Dynamics of \((n, 1)\) Strings

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Abstract

An \((n, 1)\) string is a bound state of a D-string and \(n\) fundamental strings. It may be described by a D-string with a world volume electric field turned on. As the electric field approaches its critical value, \(n\) becomes large. We calculate the 4-point function for transverse oscillations of an \((n, 1)\) string, and the two-point function for massless closed strings scattering off an \((n, 1)\) string. In both cases we find a set of poles that becomes dense in the large \(n\) limit. The effective tension that governs the spacing of these poles is the fundamental string tension divided by \(1 + (n\lambda)^2\), where \(\lambda\) is the closed string coupling. We associate this effective tension with the open strings attached to the \((n, 1)\) string, thereby governing its dynamics. We also argue that the effective coupling strength of these open strings is reduced by the electric field and approaches zero in the large \(n\) limit.

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1 Introduction

Type IIB string theory is believed to possess a non-perturbative SL(2,Z) symmetry [1, 2]. This implies that the theory contains various types of strings, labeled by relatively prime integers \(p\) and \(q\). In this scheme the fundamental string is \((1,0)\) while the basic RR-charged (Dirichlet) string is \((0,1)\). The classical solutions of type IIB supergravity, which correspond to the entire SL(2,Z) multiplet of \((p,q)\) strings were constructed in [3]. In the D-brane description [4, 5], the \((0,1)\) string is the basic D-string characterized by the Neumann boundary conditions for directions 0 and 1, and by Dirichlet boundary conditions along the remaining directions. In a subsequent development, Witten identified a \((p,q)\) string with a bound state of \(p\) fundamental and \(q\) D-strings [6]. This bound state is a particular state in the \(U(q)\) gauge theory which describes \(q\) parallel D-strings.

In [7, 8] the \((p,q)\) bound states were studied from the point of view of the non-linear Born-Infeld action [9], which describes the open strings that define the D-brane dynamics. It was found that the correct BPS formula for the \((p,q)\) string tension,

\[
T_{p,q} = T_{1,0} \sqrt{p^2 + \frac{q^2}{\lambda^2}},
\]

follows from a straightforward quantization of the non-linear action for the collective coordinate describing the electric field [8]. The situation is particularly simple for \(q = 1\) where the theory is abelian, and the action is known in detail. Here the \((n,1)\) string corresponds to a state with \(n\) units of electric flux. If we expand the exact mass formula for weak coupling \(\lambda\), we find

\[
T_{n,1} = T_{0,1} + \frac{1}{2} \lambda n^2 T_{1,0} + O(\lambda^3).
\]

This means that for small \(\lambda\) the \(n\) fundamental strings lose their energy almost entirely in the process of binding to a single D-string. Note, however, that the extra energy due to the \(n\) bound fundamental string grows as \(n^2\). It may be interesting, therefore, to study what happens for large values of \(n\) (this is where the electric field approaches its critical value [8]). This situation was first considered by H. Verlinde [10]. If the large \(n\) limit is taken first then the mass formula may be expanded as

\[
T_{n,1} = nT_{1,0} + \frac{T_{0,1}}{2n\lambda} + \ldots
\]

This implies that the effective tension of a D-string bound to a large number \(n\) of fundamental strings has decreased by a factor \(2n\lambda\) [10]. Thus, it is possible that the low-lying excitations of the bound state are described by a string with rescaled tension.

One of the motivations in [10] was that the S-duality maps a \((n,1)\) string into a \((1,n)\) string. Thus, by studying a single D-string in the near-critical electric field we may learn something about the large \(n\) limit of \(N = 8\) supersymmetric \(U(n)\) gauge theory in 1 + 1 dimensions. Another reason to be interested in this problem is the theory of confining strings describing gauge theories. It was shown in [11] that in this case one expects tensionless open
strings interacting with tensile closed strings. In this paper we show that precisely this situation occurs for a D-string in the near-critical world volume electric field. This fact has a simple qualitative explanation. Indeed, the open strings have charges attached to their ends. The electrostatic energy in a constant electric field is proportional to the length of the string. When the field is just below critical, it reduces the effective string tension almost to zero. The field above critical would tear the open string apart.

