equation of state (EOS) \[ n \]. For NSs the EOS has to cover a wide range of densities ranging from super–nuclear densities (up to 5 to 10 times normal nuclear matter density) in the star’s core down to the density of iron at the star’s surface. At present, neither heavy–ion reactions nor NS data are capable to determine the EOS accurately, and the behaviour of high–density matter is still an open and one of the most challenging problems in modern physics, containing many ingredients from different branches of physics. The theoretical determination of the EOS over such an enormous range has therefore to rely mainly on theoretical arguments and extrapolations for which no direct experimental confirmation exists. The best one can do in such a situation is a step-by-step improvement of the available models for the EOS. Since the central density of a NS is so extreme, both the Fermi momenta and the effective nucleon mass are of the order of 500 MeV, one should prefer a relativistic description \[ b_2 d_9 d_9 \].

Neutron star matter differs from the high density systems produced in heavy ion collisions by two essential features: a) Matter in high energy collisions is still governed by the charge symmetric nuclear force while neutron star matter (NSM) is bound by gravity. Since the repulsive Coulomb force is much stronger than the gravitational attraction, NSM is much more asymmetric than “standard” matter. b) The second essential difference is caused by the weak interaction time scale of \( \sim 10^{-10}\) s, which is small in comparison with the lifetime of the star, but large in comparison with the characteristic time scale of heavy ion reactions. For that reasons “normal” matter is subject to the constraints of isospin symmetry and strangeness conservation, but NSM has to obey the constraints of charge neutrality and generalized beta–equilibrium with no strangeness conservation. It is obvious from these considerations that NSM is an even more theoretical object than nuclear “normal” matter, with a rather complex structure \[ b_2 d_9 d_9 \].

Due to these features one can adopt the following structure of a neutron star: The two outer crusts of the star have crystalline structures, for which rather reliable EOSs exist in the literature. In the uniform density region \( (\rho \gtrsim 10^{14} \text{ g/cm}^3) \) of neutron star matter, one has to deal with a system of interacting baryons (i.e., \( p, n, \Lambda, \Sigma \), possible \( \Delta \)-s, etc.) and/or quarks (\( u, d, s \)
flavours, uniform or not uniform) that are in generalized $\beta$-equilibrium with leptons ($e^-, \mu^-$) \( \{\mu, \mu^-\} \). Furthermore one may encounter meson condensates \( \{\rho, \phi, \pi\} \).

Despite the differences of hot symmetric non-strange matter produced in high energy collisions and cold asymmetric charge neutral and strangeness containing matter of NSs one should combine both systems in a common theory. This is possible to a certain degree in modern field theoretical relativistic approaches, and NSs are unique systems in the sense, that they offer a test bench for the EOS of exotic NSM, which cannot be mimicked in terrestrial laboratories.

The first attempt, based on the Fermi gas model, to incorporate the role of the hyperons in NSs is due to Ambartsumyan and Saakyan \[13\], which was later improved in the nonrelativistic scheme by several authors \[14\]. The first systematic investigation in the relativistic framework was performed by Glendenning, who used the relativistic mean-field approximation with inclusion of hyperons and $\Delta$'s. \[8\]. This model was later extended by using the relativistic Hartree–Fock–approximation (RHF) \[7, 14\] and by improving the Lagrangian in the mean-field approximation (or relativistic Hartree–approximation; RH) \[2, 3, 7, 15\]. One of the most exhaustive investigations in the latter framework was performed by Schaffner and Mishustin, who used modern phenomenological interactions in the nucleon and hyperon sector, respectively. In this last model kaon condensation turned out to be unlikely \[15\].

From a microscopic standpoint such treatments are not satisfactory, since the interaction is adjusted in a phenomenological manner to properties of finite nuclei and the parameters of the Bethe–Weizsäcker mass formula. Other drawbacks are, for instance, the vanishing of the $\pi$–meson contributions in the mean-field approximation, and large $\sigma$–meson self-interactions are needed to reduce the incompressibility $K$. Some of these deficiencies do not occur in the framework of the relativistic Hartree–Fock approximation, which also resembles, in both its mathematical structure and its Lagrangian density rather closely to the microscopic relativistic Brueckner–Hartree–Fock theory, for which one–boson–exchange potentials (OBEP) are adjusted to the two–nucleon data. In order to incorporate a more microscopic description of neutron star matter, we have recently developed a model in which this mat-
ter is described for moderate densities by self-consistent RBHF-calculation
for $\beta$-stable matter ($n, p, e^-, and \mu^-$ only) \cite{9,17}. Due to technical difficulties, RBHF-calculation are presently restricted to densities up to 2–3 times nuclear matter density. Since at higher densities other baryon states become populated and RBHF-calculation with inclusion of such states are presently not feasible, we extrapolated the EOS at higher densities within the RH and/or the RHF approximation. The essential new feature of this scheme in comparison with older treatments was the different adjustment of the parametrization of the RH and RHF Lagrangians. Normally one restricts oneself to the reproduction of the properties of symmetric nuclear matter, but in the new parametrization we used the outcome of RBHF calculations of asymmetric and NS–matter as well. It turned out that the RH approximation is not so flexible in reproducing the properties of NSM (asymmetry, composition etc.) Since the Hartree approximation has been used in the vast majority of the earlier investigations, we will include this approximation in our considerations, too, which has also the advantage of greater transparency. Special attention will be paid to the RHF–approximation, where new features enter into the properties of NSM. In both approximations it is possible to incorporate more baryon states (for more details, see Refs.\cite{14,16,19} and following sections).

In order to achieve a certain degree of selfcontainment we recapitulate also the basic theory for the EOSs, and for the properties of neutron stars, i.e. structure of rotating stars in general relativity, stability of rotations (Kepler criterion, gravitation–radiation instabilities), cooling of neutron stars etc. The contribution is organized as follows. Section II is devoted to the EOS of NSM, where we discuss, after a brief theoretical review the different models of the EOSs. The next section deals with the properties of NSs and the summary is given in Section IV.

II Equation of state

a) General theory:

The EOSs of neutron star matter were determined as described in Refs.\cite{17,18}. For the two outer crusts we used EOSs taken from the literature \cite{15,20}. The general dynamics of the hadron/lepton system in the uniform region of
neutron star matter is governed by an OBE Lagrangian of the following form:

\[
\mathcal{L}(x) = \sum_{B=p,n,\Sigma,\Lambda,\Xi,\Delta^{-},\Sigma^{-},\Lambda^{-},\Xi^{-},\Delta^{+}} \mathcal{L}_B^0(x) \tag{II.1}
\]

\[
+ \sum_{M=\sigma,\omega,\pi,\rho,\delta} \left\{ \mathcal{L}_M^0(x) + \sum_{B=p,n,...,\Delta} \mathcal{L}_{B,M}^0(x) \right\} + \sum_{L=e^{-},\mu^{-}} \mathcal{L}_L(x) .
\]

The forces between the different baryons are mediated by the exchange of different mesons \( M = \sigma, \omega, \ldots \) The leptons \( e^{-} \) and \( \mu^{-} \) are treated as free particles. Furthermore one has to impose the constraints of beta-equilibrium (\( q_B \) and \( \mu_B \) denote the electric charge and chemical potential of the baryon \( B \), respectively):

\[
\mu_B = \mu_n - q_B \mu_e , \quad \mu_\mu = \mu_e . \tag{II.2}
\]

and charge neutrality. For the many-body treatment we employed the relativistic many-body Green’s function scheme. Here one has to solve – for Brueckner-type approximations – a coupled system of equations which consists of the Dyson equation for the Green’s function \( G \), the effective scattering matrix \( T \) in matter, and the equation for the self-energy \( \Sigma \) of a baryon in matter \( [7] \):

\[
(G^0)^{-1}(1,2)\Sigma(1,2)G(2,1') = \delta(1,1') \tag{II.3}
\]

\[
<12|T|1'2'> = <12|V|1'2'-2'1'> + \tag{II.4}
\]

\[
i <12|V|34 > \Lambda(34,56) <56|T|1'2'> ,
\]

with

\[
\Sigma(1,2) = - <14|T|52 > G(5,4) . \tag{II.5}
\]

For the intermediate baryon–baryon propagator \( \Lambda \), we have chosen in the RBHF approximation the Brueckner propagator. \( V \) denotes the OBE potential, which has the following structure

\[
<12|V|1'2'> = \sum_{M=\sigma,\omega,\pi,...} <12|V_M|1'2'> . \tag{II.6}
\]

The RH and RHF approximations are obtained for \( T = V \) and \( T = V - V_{ex} \), respectively. For the OBE potentials we selected the potentials A and B
constructed by Brockmann and Machleidt [23]. In the RBHF–treatment, performed in the full Dirac space, of symmetric and asymmetric nuclear matter both potentials (they differ in the strength of the tensor force, which increases from A to B) give good agreement with the nuclear matter parameters \( E/A, \rho_0, K, J \), and also the volume parameters \( L \) and \( K_{\text{sym}} \) of asymmetric matter are in accordance with the data. An illustrative comparison with other treatments is given in Table I [23].

