Type-II parametric down conversion in the Wigner-function formalism. Entanglement and Bell’s inequalities

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Abstract

We continue the analysis of our previous articles which were devoted to type-I parametric down conversion, the extension to type-II being straightforward. We show that entanglement, in the Wigner representation, is just a correlation that involves both signals and vacuum fluctuations. An analysis of the detection process opens the way to a complete description of parametric down conversion in terms of pure Maxwell electromagnetic waves.
1 Introduction

The theory of parametric down conversion (PDC) was treated, in the Wigner formalism, in an earlier series of articles\cite{1, 2, 3}. There we showed that, provided one considers the zeropoint fluctuations of the vacuum to be real, the description of radiation is fully within Maxwell electromagnetic theory. Effectively, because the Wigner function maintains its positivity, we can say that quantization is just the addition of a zeropoint radiation, and there is no need for any further quantization of the light field. In the present article we show that the same result extends, without any difficulty, from the type-I PDC case to the type-II situation.

There seems to be a widespread reluctance to accept the reality of the vacuum fluctuations, in spite of the fact that they appear, quite naturally, in the Wigner function of the vacuum state. We remark that such fluctuations have been taken seriously, within a certain school of thought, throughout the entire history of the quantum theory, following the formulation of Max Planck, originating in 1911\cite{4}. Of course, it is true that, integrated over all frequencies, they give us a vacuum with infinite energy density; why then are all photographic plates not blackened instantaneously? But all photodetectors, including even our own eyes, are very selective, not only as regards the frequency, but even also the wave vectors, of the light components they analyze. This is especially the case with the detectors commonly used in PDC experiments. So, there is a noise to subtract, but it is not infinite!

In our previous articles we indicated how the noise subtraction is made, according to the Wigner formalism, and showed how this subtraction is related to the standard calculating device, of normal ordering, used in the Hilbert-space formalism. Here we extend this analysis, in an informal manner, showing that, if we take into account the fact that all detectors integrate the light intensity over a large time window, the process of light detection, like that of light propagation, may also be described entirely in terms of real waves and positive probabilities. We are then able to see that, in terms of a purely wave description, the highly problematic concept of “entangled-photon” states of the field loses all its mystery. Entangled photons are just correlated waves! The only reason this description has taken so long to mature is that the word “classical”, in reference to the light field, is restricted in its application to Glauber-classical states \cite{5}. A discussion of the difference between classical and nonclassical effects has been given in Ref.\cite{6}. The
states which are produced when a nonlinear crystal interacts with a coherent incoming beam, and, of course, simultaneously with the vacuum, may be described using classical Maxwell theory, but there is correlation of the outgoing light beams both above and below the zeropoint level.

2 General description of parametric down conversion in the Wigner representation

Type-I parametric down-conversion, in which the correlated signal and idler beams have the same polarization, has been recently studied within the framework of the Wigner function [1, 2, 3]. The formalism is almost identical in the case of type-II PDC, in which the correlated beams leaving the nonlinear crystal are orthogonally polarized. The process of type-II PDC can be formalized in analogy with the classical Hamiltonian of [1]

$$H = \sum_{j=0,e} \sum_{k} \hbar \omega_{jk} \alpha_{j}^{\dagger} \alpha_{j} + (i \hbar g' V \sum_{k,k'} f(k, k') \exp(-i \omega_{p} t) \alpha_{ok}^{\dagger} \alpha_{ek'}^{\dagger} + c.c.),$$

$$o(e)$$ refers to the ordinary (extraordinary) rays. We have taken the origin of the coordinate system at the center of the crystal, and treated the pump beam as an intense monochromatic plane wave represented by

$$V(r, t) = (V(t) \exp[i(k_{p} \cdot r - \omega_{p} t)] + c.c.) u,$$

$$u$$ being a unit vector perpendicular to $$k_{p}$$. As the coherence time of the laser is large in comparison with most of the times involved in the process, we shall consider $$V(t)$$ as a constant. $$g'$$ is a real coupling constant defined so that the product $$g'V$$ has dimensions of frequency, and $$f(k, k')$$ is a dimensionless symmetrical function of the wave vectors inside the crystal. This function, which is related to the function $$h(k, k')$$ introduced in equation (8) of reference [7], is different from zero only when the following matching condition is fulfilled

$$k_{p} \approx k + k'.$$

As is well known [7], there is in addition a matching condition for frequency that is fulfilled much more rigorously, namely,
\[ \omega'_p = \omega_k + \omega_{k'}. \tag{4} \]