In this paper we probe the large $n$ limit of the $(n,1)$ bound state with dynamical calculations which go beyond the BPS limit. We calculate the 4-point function for transverse oscillations of the $(n,1)$ string and find that the poles indeed become dense in the limit where $n\lambda \to \infty$. The relevant tension which governs the spacing of the poles is

$$T_{\text{eff}} = \frac{T_{1,0}}{1 + (n\lambda)^2}. \quad (4)$$

In the large $n$ limit this is equal to the fundamental string tension reduced by the factor $(n\lambda)^2$. The strings that lose their tension here are the open strings attached to the $(n,1)$ string, i.e. the objects that define their dynamics. Note that (4) is different from the rescaling factor in the effective tension of the D-string,

$$T_{\text{Deff}} \sim \frac{T_{1,0}}{n\lambda^2}, \quad (5)$$

which appears in the BPS formula. We believe that the apparent difference is due to the fact that the string coupling is rescaled as well:

$$\lambda_{\text{eff}} = \lambda \frac{1}{\sqrt{1 + (n\lambda)^2}},$$

so that

$$T_{\text{Deff}} = \frac{T_{\text{eff}}}{\lambda_{\text{eff}}}$$

scales according to (5). This rescaling affects only the interaction strength of the open strings, which move in $1 + 1$ dimensions along the bound state. The interaction strength of the closed strings, which move in the bulk, is independent of the electric field.

We conclude the paper by indicating how to probe the $(n,1)$ string with massless closed strings incident from the outside. The rescaled open string tension (4) can be seen in these amplitudes as well. The amplitudes also show that, as expected, the tension of the closed strings propagating in the bulk is unaffected by the electric field on the D-string.

2 Setup

We study a bound state of $n$ fundamental strings with a D(irichlet) string [6] in type IIB string theory. From the conformal field theory point of view we need to introduce the following boundary term into the action [6, 8],

$$S_b = \oint d\sigma EX^0 \frac{\partial}{\partial \sigma} X^1,$$
where \( X^1 \) is the compact direction of length \( l \) over which the string is wrapped. \( E = \dot{A}_1 \), and the lagrangian for the collective coordinate \( A_1 \) is given by the DBI action,
\[
L = -l \frac{T_{1,0}}{\lambda} \sqrt{1 - E^2} .
\] (7)

On compact \( X^1 \) the momentum conjugate to \( A_1 \) is quantized. Taking \( n \) quanta of it we get the \((n,1)\) string tension \([8]\)
\[
T_{n,1} = T_{1,0} \sqrt{n^2 + \frac{1}{\lambda^2}} .
\] (8)

For reasons explained in the introduction, we are mostly interested in the large \( n \) limit. The expression for the electric field is \([8]\)
\[
E = \frac{n \lambda}{\sqrt{1 + n^2 \lambda^2}} .
\]
Thus, in the large \(|n|\) limit \( E \) tends to its critical value \( E_c = \pm 1 \).

The boundary interaction \((6)\) assigns a specific set of boundary conditions on the real axis. It turns out that the boundary conditions have a linear form,
\[
\tilde{X}^\mu = D^\mu_{\nu} X^\nu ,
\] (9)
where \( \tilde{X}^\mu \) and \( X^\mu \) are the antiholomorphic and the holomorphic parts of the field respectively. While it is possible to work on the upper half-plane \( \mathcal{H}^+ \), it is more convenient to use the doubling trick \([12, 13]\) where the holomorphic part of the field, \( X^\mu(z) \), is extended to the entire complex plane in the following way:
\[
\left\{ \begin{array}{l}
X^\mu(z) \to X^\mu(z) \quad z \in \mathcal{H}^+ , \\
\tilde{X}^\mu(z) \to D^\mu_{\nu} X^\nu(z) \quad z \in \mathcal{H}^- .
\end{array} \right.
\] (10)
This replacement allows us to express all correlators in terms of holomorphic variables only. All we need is to determine the form of the matrix \( D^\mu_{\nu} \). In fact, its form is well known from the analysis of the boundary state.