Up to 2–3 times equilibrium nuclear matter density we used for the EOSs the outcome of an RBHF–calculation for neutron star matter by restriction to \( p, n, e^- \), and \( \mu^- \), only. For higher densities more baryon states \( B \) become populated in \( \beta \)--stable neutron star matter. For high density systems with such a complex composition the RBHF scheme is not feasible. In order to find an extension of the microscopic RBHF–EOS to higher densities we selected both the relativistic Hartree and the relativistic Hartree–Fock approximation, where the parameters in the nucleonic sector (coupling constants etc.) are adjusted to the results of RBHF–calculations for asymmetric and NS matter.

Explicitly, the Lagrangian density for the RHF–approximation, where the forces are mediated by the exchange of \( \sigma-, \omega-, \) and \( \rho-, \) mesons, and \( \sigma \) self–interactions are included, is given by [14]:

\[
\mathcal{L}(x) = \sum_B \bar{\psi}_B(x) \left[ i \gamma^\mu \partial_\mu - m_B + g_\sigma^B \sigma(x) - g_\omega^B \gamma^\mu \omega_\mu(x) - \frac{f_\rho^B}{4m_B} \frac{\sigma^{\mu\nu}}{2} F^\rho_{\mu\nu} \right. \\
- \left. \frac{f_\pi^B \gamma^5 \gamma^\mu \tau_B \cdot \partial_\mu \pi - g_\rho^B \gamma^\mu \tau \cdot \rho_\mu(x) - \frac{f_\rho^B}{4m_B} \frac{\sigma^{\mu\nu}}{2} \tau \cdot F^\rho_{\mu\nu} \right] \psi_B(x) + \frac{1}{2} [\partial_\mu \sigma(x) \partial^\mu \sigma(x) - m_\sigma^2 \sigma^2(x)] + \frac{1}{2} [\partial_\mu \pi(x) \cdot \partial^\mu \pi(x) - m_\pi^2 \pi^2(x)] \\
- \frac{1}{4} F^\rho_{\mu\nu}(x) \cdot F^{\mu\nu,\rho}(x) + \frac{1}{2} m_\rho^2 \rho^\rho(x) \cdot \rho_\mu(x) - \frac{1}{4} F^\omega_{\mu\nu}(x) F^{\mu\nu,\omega}(x) + \frac{1}{2} m_\omega^2 \omega^\mu(x) \omega_\mu(x) \right] \\
- \frac{1}{3} m_N b_N [g_\sigma \sigma(x)]^3 - \frac{1}{4} c_N [g_\omega \sigma(x)]^4 , \quad (\text{II.7})
\]

with

\[
F^\rho_{\mu\nu}(x) \equiv \partial_\mu \omega_\nu(x) - \partial_\nu \omega_\mu(x) , \quad F^\rho_{\mu\nu}(x) \equiv \partial_\mu \rho_\nu(x) - \partial_\nu \rho_\mu(x) . \quad (\text{II.8})
\]

With respect to the saturation properties of infinite nuclear matter (INM) (see Table I) the results are identical to the RBHF–outcome. It seems that, at
present, such a procedure is the only possibility to incorporate more baryons and to establish a connection to microscopic RBHF-calculations with realistic OBE-potentials simultaneously. In this context one has to remark that the standard relativistic approach in almost all cases is based on a pure phenomenological RH–treatment, where the parameters in the nucleonic sector are adjusted to the properties of symmetric nuclear matter, so that our approach has the advantage to contain more microscopic elements (for more details, see Refs. [17], [18]). In the calculations we used an improved parametrization [18], and the parameter sets are given in Table II.

b) Comparison of the different approximations

First one has to test whether the described approximations can reproduce the properties of asymmetric and NS matter in the density range, where the RBHF–treatment is applicable (nucleons and leptons only). In Fig. 1 we compare the EOSs for different asymmetries in the different approximations. The agreement of the RBHF–EOSs with the RHF–EOSs is rather satisfactory for the whole asymmetry range. However, for the RH–EOSs the agreement is worse for larger asymmetries, furthermore the RH–EOS becomes stiffer for higher densities. This behaviour of the RH–approximation can better be inferred from Fig. 2, where the comparison for the pressures for NSM is shown. In Fig. 3 we exhibit, as a further example, the comparison with respect to the baryon/lepton composition. As discussed in Refs. [17], [18] the RHF–approximation with no $\rho$–tensor coupling gives the best agreement with the RBHF–EOS. The RHF–approximation has the additional advantage that the Lagrangian density and the mathematical structure resembles more the RBHF–scheme than the RH–approximation (no $\pi$–meson, exchange contributions etc.). For the presentation we selected the Brockmann–Machleidt potential $B [22]$: for the potential $A$ the situation is completely analogous [23].

c) Neutron star matter

a) General considerations

For the calculation of NS–properties the EOS is needed for a wide range of densities, stretching to several times of nuclear matter saturation density $\rho_0$. If one extrapolates the described scheme to the density domain of
NSM, one faces the following dilemma: It is well known that NS–properties depend strongly on the properties of the EOS near saturation, which is obvious for lighter NSs but also true for heavier stars \(^{27}\). Therefore one should use in this region an EOS, which is either based on microscopic RBHF-calculations \(^{21, 22, 27, 28}\) or phenomenological parametrizations of the RH/RHF-approximation, adjusted to nuclear data \(^{23}\). In both cases one obtains inevitably a rather stiff EOS caused by the low value of the Dirac mass of about \(0.6 \, m_N\) (correct spin–orbit splitting \(^{29}\); the RBHF–scheme gives also reasonable results for finite nuclei \(^{29, 30}\) at saturation \(^{23, 31, 32}\). The resulting meson fields are then rather large and consequently one obtains a sharp drop of the Dirac masses with increasing density. This feature is even amplified by the unavoidable occurrence of negative values for \(c_N(b_N/c_N \sim -1) \quad ^{23, 31, 32}\), which causes a nonmonotonic behaviour of the effective \(\sigma\)-mass

\[
m^{*^2} = m^{2} + g^{2} [b_N m_N < g_\sigma \sigma > + c_N < g_\sigma \sigma >^2],
\]

which increases the attraction beyond \(< g_\sigma \sigma >^0 \equiv -b_N m_N/2c_N\). As long as the composition of NSM is restricted to \(n, p, e^-\), and \(\mu^-\) only, the EOS is sufficiently stiff to reach the necessary central pressure of the star at moderate densities (see Section III.e). However if one includes more baryons in the NSM–composition the EOS becomes considerably softer and higher densities are needed to obtain sufficient central pressure. The scalar fields are therefore in this case rather large and hence negative Dirac masses for the nucleons occur in the calculation \(^{31, 33}\). One has tried to overcome this problem in a phenomenological manner using so-called stabilized \(\sigma\)-functional forms \(^{23, 34}\), for which the dangerous negative curvature of \(m^{*^2}\) is switched off. However for the familiar parametrizations PL–2 and PL–40 negative Dirac masses also still occur in NSM. Schaffner and Mishustin have circumvented this problem by the rather questionable ad hoc assumption of using always absolute values for the Dirac masses, so implicitly changing the stiffness of the EOS \(^{11}\). Another approach by which one gets now along with a reduced attraction, i.e. \(b_N, c_N > 0 (c_N \gg b_N)\) reduces the repulsion by an additional \(\omega\)-self-interaction \(^{35}\). In this manner one obtains softer EOS with smaller \(\sigma\)-fields, where negative Dirac masses do not occur. However the resulting EOS may be too soft and an additional repulsion among the
hyperons seems necessary. Furthermore the asymptotic behaviour is changed from the standard behaviour proportional to $\rho_B^{4/3}$ to that of an ideal gas ($\propto 1/p_B^{4/3}$). Unfortunately the NSM-results for this case were not applied to NS-properties in Ref.\cite{1}. For the sake of comparison we performed therefore some NS-calculations and obtained – as expected – maximal masses around 1.5 $M_\odot$. (For instance, for the parameter set TM1 we obtained 1.561 $M_\odot$ with a central energy density $\epsilon_c = 740 \text{ MeV/fm}^3$.) One might wonder why this problem was not discussed in the other investigations of NSM.

In the vast majority one prefers an EOS which is based on the standard nuclear matter parameters but with a Dirac mass of approximately 0.78 $m_N$\cite{2,3,4,5,6}. Only in this window\cite{7,8} one obtains positive values for $c_N$ (or $b_N \gg |c_N|$) and more moderate meson fields, so that the problem of negative nucleon Dirac masses is avoided. The reasons given for this choice rest on a reproduction of the effective mass ($\sim 0.83 \, m_N$)\cite{9,10,11}, however a closer inspection according to a more elaborate investigation by Celenza and Shakin shows that values of again 0.6 $m_N$ are more appropriate for the Dirac mass\cite{7} (see also Ref.\cite{12}). Furthermore one favours rather low values for the saturation density and higher values for the incompressibility in order to stiffen the EOS (see, for instance, Ref.\cite{13}). For instance, according to the Hugenholtz–vanHove theorem, $g_{\sigma N}$ varies by approximately 40% in the range of 1.3 fm$^{-1} \leq p^0_F \leq 1.42$ fm$^{-1}$\cite{14}.

