We describe a nonperturbative method for calculating the QCD vacuum and glueball wave functions, based on an eigenvalue equation approach to Hamiltonian lattice gauge theory. Therefore, one can obtain more physical information than the conventional simulation methods. For simplicity, we take the 2+1 dimensional U(1) model as an example. The generalization of this method to 3+1 dimensional QCD is straightforward.

1. Wave Functions

To study more precisely the glueball properties, one should compute not only the spectroscopy, but also the wave functions. Once the wave functions are available, the matrix elements relevant for the glueball production and decays (branch ratios) become calculable. In some sense, the wave functions [1,2] can give more physical information than the masses themselves. In this paper, we present a method for such a purpose.

We begin with the vacuum wave function in the form

\[ | \Omega > = e^{R(U)} | 0 >, \]

where \( R(U) \) consists of Wilson loops (clusters) [3], and the state \( | 0 > \) is defined by \( E_0 | 0 > = 0 \). For the glueballs, we consider here only the antisymmetric (under a plane parity transformation) lowest lying excited state, and take the wave function as

\[ | \Psi_A > = F^A(U)e^{R(U)} | 0 >, \]

where \( F^A(U) \) contains various Wilson loops with the appropriate symmetry. It has been shown [4] that we can establish a truncation scheme, which preserves the continuum limit, where the operators \( R \) and \( F^A \) are expanded in order of graphs (clusters):

\[ R = R_1 + R_2 + \cdots, \quad F^A = F^A_1 + F^A_2 + \cdots. \]

Here \( R_1 \) (or \( F^A_1 \)) is the lowest order term in \( R \) (or \( F^A \)), and is chosen to be

\[ R_1 = r_1 G_1, \quad G_1 \equiv \frac{1}{2} \sum_x \left[ U_p(x) + h.c. \right], \]

\[ F^A_1 = a_1 G^A_1, \quad G^A_1 \equiv \frac{1}{2} \sum_x \left[ U_p(x) - h.c. \right]. \]

with a coefficient \( r_1 \) (and \( a_1 \)) to be determined. Higher order clusters can be produced by solving the eigenvalue equations [4,5] for \( | \Omega > \) and \( | \Psi_A > \) order by order, from which we obtain

\[ R = r_j G_j, \quad F^A = f_j G^A_j. \quad (1) \]

Here the repeated index \( j \) implies a summation over all the clusters up to some \( n \)-th order. In previous papers [4,6–8], we have shown that this method is very efficient in obtaining scaling behavior of physical quantities. First results [8] for the glueball masses in QCD have been obtained from this method.
2. Correlation lengths

In order to obtain the correlation of the states, we first investigate the continuum limit of the clusters. Expanding $U_p$ (plaquette), $G_j$, and $G^A_j$ in order of the lattice spacing $a$, we have

$$U_p = e^{-i\phi}, \quad \Phi = a^2\mathcal{F} + \frac{a^4}{24}(D_1^2 + D_2^2)\mathcal{F} + \cdots$$

Then

$$G_j = 1 - A_j a^4 e^2 \mathcal{F}^2 - B_j a^6 e^2 (D_1^2 + D_2^2)\mathcal{F} + \cdots,$$

$$G^A_j = - X_j a^6 e^3 \mathcal{F}^3$$

$$- Y_j a^8 e^3 (D_1^2 + D_2^2)\mathcal{F} + \cdots \quad (2)$$

where $\mathcal{F} = \mathcal{F}_{12}$ is the field strength tensor, $D_1$ and $D_2$ are the covariant derivatives, and $A_j$ and $B_j$ or $X_j$ and $Y_j$ are constants, corresponding to the cluster $G_j$ or $G^A_j$, respectively. The long wave length vacuum wave function is [9,4,6]

$$|\Omega > \sim e^{-\int d^4x [\mu_0 \mathcal{F}^2 + \mu_2 \mathcal{F}(D_1^2 + D_2^2)\mathcal{F}]$$