The bosonic part of the boundary state (the fermionic part has analogous form) is \([8, 14, 15]\):
\[
|B\rangle = T_{1,0} \sqrt{1 - E^2} \prod_{j=2}^9 \delta(X^j) \exp \left[ - \sum_{n=1}^{\infty} \frac{1}{n} \alpha^\nu_{\mu} D_{\mu\nu}(E) \tilde{\alpha}^\nu_{-n} \right] |0\rangle
\] (11)
where the matrix \( D^\mu_{\nu} \) has the form of a Lorentz boost,
\[
D^\mu_{\nu} = \begin{pmatrix}
D & 0 \\
0 & -1
\end{pmatrix}
\]
where \( D^\mu_{\nu} = \frac{1}{1 - E^2} \begin{pmatrix} 1 + E^2 & 2E \\ 2E & 1 + E^2 \end{pmatrix} \).
\] (12)

It is quite clear that the matrix \( D^\mu_{\nu} \) entering the boundary state is the same matrix as the one in \((9)\) and \((10)\). This is because the boundary state satisfies
\[
\tilde{\alpha}^\nu_{-n}|B\rangle = D^\mu_{\nu} \alpha^\nu_{-n}|B\rangle ,
\] (13)
which implies the boundary condition (9).

Though throughout the paper we use conformal field theory techniques and mainly apply
the doubling trick, all the results can be derived also from the boundary state (11). Let us
review some calculations where the tensionless string can be excited.

3 The four-point amplitude

The simplest nontrivial example where one can probe the tensionless string is the four-point
amplitude for NS-states on the string world-sheet:

\[ A_4(\xi_1, p_1; \xi_2, p_2; \xi_3, p_3; \xi_4, p_4) \sim \frac{1}{\lambda} \sqrt{1 - E^2} \int \{ d\sigma_1 d\sigma_2 d\sigma_3 d\sigma_4 \} \]

\[ \langle \xi_1 \cdot V_0(p_1, \sigma_1) \xi_2 \cdot V_0(p_2, \sigma_2) \xi_3 \cdot V^{-1}(p_3, \sigma_3) \xi_4 \cdot V^{-1}(p_4, \sigma_4) \rangle \]

The vertex operators for scalar particles (the transverse modes of the string) have the form:

\[ V^j_1(p_1, z) = e^{-\phi(z)} \psi^j(z) e^{ip_1 \cdot X(z)} \]

\[ V^j_0(p_2, z) = (\partial X^j(z) + ip_2 \cdot \psi(z) \psi^j(z)) e^{ip_2 \cdot X(z)} \]

(14)

where \( j = 2, \ldots, 9 \), while the momenta \( p^\alpha \) are longitudinal, \( \alpha = 0, 1. \) \( z \) must lie on the real
axis. For the (0,1) string, the holomorphic and antiholomorphic parts of \( X \) are identical;
hence, \( X(z, \bar{z}) \) may be replaced by twice the holomorphic part:

\[ X^\alpha(z, \bar{z}) \to 2X^\alpha(z) . \]

Thus, all we need to do is replace

\[ p^\alpha \to 2p^\alpha \]

in the usual type I 4-point amplitude. In the \((n, 1)\) case we must be more careful with \( X^0 \)
and \( X^1 \). Because of the boost, the appropriate replacement is:

\[ p^\alpha \to p^\alpha + D^\alpha_\beta p^\beta . \]

(15)

Furthermore, the amplitude has to include a phase depending on the ordering of the vertex
operators, which originates from the second term in the Green function on the boundary:

\[ \langle X^\alpha(\sigma_1)X^\beta(\sigma_2) \rangle = -\frac{1}{1 - E^2} \eta^{\alpha\beta} \ln |\sigma_1 - \sigma_2| + i \frac{\pi}{2} E \frac{1}{1 - E^2} \varepsilon^{\alpha\beta} \text{sgn}(\sigma_1 - \sigma_2) . \]

(16)

Let us consider the 4-point function for back-scattering. In the center of mass frame the
momenta are

\[ p^\alpha_1 = \left( \begin{array}{c} p \\ p \end{array} \right) , \quad p^\alpha_2 = \left( \begin{array}{c} p \\ -p \end{array} \right) , \quad p^\alpha_3 = \left( \begin{array}{c} -p \\ p \end{array} \right) , \quad p^\alpha_4 = \left( \begin{array}{c} -p \\ p \end{array} \right) . \]
Let us position $V_2$ and $V_4$ at 0 and 1, $V_3$ at $\infty$, and integrate over the position of $V_1$ from 0 to 1. The integral is of the well-known type I form:

$$\frac{1}{\lambda} \sqrt{1 - E^2} \frac{\Gamma(p_1 \cdot p_2) \Gamma(p_1 \cdot p_4)}{\Gamma(p_1 \cdot p_2 + p_1 \cdot p_4 + 1)} K(\xi_1, p_1; \xi_2, p_2; \xi_3, p_3; \xi_4, p_4)$$

where $K$ is the kinematic factor:

$$K(\xi_1, p_1; \xi_2, p_2; \xi_3, p_3; \xi_4, p_4) = -p_2 \cdot p_3 p_2 \cdot p_4 \xi_1 \cdot \xi_2 \xi_3 \cdot \xi_4 - \frac{1}{4} p_1 \cdot p_2 (\xi_1 \cdot p_4 \xi_3 \cdot p_2 \xi_2 \cdot \xi_4 +$$

$$\xi_2 \cdot p_3 p_4 \cdot p_1 \xi_1 \cdot \xi_3 + \xi_1 \cdot p_3 \xi_4 \cdot p_2 \xi_2 \cdot \xi_3 + \xi_2 \cdot p_4 \xi_3 \cdot p_1 \xi_1 \cdot \xi_4) +$$

$$+ \left\{ 1, 2, 3, 4 \rightarrow 1, 3, 2, 4 \right\} + \left\{ 1, 2, 3, 4 \rightarrow 1, 4, 3, 2 \right\}$$

but with all momenta replaced according to (15). We should also take into account the phases from the second term in (16). For $V_2$ at 0 and $V_4$ at 1, the phase is $\exp \left( 2\pi i \frac{E_1^2}{1 - E^2} \right)$. For $V_4$ at 0 and $V_2$ at 1, the phase is $\exp \left( -2\pi i \frac{E_1^2}{1 - E^2} \right)$. The complete s-channel amplitude is

$$A_4 \sim \left( -\frac{\sqrt{1 - E^2}}{\lambda} \cos \left( 2\pi \frac{E_1^2}{1 - E^2} \right) \right) \frac{s^2 \Gamma(s) \Gamma(-s)}{\pi s} \xi_1 \cdot \xi_3 \xi_2 \cdot \xi_4$$

with $s$ given by

$$s = p_{1\mu} (\delta^\mu_\lambda + D^\mu_\lambda) \left( \delta^\lambda_\nu + D^\lambda_\nu \right) p_2^\nu = -\frac{8p^2}{1 - E^2}$$

Comparing to the $n = 0$ case we see that $s$ is rescaled by the factor

$$\frac{1}{1 - E^2} = 1 + (n\lambda)^2.$$

Thus, as $n\lambda$ increases the poles in the actual kinematical variable, $-8p^2$, become denser. This is equivalent to a rescaling of the tension of the fundamental string,

$$T_{eff} = T_{1,0}(1 - E^2) = \frac{T_{1,0}}{1 + (n\lambda)^2}.$$

In the large $n$ limit the poles become infinitely dense, and the process is governed by a kind of tensionless string. The strings that lose their tension are the fundamental open strings attached to the $(n, 1)$ string, i.e. the objects that define the dynamics in the D-brane theory. Thus, the massive states of such strings give rise to a tower of low energy excitations of the $(n, 1)$ bound state, with spacing of order

$$\delta E \sim \frac{1}{n\lambda \sqrt{\alpha'}}$$

in the large $n$ limit.
Is this the only effect of the electric field? We believe that the answer is no: the string coupling gets rescaled as well. Indeed, it is tempting to identify the disk amplitude with no insertions,

\[ \frac{T_{1,0}}{\lambda} \sqrt{1 - E^2}, \]

with \( T_{\text{eff}}/\lambda_{\text{eff}} \). Using (19), we find

\[ \lambda_{\text{eff}} = \lambda \sqrt{1 - E^2} = \frac{1}{\sqrt{\frac{1}{n^2} + n^2}}. \]  

(21)

Another argument in favor of this rescaling is that an insertion of an extra hole into the world sheet carries a factor \( \sqrt{1 - E^2} \) from the normalization of the boundary state, in addition to the obvious factor \( \lambda \). Thus, \( \lambda_{\text{eff}} \) is the effective hole counting parameter (it is the effective coupling constant squared for the open strings that move in 1 + 1 dimensions).\(^2\) Each handle, on the other hand, introduces a factor \( \lambda^2 \) independent of \( E \). Thus, the interaction strength of the closed strings, which move in the bulk, is independent of the electric field.