Common to all these approaches is to impose a certain behaviour of the NSM-EOS in the NSM-domain, which is not known. Controlled is this behaviour by an additional parameter (large Dirac mass; switching off parameter; coupling constant of the vector self–interaction etc.\cite{15,16,17,18,19,20}). In order to overcome this dilemma, namely to keep the connection to the EOS in the vicinity of nuclear saturation and the described unpleasant features of the NSM-EOS with additional baryons one is also forced in our approach to implement a working extrapolation hypothesis controlled by an additional parameter. Also RBHF-calculations in symmetric matter favour higher Dirac masses in this density region\cite{21}. In order to keep the scheme as simple as possible we extrapolate as follows: We maintain the dynamics till the maximum of the effective $\sigma$-mass and extrapolate beyond this point via a Lagrangian of the same structure, but with new self–interaction couplings
and a modified $\sigma$-mass:

$$m_{\sigma}^2 \rightarrow m_{\sigma}'^2 = m_{\sigma}^2 + \frac{g_{\sigma}^2}{4c_N} m_N^2 b_N \left( \frac{c_N}{c_N} - 1 \right), \quad (II.10)$$

$$b_N \rightarrow b_N' = \frac{b_N}{c_N}; \quad c_N \rightarrow c_N' = \alpha. \quad (II.11)$$

For this model the effective $\sigma$-mass and its derivative agree with the microscopic model at the transition point (see Eq. II.9). $\alpha = c_N$ gives the original dynamics; $\alpha = 0$ results in a linear Walecka model, which turns out to be insufficient (negative Dirac masses). With $\alpha > 0$ one can now control the stiffness. One should remark in this context that the main effect of the extrapolated dynamics is a more moderate drop of the Dirac masses beyond four times nuclear matter saturation density in accordance with Ref. [58] (see Fig. 4). With respect to the pressure, the differences against the original dynamics are rather small ($10\%$) in the lower part of the NSM-density domain and become very small in the high-density NSM-region, where the unchanged $\omega$-meson repulsion dominates. We have also tested the pure linear or quadratic extrapolations, i.e. $b_N' = b_N/2, c_N' = 0$ or $b_N' = 0, c_N' = -c_N; m_{\sigma}' = m_{\sigma}$, but their results can be incorporated closely in the scheme selecting special $\alpha$-values [58].

3) Results and discussion

Essentially three main features characterize the properties of NSM: a) the stiffness of the EOS b) the relative coupling constants of hyperons, c) the chosen many-body approximation. All three points are correlated, but nevertheless we will try to separate them to a certain degree in order to extract some insight into the structure of the problem.

The influence of the first point is obvious, since the necessary central pressure for stable stars can be reached earlier for larger stiffnesses. Important is the second issue. The relative hyperon couplings are not well known, since the information from hypernuclei data (for more details, see, Refs. [59, 60]) permits a wide bandwidth (for instance, $0.4 \leq x_{H\sigma} \equiv g_{H\sigma}/g_{N\sigma} \leq 0.8$ in the relativistic mean field approximation). Therefore a number of choices were made in the literature, reaching from universal coupling, which gives a first insight, to ratios motivated by the quark model [61, 62, 63, 64, 65, 66, 67, 68]. A
suitable first choice in the latter case is to use the SU(6) symmetry for the vector couplings i.e.,

\[
\frac{1}{3} g_{\omega N} = \frac{1}{2} g_{\omega \Lambda} = \frac{1}{2} g_{\omega \Xi} = g_{\omega \Xi} \; ; \; g_{\rho N} = g_{\rho \Xi} = g_{\rho \Xi} \; , \; g_{\rho \Lambda} = 0 , \tag{II.12}
\]

and to fix the $\sigma$–coupling according to the potential depth of the hyperon–particle in nuclear matter ($\sim -30$ MeV). According to these uncertainties we will investigate the problem in a more systematic manner by making several choices for the $\sigma$–hyperon coupling and fix the $\omega$–coupling by means of the hyperon potential depth \cite{112}. The selected choices for the EOSs are described in Table III. The hyperon couplings are given in Table IV.

The general trend is that the EOSs become softer with decreasing couplings, because then the conversion of nucleons into hyperons is energetically favourable, due to the smaller repulsive force dominating in the high–density domain of NSM. A special feature of the Hartee approximation are large $\rho$–couplings, which are necessary for the adjustment of the symmetry energy \cite{113}. For that reason the charge–favoured (isospin unfavoured) $\Delta^-$ does not occur (or play a minor role; see discussion of the RHF–approximation) and therefore is usually neglected a priori in this approximation \cite{114,115,116,117,118}. For the $\Sigma^-$ the charge compensation still dominates and so the $\Sigma^-$ occurs rather early. Increase of the $\rho$–coupling, for instance like $g_{\rho \Xi} = 2 g_{\rho N}$, would reverse the onset of $\Sigma^-$ and $\Sigma^+$ \cite{118}. A further and more severe point is the selection of the many–body approximation. For the Hartree approximation we obtain compositions and stellar properties which are familiar to investigations performed earlier in this framework. This approximation is characterized as a high density approximation with a relatively stiff EOS containing no exchange contributions. For the latter reason the coupling constants are larger than in the RHF–scheme. Two examples for the composition (universal coupling and SU(6) scheme) are given in Figs. 5 and 6, from which one can infer the dependence on the hyperon couplings. For the universal coupling the attraction is more favoured, which reduces the Dirac masses. Therefore the onset of the hyperons occurs earlier and also $\Delta^-$'s are possible. This causes a lower pressure at lower densities, however for high densities the stronger repulsion causes then a stiffer EOS (see Fig. 7). For these reasons one should obtain smaller NS–masses for the universal coupling for smaller central densities than for the SU(6)–coupling. For higher densities
the situation is reversed (see Sec. IIIe).

Due to the additional degrees of freedom the RHF–EOS is softer than the corresponding RH–EOS in a large density domain. A special new feature of the RHF–approximation is that the onset of a particular baryon species in NSM is, as in the RH–approximation, solely controlled by the Hartree term. Due to the lower couplings the participation of the other baryons is more enhanced and in general the onset of the hyperons occurs earlier than in the RH–scheme, resulting in a quite different baryon/lepton composition and a softer EOS. A further characteristic difference is caused by the smaller \( \rho \)-coupling constants in the RHF–theory (see Table II). Responsible for this decrease of the coupling constants \( g_\rho \) are – as in the case of realistic OBE–potentials in the RBHF–theory – the exchange contributions, which contribute to the symmetry energy. For that reason the charge–favoured but isospin unfavoured \( \Delta^- \) plays now – as in former nonrelativistic many–body approximations with correlations \( [8] \) – an important role in the composition. One might further wonder that one has not included the \( \omega \)–tensor coupling for the hyperons, since, for instance, according to the quark model \( f_\omega^\Delta / g_\omega^\Delta \) becomes \(-1\) for the \( \Lambda \) hyperon. For that reason we included this term in a test calculation \( [5,6] \) but the impact on the EOS was rather small and can therefore be neglected in comparison with the other uncertainties.

If one would use naively the same hyperon couplings as in the RH–approach one obtains compositions – shown for the SU(6) in Fig.8 – where the \( \Delta \)'s occur rather early (for universal coupling the baryons occur in the order \( \Sigma^- \), \( \Lambda \), \( \Delta^- \) with a high contribution of \( \Delta \)'s at higher density \( [9,5] \)). For that reason the EOSs are rather soft at moderate densities and should cause a weak increase of the NS–mass in the lower part of the central energy–density region resembling in this part the behaviour of NS–masses calculated with EOSs with rather low incompressibilities \( [1,5,12] \) (see Fig.17). The strong abundance of the \( \Delta \)'s in such EOSs is caused, as discussed before, by the early onset of the charge-favoured \( \Delta^- \), which is not so strongly isospin hindered in the RHF–approximation. The onset of hyperons in this model may be overestimated, since it is controlled by their Hartree contribution solely. A special feature of the RHF–theory seems to be that the Fock contributions for the scalar part of the self–energy approximately cancel each other but for the vector part they amount to 50\% of the total value \( [1,8] \). For that reason
the hyperon couplings taken from the RH–approach do not reproduce the
potential depths of the hyperons in nuclear matter in the RHF–scheme (at-
traction too strong) and hence favour the hyperons. If one now corrects this
deficiency by adjusting the hyperon couplings ($x_{\sigma H}$ (RHF) < $x_{\sigma H}$ (RH); see
Table IV), the composition becomes now dominated by the $\Delta$’s (see Fig. 9).
The occurrence of the hyperons ($\Lambda$–hyperon) is rather late even for weak
couplings (for instance, for $x_{\omega H} = 0.5$ at $\rho \sim 0.5$ fm$^{-3}$). The peculiar behaviour
of the pressure disappears now and consequently the NS–masses as function
of the central energy–density give now the standard strong increase for
moderate densities (see Fig. 19, 20). The models treated so far involve the
assumption that the $\Delta$–coupling agrees with the nucleon coupling. But ac-
cording to investigations of ter Haar and Mal lief [13], the $\Delta$–mass does not
change very much from the vacuum case, indicating smaller $\Delta$–couplings,
which would delay the onset of the $\Delta$s. In a recent treatment by R. Rapp et
al. [14], the choice $g_{\Delta N} = g_{\Delta N} = 0.625 g_{NN}$ was recommended, but one could
use as an option also this choice for the attraction only. For the first case
one gets a slight shift of the $\Delta$–threshold towards higher densities (see Fig. 10)
and the EOS becomes much softer due to the lower repulsion. This can be in-
ferred from Fig. 20, where the maximum star mass is approximately 1.5 $M_\odot$.
In the last case one deals with a strong repulsion combined with moderate
masses for the $\Delta$’s. For that reason one expects a minor role to be played by
$\Delta$s. The calculation of the chemical potentials shows that the attraction is
just a little bit to small for the occurrence of the $\Delta$’s and one gets a pattern familiar from the RH–treatment (see Fig. 11). If one reproduces the same
$\Delta$–potential depths as in the RH–scheme, the $\Delta^-$ is still preferred at lower
densities, but hyperons play now a more significant role due to their lower
repulsion and dominate the high density region (see Fig. 12). With respect to
the adjustment of the potential depths of the baryons, this RHF–model cor-
responds exactly to the treatment within the RH–scheme. As in the case of
nucleons, the different assumptions about the behaviour of the $\Delta$–Dirac mass
in matter influence the EOS only weakly. For instance, in the SU(6)–scheme for
the hyperons the pressure increases only slightly by going from RHF 8 via
RHF 9 to RHF 1. More severe are changes of the repulsion of the baryons.
For example, one obtains by increasing the hyperon repulsion by going from
RHF 1 to RHF 4 ($x_{\omega H} = 0.8$) pressure increases of $\sim 150$ MeV/fm$^3$, compared
with $\sim 30$ MeV/fm$^3$ in the case before with constant hyperon couplings [18].