According to Eqs. (1) and (2),

$$\mu_0 = r_j A_j a^2 e^2, \quad \mu_2 = r_j B_j a^4 e^2,$$

from which we obtain the correlation length between the field strengths of the vacuum $\xi_v$

$$\xi_v = a \sqrt{r_j B_j / r_j A_j}.$$

Similarly, the long wavelength anti-symmetric glueball operator is

$$F^A(U) \sim - \int d^4x [\mu_0^A \mathcal{F}^3 + \mu_2^A \mathcal{F}^2 (D_1^2 + D_2^2)\mathcal{F}]$$

and

$$\mu_0^A = f_j X_j a^4 e^3, \quad \mu_2^A = f_j Y_j a^6 e^3.$$

The correlation length in the anti-symmetric glueball state $\xi_A$ is

$$\xi_A = a \sqrt{f_j X_j / f_j X_j}.$$

At this point, it should be noted that the results above are very general in the sense that the spatial dimension and the gauge group of the theory are not specified.

For illustration and simplicity, we consider here a 2+1 dimensional U(1) model. It is well known [10] that the theory is confining for all non-vanishing coupling constant. When $a$ goes to zero, the glueball mass $Ma$ is expected to decrease exponentially as

$$M^2 a^2 \sim \frac{c_1}{g^2} \exp \left(-\frac{c_2}{g^2}\right),$$

where $c_1$ and $c_2$ are some constants. Hence, in the scaling region

$$\mu_0 = 2 \ln(\xi_v A_j) = 2 \ln(\mu_0 M c_{\xi_v}^{-\frac{1}{2}}) + c_2\beta,$$

$$\mu_2 = \frac{2}{3} \ln(g^{-1} r_j B_j) = \frac{2}{3} \ln(\mu_2 M c_{\xi_v}^{-\frac{3}{2}}) + c_2\beta,$$

$$\xi_v = \ln(\beta r_j B_j / r_j A_j) = \ln(\xi_v^2 M^2 c_{\xi_v}^{-1} A_j) + c_2\beta,$$

$$\mu_0^A = \frac{4}{5} \ln(g^{-\frac{1}{2}} f_j X_j) = \frac{4}{5} \ln(\mu_0^A M^2 c_{\xi_A}^{-\frac{1}{2}}) + c_2\beta,$$

$$\mu_2^A = \frac{4}{5} \ln(g^{-\frac{3}{2}} f_j Y_j) = \frac{4}{5} \ln(\mu_2^A M^2 c_{\xi_A}^{-\frac{3}{2}}) + c_2\beta,$$

$$\xi_A = \ln(\beta f_j X_j / f_j Y_j) = \ln(\xi_A^2 M^2 c_{\xi_A}^{-1} A_j) + c_2\beta.$$

In the continuum limit, $\mu_0$, $\mu_2$, $\mu_0^A$, $\mu_2^A$, $\xi_v$, $\xi_A$ and $M$ should be constants, which means the curves of $\xi_0$, $\xi_2$, .... against $\beta$ will be straight line with the expected slope in the scaling region.

In Figs. 1 and 2, we present the results for $\mu_0$ and $\mu_2$ against $\beta$ from the 2nd to 5th order. In Fig. 3, we also show the result for $\mu_0^A$ from the 2nd order through 4th order against $\beta$. All the curves show a nice exponential behavior and a clear trend towards convergence. In particular, the slope of $\mu_0$ is well consistent with the spectrum. For $\mu_2, \mu_0^A, \mu_2^A, \xi_v$ and $\xi_A$, we expect that when the order increases, their slopes would approach their correct theoretical values.

In conclusion, the method described above has proved to be very useful for calculating the vacuum and glueball wave functions and correlation lengths. The calculations in 3+1 dimensional QCD is in progress and the results will be reported elsewhere.
Figure 1. $\mu_0\beta$ as a function of $\beta$

Figure 2. $\mu_2\beta$ as a function of $\beta$

Figure 3. $\mu_A\beta$ as a function of $\beta$

REFERENCES