Note that in the large \( n \) limit \( \lambda_{\text{eff}} = 1/n \) independent of \( \lambda \). Let us consider performing a type IIB S-duality transformation to the \((1,n)\) bound state described by the supersymmetric \( U(n) \) gauge theory. In this theory we expect that

\[ g_{YM}^2 \sim \frac{1}{\lambda_{\text{eff}}} \sim n. \]

(22)

The effective gauge coupling should be the ‘t Hooft coupling,

\[ g_{YM}^2 = g_{YM}^2 n. \]

(23)

Thus, consistency requires that \( g_{YM} \) is independent of \( n \). This is indeed the behavior of the \( U(n) \) gauge coupling [10]. These arguments appear to support the scaling of the effective open string coupling that we have found for the \((n,1)\) bound state.

We may think of \( T_{\text{eff}}/\lambda_{\text{eff}} \) as the effective D-string tension. With this definition, we find that the rescaling factor is \( 1/(n\lambda) \). This is in qualitative agreement with the result of [10] but differs by a factor of \( 2.\(^3\)\)

\(^2\)Using the Born-Infeld action as the low-energy effective action for the open strings, we have checked explicitly that the \( l \) loop correction to the 4-point function is of order \( \lambda_{\text{eff}}^l \) times the tree level 4-point function.

\(^3\)In fact, the two definitions do not necessarily have to agree: in [10] the energy per unit length of a D-string bound to \( n \) fundamental strings was obtained from the energy of the bound state. We, on the other hand, are examining the lagrangian per unit length. The qualitative agreement is manifest, however.
4 Other amplitudes

Another example is scattering of a massless closed string off a \((n, 1)\) string. Again, we can track the \(D\) factors in the amplitude and substitute (12) into the final expression [13, 16, 17]:

\[
A_2 \sim \frac{1}{\lambda} \sqrt{1 - E^2} \int \{dz_{1,2}\} \epsilon_{1\mu\lambda}D_\nu^\lambda \epsilon_{2\sigma\eta}D_\mu^\sigma(V_{-1}^\nu(Dp_1, \bar{z}_1)V_{1}^\mu(p_1, z_1)V_{0}^\sigma(p_2, z_2)V_{0}^\nu(Dp_2, \bar{z}_2))
\]

(24)

We have already defined the vertex operators in (14). Calculations lead to the answer [12]:

\[
A_2 = \frac{1}{\lambda} \sqrt{1 - E^2} \frac{\Gamma(s)\Gamma(t)}{\Gamma(1 + s + t)} (sa_1 + ta_2)
\]

(25)

where \(a_1, a_2\) are polarization-dependent kinematic factors:

\[
a_1 = \text{Tr}(\epsilon_1 \cdot D) p_1 \cdot \epsilon_2 \cdot p_1 - p_1 \cdot \epsilon_2 \cdot D \cdot \epsilon_1 \cdot p_2 - p_1 \cdot \epsilon_2 \cdot \epsilon_1^T \cdot D \cdot p_1 - p_1 \cdot \epsilon_2^T \cdot \epsilon_1 \cdot D \cdot p_1
\]

\(-p_1 \cdot \epsilon_2 \cdot \epsilon_1^T \cdot p_2 + \frac{s}{2} \text{Tr}(\epsilon_1 \cdot \epsilon_2^T) + \{1 \leftrightarrow 2\}\)

\[
a_2 = \text{Tr}(\epsilon_1 \cdot D) (p_1 \cdot \epsilon_2 \cdot D \cdot p_2 + p_2 \cdot \epsilon_2 \cdot p_1 + p_2 \cdot D \cdot \epsilon_2 \cdot p_2) + p_1 \cdot D \cdot \epsilon_1 \cdot D \cdot \epsilon_2 \cdot D \cdot p_2
\]

\(-p_2 \cdot D \cdot \epsilon_2 \cdot \epsilon_1^T \cdot D \cdot p_1 + \frac{s}{2} \text{Tr}(\epsilon_1 \cdot \epsilon_2 \cdot D) - \frac{s}{2} \text{Tr}(\epsilon_1 \cdot \epsilon_2^T)
\]

\(-\frac{s + t}{2} \text{Tr}(\epsilon_1 \cdot D) \text{Tr}(\epsilon_2 \cdot D) + \{1 \leftrightarrow 2\}\).