In conclusion we have constructed and discussed EOSs of NSM, which, in contrast to former investigations, are tight to the outcome of microscopic RBHF-calculations. The extrapolation to higher densities, where more baryons participate, was performed in the RH- and RHF-scheme. For the density domain of NSM, where the EOS is (completely) unknown, we were confronted as in all treatments with a complex composition with the necessity to invoke assumptions about the behaviour, especially with respect to the density dependence of the Dirac masses, controlled by an additional parameter. In this context we would like to emphasize again that so far almost all relativistic models have considered the rather special case of a RH–EOS based on a large Dirac mass at saturation ($\sim 79$ $m_N$). With respect to the couplings of the hyperons and $\Delta$'s large uncertainties exist. For the hyperons the relative ratios of the $\sigma$– and $\omega$–couplings were fixed by the potential depths in nuclear matter. (An absolute fixing from hypernuclei data is not possible at present [20].) For the $\Delta$–coupling we invoked several assumptions, which take into account the smaller decrease of the $\Delta$–mass in matter.

The case of the RH–treatment resembles in its basic features to earlier investigations in this framework, which were based on more or less phenomenological treatments. Their distinguishing features are a high hyperon content (high strangeness), stiffer EOSs, and a strong $\Delta$–suppression. In the RHF–scheme the $\Delta$'s play in general due to their smaller $g_\rho$–coupling an important role, and the hyperons are more suppressed. The detailed composition and stiffness of the EOS depends on the Dirac mass of the $\Delta$, for which we selected four different choices.

We are aware of the fact that the described uncertainties open the gates to a realm of options, some of which might be rather unfamiliar in comparison with standard treatments. Of course we could not explore and present all the hypotheses, but we have tried to select illustrative samples, from which the trends can be extracted (for more details, see Ref. [18]). The consequences for NSs will be discussed in the following sections.
III Neutron star properties

a) General considerations

The theoretical description of a neutron star is governed by the following conditions, which can be roughly estimated, for instance, on the basis of the Fermi gas or the hard core model for the EOS: i) General Relativity has to be taken into account for the determination of the gross properties of a star with approximately one solar mass \( M_\odot \) and a radius \( R \) of approximately 10 km, since the relativistic effects (change of the metric etc.) for such objects are of the order

\[
\frac{2M}{Rc^2} \sim 0.3 .
\]

(ii) For the EOSs one can use relativistic treatments within a Minkowski metric since the spacing of baryons in the star \( R = r_0 A^{1/3}; r_0 \sim 0.5 \text{ fm}, A \sim 10^{57} \) is of the order of \( 10^{-19} \). iii) As explained before, due to the high densities one should favour relativistic EOSs for NSM.

Roughly one can cast the EOSs in terms of stiff or soft equations. The “stiff” equations give a maximum mass (Oppenheimer–Volkoff limit \( M_{OV} \)) of about twice the solar mass, and limiting rotational periods larger than 1.5 ms. For “soft” EOSs the estimates are \( M_{OV} = 1.5 M_\odot \) and \( P_{\text{min}} = 0.6 \text{ ms} \). Hence, the detection of sub–millisecond or heavy pulsars could discriminate the (theoretical) EOSs. Furthermore the millisecond pulsars, explained according to the present understanding within the so–called recycling model, are very fast rotating objects (at birth at least 0.5 ms) \[1\]. Consequently it is important to include the (fast) rotation into the theoretical treatment of NSs.

b) Theoretical treatment of rotating and deformed stars

Due to these conditions one is faced with the problem to determine Einstein’s curvature tensor \( G_{\mu\nu} \) for a massive star \( (R_{\mu\nu}, g_{\mu\nu}, \text{ and } R \text{ denote the Ricci tensor, metric tensor, and Ricci scalar, respectively}) \).

\[
G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu} (\epsilon, P(\epsilon)) .
\]

A necessary ingredient for solving (III.2) is the energy–momentum tensor density \( T_{\mu\nu} \), for which knowledge of the (relativistic) EOS, i.e. pressure \( P \) as function of the energy density \( \epsilon \) is necessary.
For a spherically symmetric and static star, the metric has the famous Schwarzschild form \((G = c = 1)\):

\[
    ds^2 = -e^{2\phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) ,
\]

where the metric functions are given by:

\[
    e^{2\Lambda(r)} = (1 - \gamma(r))^{-1} , \quad (\text{III.4})
\]

\[
    e^{2\phi(r)} = e^{-2\Lambda(r)} = (1 - \gamma(r)) \quad \text{for} \quad r > R_{\text{star}}, \quad (\text{III.5})
\]

with

\[
    \gamma(r) = \begin{cases} 
    \frac{2M(r)}{r} & r < R_s \\
    \frac{2M_s}{r} & r \geq R_s
    \end{cases}
\]

Einstein’s equations for a static star reduce then to the familiar Tolman–Oppenheimer–Volkoff equation (TOV) \([7, 11, 14]:\)

\[
    \frac{dP(r)}{dr} = -\frac{1}{r^2} (\epsilon(r) + P(r)) \left( M(r) + 4\pi r^3 P(r) \right) e^{-2\Lambda(r)} , \quad (\text{III.7})
\]

where the gravitational mass \(M(r)\) contained in a sphere with radius \(r\) is determined via the energy–density \(\epsilon(r)\) by:

\[
    M(r) = 4\pi \int_0^r \epsilon(r) r^2 dr . \quad (\text{III.8})
\]

The metric function \(\phi(r)\) obeys the differential equation

\[
    \frac{d\phi}{dr} = -\frac{1}{\epsilon(r) + P(r)} \frac{dP}{dr} , \quad (\text{III.9})
\]

with the boundary condition

\[
    \phi(r = R_s) = \frac{1}{2} \ln(1 - \gamma(R_s)) . \quad (\text{III.10})
\]

For a given EOS i.e. \(P(\epsilon)\), one can now solve the TOV equation by integrating them for a given central energy density \(\epsilon_c\) from the star’s centre to the star’s radius, defined by \(P(R_s) = 0\).
More complicated is the case of rotating stars, where due to the rotation changes occur in the pressure, energy density, etc. The energy–momentum density tensor $T_{\mu\nu}$ takes the form $(g^{\mu\nu}u_\mu u_\nu = -1)$:

$$T_{\mu\nu} = T^0_{\mu\nu} + \Delta T_{\mu\nu},$$

(III.11)

with

$$T^0_{\mu\nu} = (\epsilon + P)u_\mu u_\nu + Pg_{\mu\nu},$$

(III.12)

$$\Delta T_{\mu\nu} = (\Delta \epsilon + \Delta P)u_\mu u_\nu + \Delta Pg_{\mu\nu}.$$  

(III.13)

$P, \epsilon$, and $\rho$ are quantities in a local inertial frame comoving with the fluid at the instant of measurement. For the rotationally deformed, axially-symmetric configurations one assumes a multipole expansion up to second order ($P_2$ denotes the Legendre polynomial):

$$\Delta P = (\epsilon + P)(p_0 + p_2 P_2(\cos \theta)),$$

(III.14)

$$\Delta \epsilon = \Delta P \frac{\partial \epsilon}{\partial P},$$

(III.15)

$$\Delta \rho = \Delta P \frac{\partial \rho}{\partial P}.$$  

(III.16)

For the rotating and deformed star with the rotational frequency $\Omega$ one has now to deal with a generalized Schwarzschild metric, given by:

$$ds^2 = -e^{2\nu(r,\delta,\Omega)}dt^2 + e^{2\psi(r,\delta,\Omega)}(d\varphi - \omega(r, \Omega)dt)^2 + e^{2\mu(r,\delta,\Omega)}dr^2 + \mathcal{O}(\Omega^3).$$

(III.17)

Here, $\omega(r)$ denotes the angular velocity of the local inertial frame, which – due to the dragging of the local system – is proportional to $\Omega$.

The metric functions of Eq. (III.17) which correspond to stationary rotation and axial symmetry with respect to the axis of rotation are expanded up to second order as (independent of $\phi$ and $t$):

$$e^{2\nu(r,\delta,\Omega)} = e^{2\nu(r)}[1 + 2(h_0(r, \Omega) + h_2(r, \Omega)P_2(\cos \phi))],$$

(III.18)

$$e^{2\psi(r,\delta,\Omega)} = r^2 \sin^2 \theta [1 + 2(v_2(r, \Omega) - h_2(r, \Omega))P_2(\cos \theta)],$$

(III.19)

$$e^{2\mu(r,\delta,\Omega)} = r^2 [1 + 2(v_2(r, \Omega) - h_2(r, \Omega))P_2(\cos \theta)]$$

(III.20)

$$e^{2\lambda(r,\delta,\Omega)} = e^{2\lambda(r)} \left[1 + \frac{2}{r} \frac{m_0(r, \Omega)G + m_2(r, \Omega)P_2(\cos \theta)}{1 - \gamma(r)}\right].$$

(III.21)
The angular velocity in the local inertial frame is determined by the differential equation

$$\frac{d}{dr} \left( r^4 a(r) \frac{d\omega}{dr} \right) + 4 r^2 \frac{da(r)}{dr} \omega(r) = 0 \quad \text{for } r < R, \quad (\text{III.22})$$

where $\omega(r)$ is regular for $r = 0$ with $\frac{d\omega}{dr} = 0$. $a(r)$ abbreviates

$$a(r) \equiv e^{-\phi(r)} \sqrt{1 - \gamma(r)} \quad \text{(III.23)}$$

Outside the star $\omega(r, \Omega)$ is given by:

$$\omega(r, \Omega) = \Omega - \frac{2}{r^3} J(\Omega) \quad \text{for } r > R. \quad (\text{III.24})$$

The total angular momentum is defined by:

$$J(\Omega) = \frac{R^4}{6} \left( \frac{d\omega}{dr} \right)_{r=R} \quad \text{(III.25)}$$

From the last two equations one obtains then an angular frequency $\Omega$ as a function of central angular velocity $\omega_c = \omega(r = 0)$ (starting value for the iteration):

$$\Omega(\omega_c) = \omega(R) + \frac{2}{R^3} J(\Omega) \quad \text{(III.26)}$$

Due to the linearity of Eq. (III.22) for $\omega(r)$ new values for $\omega(r)$ emerge simply by rescaling of $\omega_c$. The momentum of inertia, defined by $I = \frac{J}{\Omega}$, is given by $a(R) = 1$:

$$I = \frac{J(\Omega)}{\Omega} = \frac{8\pi}{3} \int_0^R dv \ r^4 \frac{\epsilon + P}{\sqrt{1 - \gamma(r)}} \frac{\omega - \Omega}{\omega} e^{-\phi} \quad \text{(III.27)}$$

Relativistic changes from the Newtonian value are caused by the dragging of the local systems, i.e. $\frac{\omega}{\Omega}$, the redshift ($e^{-\phi}$), and the space-curvature $\left(1 - \gamma(r)\right)^{-1/2}$. For slowly rotating stars with low masses, one can neglect the dragging ($\frac{\omega}{\Omega} \to 1$) and rotational deformations, but we would like to emphasize that the described treatment is not restricted by low masses and/or slow rotations.
If one has determined $\omega(r)$, one solves in the next step the coupled mass monopole equations ($\ell = 0$) for $m_0, p_0$ (= monopole pressure perturbation) and $h_0$ (for details, see Refs. [4, 7, 28]). The quadrupole distortions $h_2$ and $v_2$ ($\ell = 2$) determine the star’s shape (see Refs. [3, 7, 46]). After the determination of the distortion functions, one can express the surfaces of constant density, the star’s eccentricity $\frac{p}{2} e$, and the mass quadrupole moment of the star as follows:

$$r(\theta) = r + \xi_0(r) + \left\{ \xi_2(r) + r \left[ v_2(r) - h_2(r) \right] \right\} P_2(\cos \theta), \quad (\text{III.28})$$

$$\epsilon = \sqrt{1 - \left( \frac{R_p}{R_{eq}} \right)^2}, \quad (\text{III.29})$$

$$\Pi = \frac{8}{5} A_2 \left( \frac{\gamma_4}{2} \right)^2 + \left( \frac{J}{R_s} \right)^2. \quad (\text{III.30})$$

$A_2$ denotes an integration constant and $\xi_i$ is defined by ($i = 0, 2$):

$$\xi_i = -p_i (e + P) \left( \frac{\partial P}{\partial r} \right)^{-1}. \quad (\text{III.31})$$

c) Stability criteria

An intriguing problem in the physics of NSs is the question, whether the used EOSs are in accordance with the observed data, so supplying a test for the theoretical and partly speculative EOSs for highly compressed matter. Unfortunately, as far as gross properties (radii, masses) of NSs are concerned most of the nonrelativistic and relativistic EOSs are able to reproduce these gross properties (For a test of 25 different EOSs, see Refs. [4, 7, 46, 47, 56]).

A more decisive criterion may be the stability of a star against rotation. Since no trivial stability criteria are known for rotating configurations in general relativity, we consider first Kepler’s criterion, which sets an absolute upper limit on a star’s rotation. The resulting Kepler frequency, $\Omega_K$, beyond which instability sets in due to mass shedding at the equator, is given for the generalized Schwarzschild metric as solution of ($\psi' \equiv \frac{\delta \psi}{\delta r}$, et c.) [4, 7, 46, 47, 56]

$$\Omega = \left[ e^{\psi'(\Omega) - \psi(\Omega)} V(\Omega) + \omega(\Omega) \right]_{\psi, \Omega = \Omega_K}, \quad (\text{III.32})$$
with

$$V(\Omega) := \left[ \frac{\omega(\Omega)}{2 \psi(\Omega)} e^{\psi(\Omega)-\nu(\Omega)} \right] + \sqrt{\frac{\nu(\Omega)}{\psi(\omega)}} + \left( \frac{\omega(\Omega) e^{\psi(\Omega)-\nu(\Omega)}}{2 \psi(\Omega)} \right)^2, \quad_{\text{eq, } \Omega=\Omega_K} \tag{III.33}$$

to be evaluated at the equator. $V$ denotes the orbital velocity of a comoving observer at the equator relative to a locally non-rotating observer. Neglection of the distortions ($h_2, v_2 << 1$) and of the dragging of the local inertial frames ($\omega = \Omega$) gives

$$V_{eq} = \sqrt{\frac{\gamma_{eq}}{2}} \frac{1}{\sqrt{1 - \gamma_{eq}}} \to R_{eq} \Omega_c \quad \text{or} \quad \gamma_{eq} \to 1, \tag{III.34}$$

with

$$\Omega_c \equiv \sqrt{\frac{\dot{M}_s}{R_s^3}}. \tag{III.35}$$

The Newtonian result is recovered by using a flat space–time–geometry. The Kepler frequency of the heaviest neutron star can be obtained from the mass and radius of the most massive nonrotating neutron star as

$$\Omega_K = \frac{2}{3} \sqrt{\frac{\dot{M}_s}{R_s^3}}, \tag{III.36}$$

which has the advantage that one needs only the input from a static non-rotating star model.

As mentioned before the Kepler criterion gives only an upper limit. Gravitational–wave reaction instabilities of a rotating star are likely to lower the star’s maximum rotational frequency below $\Omega_K$. Since the theory is rather lengthy, we refer for details to Refs. [5, 6, 7, 8, 9, 51, 52]. The critical frequency for a particular instability mode ($m = 2, 3 \ldots$) is given by:

$$\Omega_m^\nu = \frac{\omega_m(0)}{m} \left\{ a_m(\Omega_m^\nu) \left( \frac{\tau_{\gamma,m}}{\tau_{\gamma,m}} \right) \right\} \tag{III.37}$$

where $\nu$ denotes the shear viscosity, depending on temperature $T$. The expressions for the damping time scales $\tau$ for gravitational radiation reactions
\[(\tau_{\beta,m}), \text{viscous damping (} \tau_{\nu,m} \text{), and the surface mode } \omega_m \text{ can be found in Refs. } [5, 7, 9, 10].\]

d) Further quantities

For completeness we also give the expressions for the redshifts, the injection energy, and the stability parameter.