The kinematical invariants are \(t = p_1 \cdot p_2\) and

\[
s = p_1\mu D_\nu^\mu p_1^\nu = \frac{2}{1 - E^2} p_2^2,
\]

(26)

where

\[
p_2^2 = (p_1^1)^2 - (p_1^0)^2 = (p_2^1)^2 - (p_2^0)^2.
\]

(27)

The rescaling of the kinematical variable \(s\) is the same as in (18). Thus, we find that

\[
s = \frac{2p_2^2}{1 + (n\lambda)^2},
\]

which leads to dense poles in the actual kinematical variable, \(p_2^2\), for large \(n\lambda\). These poles correspond to excitations of the \((n, 1)\) string by attaching to it excited open strings [17]. Thus, we find further evidence that such open string have effective tension (4) and become tensionless in the large \(n\) limit. It is also clear that the kinematical variable \(t\) is not rescaled; hence, as expected, the electric field does not affect the tension of the closed strings propagating in the bulk.

Let us consider the simplest case: scattering of gravitons polarized transversely to the string [17]. Then (25) simplifies to

\[
A_2 \sim \sqrt{n^2 + \frac{1}{\lambda^2} \frac{\Gamma(1 + s)\Gamma(t)}{\Gamma(1 + s + t)} p_2^2 \epsilon_1 \cdot \epsilon_2}.
\]

(28)
In the gravitational lensing \((t \to 0)\) limit, this becomes
\[
\frac{1}{t} \sqrt{n^2 + \frac{1}{\lambda^2} p_1^2 \varepsilon_1 \cdot \varepsilon_2} .
\] (29)

Note that the amplitude is proportional to the \((n, 1)\) string tension. Now we compare with supergravity, where the metric around the string is given by
\[
ds^2 = A^{-\frac{3}{4}}(-dt^2 + (dx^1)^2) + A^{\frac{1}{4}} dx \cdot dx ,
\] (30)
\[
A(x) = 1 + \frac{T_{n,1}}{3x^6} .
\]

From the methods of [16] it is clear that, since the long-range tail of the metric perturbation is proportional to the tension, so is the coefficient of \(1/t\). Thus, we find complete agreement between (29) and the corresponding supergravity result.

A strategy similar to the one used on the 4-point and 2-point amplitudes calculated above works also for calculating conversion of an incident closed string to a pair of open strings running along the \((n, 1)\) string. The amplitude in terms of \(D_{\mu\nu}\) is given in [13], and all we need is to substitute the new form of the matrix and multiply by the overall factor \(\sqrt{1 - E^2}\).

## 5 Conclusion

In this paper we investigated the physics of \((n, 1)\) bound states which consist of \(n\) fundamental strings bound to a D-string. In the D-brane theory the dynamics of the bound state is described by open strings whose charged end-points move in \(1 + 1\) dimensions subject to an electric field along the D-string. Due to the electric field the tension of these open strings, as well as their coupling strength, become effectively reduced and approach zero in the large \(n\) limit.

The transverse size of the bound state is of the order
\[
\sqrt{\alpha'_{\text{eff}}} \sim \frac{\sqrt{\alpha'}}{\sqrt{1 - E^2}} \sim \sqrt{\alpha'} \sqrt{1 + (n\lambda)^2} .
\] (31)

This means that, for large \(n\), the transverse size grows as \(n\lambda \sqrt{\alpha'}\). The growth with \(n\) is indeed suggestive of having \(n\) constituents. Thus, the bound state becomes very thick, even compared to the string scale. In studying the excitations of this thick string, we were led to the conclusion that their effective coupling constant squared, \(\lambda_{\text{eff}}\), decreases as \(1/n\). It would be interesting to understand this phenomenon better.

What happens in the large \(n\) limit of \((n, 1)\) bound states can perhaps be regarded as a prototype for the essential phenomenon in the theory of confining strings: the open strings become tensionless while the closed strings remain tensile [11]. We hope that this analogy can be pursued further.
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