The frequency shifts of light emitted at the equator in backward (b) and forward direction (f) is given by [5, 7, 9]:

\[
z_{\nu/b}(\Omega) = e^{-\nu(\Omega)} \left( \frac{1 \pm \omega(\Omega)}{1 \mp V(\Omega)} \right) - 1. \quad (\text{III.38})
\]

For the redshift at the pole one gets:

\[
z_{\nu/p}(\Omega) = e^{-\nu(\Omega)} - 1. \quad (\text{III.39})
\]

The so-called injection energy is defined as:

\[
\beta(\Omega) \equiv e^{2 \nu(\Omega)} \bigg|_{\text{pole}} = \frac{1}{(z_{\nu}(\Omega) + 1)^2}. \quad (\text{III.40})
\]

For discussing the stability of rotating stars it is useful to define the stability parameter:

\[
t(\Omega) := \frac{T(\Omega)}{|W(\Omega)|}, \quad (\text{III.41})
\]

where \(T\) denotes the rotational and \(W\) the gravitational energy of the star.

e) Results (gross properties)

As a first example for the influence of the rotation we show in Figs. 13 and 14 the NS–mass versus central energy density and the radius–mass relation for a composition with \(p, n, \epsilon^-\), and \(\mu^-\) only, for nonrotating stars and stars rotating with their Kepler frequency. As expected and discussed before one reaches, due to the large stiffness of the EOS, the maximum mass at lower central densities. The rotation can increase the mass by more than half a solar mass. In this context one may remark that relativistic EOS of nuclear matter are in general stiffer than their nonrelativistic counterparts [7, 9, 10]. If one includes now hyperons in the relativistic NSM–EOSs one obtains a
softening of the EOS, so that the gross properties of NSs may not differ too much from NS–calculations with nonrelativistic treatments with a pure nucleonic/leptonic composition. Only the protons concentration are not the same in both cases, since the different behaviour of the symmetry energy lowers the proton contribution in the nonrelativistic case, so suppressing the so-called direct Urca-process [53, 54, 55].

Next we turn to the more interesting case of NSM including more baryons. For EOSs in the relativistic Hartree scheme one obtains – as expected – still sufficiently large enough NS–masses, which decrease – as discussed before – for weaker hyperon couplings. This behaviour is exhibited in Figs. 15 and 16. So far the results comply with the familiar pattern. A limit for the relative hyperon couplings, in this scheme, is given by the SU(6) choice, since otherwise the NS–masses become too small. This result compares with our NS–calculation with the NSM–EOS TM1, where we obtained a maximum star mass of 1.56 M_⊙ (see Section II). Furthermore we find the expected peculiar behaviour for the universal coupling in the lower energy–density region (see discussion in Section II). This can also be seen in Table V, where the NS–properties for a fixed mass of 1.4 M_⊙ are given. They show with exception of the universal coupling the expected behaviour with increasing hyperon–couplings. For comparison we include also the outcome for nonrelativistic calculations without hyperons [22], which demonstrates clearly the softening of the relativistic EOS due to the hyperons (see also Figs. 13, 15, 16), which become even softer than the nonrelativistic EOS without hyperons.

More interesting are the RHF–EOSs. In the case of using the hyperon couplings of the RH–treatment and g_N = g_Δ one obtains for rather low densities an onset of Δ^- and hyperons, where the Δ^- and the Σ^- play a decisive role (neglecting the Δ^- does not change the EOS significantly, since then the Σ^- occur earlier [48]), and the EOS becomes rather soft for lower densities, but for higher densities the pressure rises again more strongly. As a consequence the gravitational NS–mass as a function of central energy density should show a flat plateau before it rises again in the standard pattern, but at higher densities than in the RH–EOS case. Furthermore one expects relatively low star masses due to the soft EOS.

This behaviour is exhibited in Fig. 17, where the gravitational NS–mass is given as function of the central density. Reasonable star masses demand
larger hyperon couplings. The flat behaviour in the lower domain resembles – as one could have expected – to the case of rather soft EOSs with low incompressibility (see, for instance, the BCK–EOS \([23]\), where one obtains maximal NS–masses of approximately \(1 \, M_\odot\) \([24]\)). In Fig. 18 we exhibit the limiting Kepler frequencies. As expected the softer RHF–EOSs permit lower rotational periods. However inclusion of the gravitational–radiation instabilities shows that the Kepler frequency is only an upper limit on the critical frequency and the limiting periods increase of about \(30\%\) \((12,16)\) (for more details, see Ref. \([18]\)). If one uses RHF–EOSs, where the ratio of hyperon couplings is adjusted in the RHF–scheme (see Section II), the hyperons are not so easily produced and the resulting EOSs become stiffer (see Section II). In the presentation we will restrict ourselves to the interesting cases of the smallest hyperon couplings compatible with star masses around \(1.5 \, M_\odot\). The resulting star masses as function of the central energy density are shown in Figs. 19 and 20. By comparison with Fig. 17 we infer that these improved EOSs, where the hyperon potential depths are treated correctly, lead to masses, which are better in accordance with the data, since increase of the hyperon couplings and rotation lead to even higher star masses. EOSs with weak \(\Delta\)-repulsion are rather soft and give only for strong hyperon couplings and high rotation frequencies mass values around \(1.5 \, M_\odot\) (see Figs. 19,20).

One may illuminate the situation further by comparing the properties for a typical NS of \(1.4 \, M_\odot\) (see Tabs. V,VI). By the given arguments one should obtain increasing central densities by going from the stiff RH–EOS (RH1) to the softest RHF–EOSs with low Dirac masses for both the deltas and hyperons (RHF12,13). For the properly adjusted RHF–EOSs (RHF8,9) the star parameters differ not significantly. Here the higher \(\Delta\)-masses suppress the influence of the \(\Delta\)'s below \(\epsilon \sim 500 \, \text{MeV/fm}^3\) and one obtains sufficient pressure to reach a mass of \(1.4 \, M_\odot\) earlier (see also Fig. 19). Decrease of the \(\Delta\)-mass according to universal coupling of the \(\Delta\)'s gives an increase of the central energy by a factor 2 (RHF1 compared with RH1). Finally we illustrate in Table VII the influence of rotation. Shown are the results for a NS with the same baryon number. The mass changes due to rotation are relatively small for constant baryon number (for fixed central energy density see Refs [4,7,24]).

\(f)\) Cooling properties
Another decisive test for the EOS may be the cooling history of a NS. In Ref. [25] we have already calculated and discussed the thermal evolution of various models with different EOSs and different involved processes. Here we show and compare the cooling tracks of NS models constructed for the three EOSs RH1, RHF1, and RHF8, as well as the RHF-EOS without hyperons (see Fig. 22). During about the first million years the NS cools mainly by emission of neutrinos. One classifies the neutrino processes into slow and enhanced ones, according to whether two or only one baryon is participating. Enhanced processes cause a temperature inversion in young stars, i.e. the interior of the star becomes much cooler than the crust. Depending on the crust thickness the cooling wave formed by the temperature gradient reaches the surface and causes the sharp decrease of the surface temperature after about 30 years (see the three broken curves in Fig. 22).

For the EOSs considered here the only possible enhanced cooling processes are the nucleon
\[ n \to p + l^- + \bar{\nu}_l \]  
and the hyperon direct Urca processes
\[ \Sigma^- \to \Lambda + l^- + \bar{\nu}_l \]  
and
\[ \Lambda \to p + l^- + \bar{\nu}_l , \]  
as well as their inverse reactions. Here, \( l \) denotes electrons and muons. The nucleon direct Urca process is only possible, if the proton fraction exceeds some critical value of about 11 % for a pure nucleonic/electron composition of the neutron star matter, since otherwise energy and momentum conservation cannot be fulfilled simultaneously. If hyperons and muons are taken into account this value rises slightly to approximately 13 %. Similar constraints have to be considered for the hyperon direct Urca processes. It is obvious that the resulting thermal evolution depends strongly on the EOS. It seems to be a general feature that non-relativistic EOSs have proton fractions below this critical value [26], whereas relativistic EOSs allow for the nucleon direct Urca process. The critical masses above which the hyperon direct Urca processes are possible are approximately equal to 1.3 \( M_\odot \) for all three EOSs studied in this section. This seems to be surprising, since \( \Lambda \) and \( \Sigma^- \) appear
beyond \( n \sim 0.7 \text{ fm}^{-3} \) in the case of RHF1 (see Fig. 9), and already beyond \( n \sim 0.3 \text{ fm}^{-3} \) in the cases of RH1 and RHF8 (see Figs. 6 and 11). This higher threshold density is however compensated by the smaller incompressibility of RHF1 compared to RH1 and RHF8 (see Sect. II). The used slow neutrino processes, as well as the processes in the crust of the NS, are discussed in greater detail in Ref. [56].

The cooling behaviour of a NS is also influenced by the appearance of superfluid phases. If neutrons or protons become superfluid the neutrino emissivity of the nucleonic processes, the thermal conductivity and the heat capacity are reduced by an approximately exponential factor \( \exp(-\Delta/kT) \), where \( \Delta \) denotes the gap energy (see Table IV of Ref. [56] for the used gap energies).

The observational data are described in Ref. [58] (see Table 2 in Ref. [58]). The obtained effective surface temperature depends crucially on whether a magnetized hydrogen atmosphere is used or not. Since the photon flux, measured solely in the X-ray energy band, does not allow to determine what atmosphere one should use, we consider both the blackbody model (solid error bars in Fig. 22) and the hydrogen-atmosphere model (dashed error bars). The plotted errors represent the 3\( \sigma \) error range due to the small photon fluxes.

All models exhibit enhanced cooling via the nucleon direct Urca process. However this process is suppressed below the critical temperature for the superfluid phase transition. Since the process (111338) is not suppressed by superfluidity, the surface temperature of these models (see broken curves) is much smaller than the one of the model without hyperons (solid curve). The observed data can almost perfectly be described by the latter model, provided one assumes that the pulsars have no hydrogen atmosphere (except PSR 1055-52, which could be explained by internal heating; see, e.g. Refs. [59, 60]). However, if some of the pulsars prove to have a hydrogen atmosphere, these models seem to be too hot. Whether the observed pulsars have a hydrogen atmosphere could be decided if one considers multiwavelength observations, as suggested by Pavlov et al. [61]. With respect to cooling properties one seems to get along with simpler relativistic EOSs without hyperons. However the weaker temperature drop in this case is caused by the superfluidity of nucleons, which cannot be included, at present, for the other
baryons. Inclusion of this effect for hyperons may shift the curves towards the observed values, since the process \( \text{n}_2 \) would be suppressed, too.

Please note that we have considered only some of the possibilities of neutron star cooling. Additional processes as internal heating \( \text{II} \) or intermediate neutrino processes \( \text{II} \) may yield different cooling tracks. This is also true for the effect of accreted atmospheres investigated in Refs. \( \text{II} \).

### IV Summary

The goal of this investigation is to incorporate “parameter-free” microscopic relativistic Brueckner–Hartree–Fock calculations of nuclear matter in the investigation of neutron star matter. In a first step we extended the RBHF–theory of asymmetric nuclear matter to neutron star matter, consisting of neutrons, protons and leptons, which has to obey the constraints of charge neutrality and generalized \( \beta \)-equilibrium. Since for higher densities more baryons (hyperons etc.) have to be included, for which, at present, microscopic RBHF–calculations are not feasible, we extended the scheme by utilizing either the relativistic Hartree– or Hartree–Fock–approximation, in which the other baryons can be incorporated. The coupling constants of these schemes were adjusted in the nucleonic sector to the outcome of the RBHF–calculations of NSM around saturation densities. In this manner we obtained a good description of NSM near saturation, which is essential for lighter neutron stars and also important for heavier stars. As long as one restricts the composition to \( p, n, e^- \), and \( \mu^- \) only, one obtains in this framework, due to the rather stiff EOS, Oppenheimer–Volkoff star masses around \( 2.2 \, M_\odot \) and minimum rotation frequencies slightly above 1 ms. However if one incorporates more baryons in the scheme one obtains a considerable softening of the NSM–EOS, which leads inevitably, as in the case of realistic phenomenological parametrizations of the nuclear Lagrangian, to negative Dirac masses for the nucleons in NSM. This drawback is the result of the necessity to reproduce a rather low nucleon Dirac mass at saturation, which leads to a peculiar feature of the mean field approximation, namely that such Lagrangians cause a strong decrease of the effective \( \sigma \)-mass at higher densities, resulting in a steep decrease of the Dirac masses. Since the behaviour of the Dirac masses
at high densities are (completely) unknown in NSM, we extrapolated the Lagrangian in these domains by a slightly changed dynamics, where the decrease of the Dirac mass is not so severe, which is also in accordance with RBHF-results in nuclear matter. The resulting pressure changes are rather small. Within the RH-framework we obtained then compositions of NSM which are more or less in accordance with former investigations, which use a priori in the whole domain Lagrangians with increasing effective $\sigma$-masses. For the hyperons we used different coupling strengths but the ratios of the $\sigma-\omega$-couplings were fixed by utilizing the potential depths of the hyperons in nuclear matter. Due to large $\rho$-coupling in the RH-approximation the $\Delta$'s are negligible in this framework. The Oppenheimer–Volkoff (OV) star mass reaches from 1.5 $M_\odot$ to 2.4 $M_\odot$ depending on the hyperon couplings and the rotation frequencies. More complicated is the situation for RHF–EOSs. Here one is confronted with smaller $\rho$-couplings, which favours $\Delta$'s in the composition, and the fact that the potential depths of the hyperons in nuclear matter are solely determined by their Hartree contributions. As discussed in details in Section II, the resulting compositions depend now strongly on both the assumptions about the hyperon couplings and the assumed behaviour of the $\Delta$–Dirac masses in NSM. As long as one assumes for the $\Delta$'s the same coupling as for the nucleons, the $\Delta$'s play a dominant role in the composition. The EOSs are relatively soft and the minimal Oppenheimer–Volkoff mass is around 1.5 $M_\odot$ for weak hyperon couplings. This mass drops even to 1.3 $M_\odot$ for strong hyperon couplings if one decreases the relative $\Delta$–strength generally to 0.625. Increase of the $\Delta$–repulsion gives compositions not so different from the RH-approximations with minimum OV–masses of 1.6 $M_\odot$.

In general we can conclude in accordance with earlier findings that the EOSs based on hadronic theories of matter are capable of accommodating the gross properties as well as rotational periods of all pulsars known to date.

Acknowledgements
We would like to thank M. Mareš, J. Schaffner, W. Wambach and W. Weise for helpful comments and clarifying remarks.
Table captions

**Table I:** Saturation properties of infinite nuclear matter. RBHF–A and RBHF–B denote the RBHF–results for the Brockmann potentials A and B, calculated in the full Dirac space (for details, see Ref. [21]). For comparison we added some results of more phenomenological relativistic Hartree calculations (NL1, NL–SH) [22,23] and phenomenological non-relativistic calculations. SkM* and S III denote two well known Skyrme forces, TF 96 is a recent Thomas–Fermi calculation [24]. Also included are two nonrelativistic microscopic variational calculations [25].

**Table II:** Parametrizations of the RH– and RHF–Lagrangian adjusted to the RBHF–calculations. For the masses the following values were selected (MeV): $m_N = 939$, $m_\sigma = 550$, $m_\omega = 738$, $m_\pi = 138$, $m_\rho = 770$ ($g_\pi = 1.00265$, $f_\omega^2/4\pi = 0.08$). The parametrizations are labelled as follows: RHA = RBHA etc. for the potential A (effective mass at the Fermi surface $\tilde{m} = 617.8$ MeV (A); 621.8 MeV (B)).

**Table III:** EOSs for the different density regions of a NS (1 MeV/fm$^3$ corresponds to 1.783 $\times$ $10^{12}$ g/cm$^3$). For the density region above 20 MeV/fm$^3$ we use the parametrizations of the RBHF–calculations in the frame of the RH– and RHF–approximation, respectively (OBE–potential B of Brockmann and Machleidt). The parameters for the nucleonic sector are given in Table II. For the population of the more massive baryons the 12 lowest lying ones are allowed for.

**Table IV:** Relative coupling strengths of the hyperons in the different approximations (see text). The ratios $x_{\alpha H}/x_{\omega H}$ are adjusted to the binding energy of the hyperon in nuclear matter. For RHF11 - RHF14 the RH–couplings are used. The universal coupling is defined by $g_N = g_H = g_\Delta$. The EOSs for RHF7, RHF11 and RHF12 are too soft for obtaining OV–masses around 1.5 $M_\odot$ (see Fig.17). In these models larger hyperon couplings are needed, for instance, by going from RHF7 to RHF10 $M_{OV}$ increases approximately to 1.5 $M_\odot$ (at Kepler frequency).

**Table V:** Comparison of the properties of a static, spherical NS of mass 1.4 $M_\odot$ for different RH–models. $\epsilon_c(P_c)$ denotes the central energy
density (pressure). The *amu* mass $M_A$ minus the gravitational mass $M_G$ is effectively the binding energy liberated when the NS is formed. $R$ denotes the star’s radius, $\Delta_c$ stands for the stellar crust using $2.4 \times 10^{14}$ g cm$^{-3}$ as the boundary, $I$ denotes the moment of inertia and $z$ the surface redshift. For a comparison we included two nonrelativistic models without hyperons (see Table I) [23].

**Table VI:** Comparison of the properties of a static, spherical NS of mass $M = 1.4M_\odot$ for different RHF models. Labels as in Table V.

**Table VII:** Properties of rotating neutron star models of rotational period $P = 1.4$ ms and the same baryon number as the nonrotating star with $M = 1.4M_\odot$, calculated for different EOSs. The entries are: central energy density $\epsilon_c$; equatorial and polar radii, $R_{eq}$ and $R_p$, respectively; moment of inertia $I$; stability parameter, $t$; injection energy $\beta$; redshift of the pole, $z_p$; eccentricity, $e$ [49]; quadrupole moment, $\Pi$. The gravitational mass increase due to rotation is rather small.
Figure captions

Fig. 1: EOSs for asymmetric nuclear matter. Compared are the EOSs for the Brockmann–Machleidt potential B for a fixed asymmetry $\delta$ in the RBHF–approximation (RBHF–B: full curves) with the treatment in the RH (RHB; dotted curves) and RHF (RHF B 1: dashed curves)–scheme, respectively.

Fig. 2: Comparison of the EOSs for neutron star matter composed of $n$, $p$, $e^-$, and $\mu^-$ (RBHF–B, RHB, RHF B 1; Brockmann potential B). The upper branches correspond to the case where myons are neglected.

Fig. 3: Comparison of the nucleon/lepton composition of neutron star matter (potential B). The upper branches correspond to the case without myons.

Fig. 4: Comparison of the Dirac masses with zero charge: The full curves show the behaviour in the original scheme. The dotted curves give the extrapolation according to Eqs.(III.10), (III.11). The selected case is the RH–approach with SU(6) couplings (RHF11, $\alpha = 0.02$) (see Table IV). The other cases are rather similar. The masses differ hardly for the different isospin states.

Fig. 5: Relative baryon/lepton populations in the relativistic Hartree scheme for universal coupling of the hyperons and deltas (RHI; $\alpha = 0.015$)

Fig. 6: Relative baryon/lepton populations in the relativistic Hartree scheme for hyperon–SU(6)–coupling of the vector mesons (RHI; $\alpha = 0.015$). The relative $\sigma$–couplings of the hyperons are adjusted to the corresponding potential depths in nuclear matter.

Fig. 7: Dependence of the EOS on the choice of the relative $\sigma$–meson–hyperon coupling $x_{\sigma H}$ (relativistic Hartree EOS, potential B, $\alpha = 0.015$). Compared are the choices $x_{\sigma H} = 0.7, 0.8$ with the so–called SU(6) parametrization and the universal coupling (see Table IV). For the first two cases the (larger) hyperon–vector couplings are adjusted to the potential depths in infinite matter (for $x_{\sigma H} = 0.9, x_{\omega H} > 1$). For the SU(6) parametrization the vector meson couplings are fixed by...
the quark picture and the (smaller) $\sigma$–couplings are adjusted to the potential depths ($x_{\sigma H} \leq x_{\omega H}$). Due to the stronger $\sigma$–coupling in the universal case one gets for lower densities a softer EOS, otherwise one confirms the expectation that smaller couplings softens the EOS (see text).

Fig. 8: Relative baryon/lepton populations in the RHF–approximation for relative hyperon couplings taken from the RH–approximation in the SU(6) case (RHF1; $\alpha = 0.02$)

Fig. 9: Relative baryon/lepton population in the RHF–approximation (RHF1; $\alpha = 0.02$). The $\sigma$–couplings are adjusted to the hyperon potential depths (see Table IV). Increase of the hyperon coupling would give a later onset of the hyperons, for instance, $\Lambda$ occurs at $\rho \sim 0.8$ fm$^{-3}$.

Fig. 10: Relative baryon/lepton population for NSM in the RHF–approximation with reduced $\Delta$–coupling ($x_{\omega H} = SU(6)$, $x_{\sigma H}$ adjusted, (RHF7; $\alpha = 0.02$), $x_{\Delta A} = x_{\Delta N} = 0.625 \left[\frac{3}{5}\right]$. The onset of the $\Delta$s/hyperons is now shifted to higher/lower densities than for $g_{\Delta} = g_N$.

Fig. 11: Relative baryon/lepton population for NSM in the RHF–approximation (RHF8) with fixed $x_{\sigma \Delta} = 0.625$ and $x_{\omega \Delta} = 1$. The hyperon couplings are adjusted to the hyperon–potential depths.

Fig. 12: Relative baryon/lepton population for NSM in the RHF–approximation (RHF9) with adjusted delta– and hyperon couplings to the potential depths.

Fig. 13: Gravitational star mass (in units of solar Mass $M_\odot$) as a function of central energy density for star models constructed from EOSs with $p, n, e^-$, and $\mu^-$. The upper curve corresponds to (deformed) stars rotating at their Kepler frequency. The RHF– and RH–curves are very close, since the pressure differs not much.

Fig. 14: Neutron star radius versus mass. Shown are sequences of stars rotating at their Kepler frequencies and at zero frequency ($p, n, e^-$, and $\mu^-$ only). The minimal periods are $\sim 1$ ms for $M = 1.5M_\odot$. 

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Fig. 15: Dependence of the gravitational star mass on the relative meson–hyperon coupling. The non rotating, spherical star families are given as function of the central energy density (RH–EOSs, $\alpha = 0.015$; all baryons included). The maximum star mass increases with stronger hyperon couplings. The peculiar behaviour for the universal coupling for lower densities is explainable by the stronger attraction (see Figs. 5, 7 and text).

Fig. 16: Increase of the gravitational star mass for rotating stars: Shown are the star families as in Fig. 15, but now for deformed stars rotating at their Kepler frequency. The radii are approximately 12 (15) km for nonrotating (rotating) stars at $M_{NS} = 1.5 M_\odot$.

Fig. 17: Gravitational NS mass for nonrotating stars versus central energy density for RHF–EOSs with universal couplings and couplings from the RH–treatment. The peculiar behaviour for low densities corresponds to the early onset of baryons in this model (see text). At their Kepler frequency theOV–mass ranges from $\sim 1.95 M_\odot$ (universal) to $\sim 1.5 M_\odot$ (SU(6)–model); for $M = 1.4 M_\odot$ the radius stretches in the range 9 km (nonrotating) to 11 km (Kepler rotating).

Fig. 18: Limiting rotational Kepler periods of pulsars versus NS mass for the models described in Figs. 16 and 17. The shaded area covers the range of observed periods and masses.

Fig. 19: Gravitational NS mass for spherical nonrotating stars versus central energy density. Compared are the star families for EOSs in the RHF–scheme for different $\Delta$–couplings for the weakest hyperon coupling compatible with $M_{OV} \sim 1.5 M_\odot$. The ratio of the hyperon couplings is adjusted to the hyperon potential depths in nuclear matter.

Fig. 20: Gravitational NS mass for nonspherical stars rotating at their Kepler frequency versus central energy density. Compared are the same EOSs as in Fig. 19.

Fig. 21: Limiting rotational Kepler periods of pulsars versus NS mass for the models described in Figs. 19, 20. Inclusion of gravitation–radiation instabilities increases the limiting period by approximately 30% (for
$M = 1.5 \, M_\odot$ \cite{[18]=55}. The shaded area covers the range of observed periods and masses \cite{[56]=55}.

**Fig. 22:** Cooling of $M = 1.4 \, M_\odot$ models for different EOSs. The surface temperature obtained with a blackbody- (magnetic hydrogen-) atmosphere are marked with solid (dashed) error bars labeled by the respective pulsar’s position.
### TABLE I

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¹ taken from Ref. [2],
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### TABLE V

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<td>0.213</td>
<td>0.229</td>
<td>0.271</td>
<td>0.254</td>
</tr>
</tbody>
</table>

### TABLE VI

<table>
<thead>
<tr>
<th>Quantity</th>
<th>RHF1</th>
<th>RHF6</th>
<th>RHF8</th>
<th>RHF9</th>
<th>RHF12</th>
<th>RHF13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_c ,(10^{14} \text{g/cm}^3)$</td>
<td>16.3</td>
<td>16.8</td>
<td>9.3</td>
<td>10.1</td>
<td>21.8</td>
<td>17.8</td>
</tr>
<tr>
<td>$P_c ,(10^{24} \text{dyn/cm}^2)$</td>
<td>29.52</td>
<td>33.59</td>
<td>10.09</td>
<td>11.91</td>
<td>45.2</td>
<td>35.47</td>
</tr>
<tr>
<td>$M_G/M_\odot$</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>$(M_A - M_G)/M_\odot$</td>
<td>0.21</td>
<td>0.25</td>
<td>0.16</td>
<td>0.16</td>
<td>0.216</td>
<td>0.219</td>
</tr>
<tr>
<td>R (km)</td>
<td>9.86</td>
<td>9.41</td>
<td>13.15</td>
<td>12.8</td>
<td>9.31</td>
<td>9.59</td>
</tr>
<tr>
<td>$\Delta_c$ (km)</td>
<td>0.8</td>
<td>0.81</td>
<td>2.4</td>
<td>2.25</td>
<td>0.88</td>
<td>0.98</td>
</tr>
<tr>
<td>$I(10^{44} \text{ g cm}^2)$</td>
<td>10.43</td>
<td>9.86</td>
<td>16.11</td>
<td>15.16</td>
<td>9.25</td>
<td>9.80</td>
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<tr>
<td>$z$</td>
<td>0.312</td>
<td>0.335</td>
<td>0.208</td>
<td>0.215</td>
<td>0.341</td>
<td>0.325</td>
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</table>
**TABLE VII**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>RHI</th>
<th>RH5</th>
<th>RHF1</th>
<th>RHF8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_c$ (MeV/fm$^3$)</td>
<td>403.12</td>
<td>479.40</td>
<td>869.83</td>
<td>438.85</td>
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<tr>
<td>$R_{eq}$ (km)</td>
<td>13.72</td>
<td>12.95</td>
<td>10.14</td>
<td>14.49</td>
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<tr>
<td>$R_p$ (km)</td>
<td>12.11</td>
<td>11.68</td>
<td>9.65</td>
<td>12.6</td>
</tr>
<tr>
<td>$\log I/(g,cm^2)$</td>
<td>45.17</td>
<td>45.13</td>
<td>45.00</td>
<td>45.19</td>
</tr>
<tr>
<td>$t$</td>
<td>0.036</td>
<td>0.03</td>
<td>0.016</td>
<td>0.038</td>
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<tr>
<td>$\beta$</td>
<td>0.658</td>
<td>0.646</td>
<td>0.572</td>
<td>0.672</td>
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<tr>
<td>$z_p$</td>
<td>0.232</td>
<td>0.244</td>
<td>0.323</td>
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<tr>
<td>$\epsilon$</td>
<td>0.47</td>
<td>0.43</td>
<td>0.31</td>
<td>0.49</td>
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<tr>
<td>$\Pi$ (km$^3$)</td>
<td>7.27</td>
<td>5.14</td>
<td>1.51</td>
<td>8.32</td>
</tr>
</tbody>
</table>
References