On the other hand, \( \alpha_{ok}(\alpha_{ek'}) \) is the field amplitude for the mode with wave number \( k \) (\( k' \)) corresponding to the ordinary (extraordinary) field, which is represented as a sum of two mutually conjugate complex \( c \)-numbers

\[
E_j(r, t) = E_j^{(+)}(r, t) + E_j^{(-)}(r, t), \tag{5}
\]

\[
E_j^{(+)}(r, t) = i \sum_k \left( \frac{\hbar \omega_{jk}}{2L^3} \right)^{\frac{1}{2}} \epsilon_{jk} \alpha_{jk}(t) \exp(i k \cdot \mathbf{r}), \quad j = o, e, \tag{6}
\]

where \( L^3 \) is the normalization volume and \( \epsilon_{jk} \) is a polarization vector. Eqs. (5) and (6) correspond to the Heisenberg picture, where all time dependence goes in the field amplitudes \( \alpha_{jk}^*(t) \) and \( \alpha_{jk}(t) \). For a free field this dependence has the form

\[
\alpha_{jk}(t) = \alpha_{jk}(0) \exp(-i \omega_{jk} t), \tag{7}
\]

but for interacting fields it is complicated and contains all the dynamics of the process.

The evolution of the Wigner field amplitudes \( \alpha_{jk}(t) \) is directly given by the Hamilton (canonical) equations of motion taking \( \sqrt{\hbar} \alpha_{jk}(t) \) as coordinates and \( \sqrt{\hbar} \alpha_{jk}^*(t) \) as canonical momenta. For instance, we get for the extraordinary field amplitude \( \alpha_{ek} \):

\[
\dot{\alpha}_{ek} = -i \omega_{ek} \alpha_{ek} + g' V \sum_{k'} f(k, k') \exp(-i \omega_{p} t) \alpha_{ok'}^*, \tag{8}
\]

and a similar expression holds for the ordinary field amplitude by exchanging index \( e \) with \( o \).

In order to calculate \( \alpha_{ek}(t) \) for all \( t \) we shall take into account that the amplitude \( \alpha_{ek}(t) \) evolves as a free-field mode before entering the crystal and after coming out. We shall integrate (8) from \( t = -\Delta t \) to \( t = 0 \), where \( \Delta t \) is the time taken for the radiation to cross the crystal. The initial condition is \( \alpha_{ek}(-\Delta t) = \alpha_{ek}^{(vac)}(-\Delta t) \), where \( \alpha_{ek}^{(vac)}(-\Delta t) \) is the field amplitude of the mode \( k \) in the incoming vacuum field.

To second order in the coupling constant \( g' \), that is taking the second term of the right side of (8) as a perturbation and retaining terms up to
order $g'^2$, we get (putting $g'\Delta t \equiv g$)

$$
\alpha_{ek}(0) = \alpha_{ek}^{(vac)}(0) + gV \sum_{k'} f(k, k') u\left[\frac{\Delta t}{2}(\omega_p - \omega_{ek} - \omega_{ek'})\right] \alpha_{ek'}^{(vac)}(0) 
+ g^2|V|^2 \sum_{k'} \sum_{k''} f(k, k') f^*(k', k'') u\left[\frac{\Delta t}{2}(\omega_{ek'} + \omega_{ek''} - \omega_p)\right] 
\times u\left[\frac{\Delta t}{2}(\omega_{ek''} - \omega_{ek})\right] \alpha_{ek''}^{(vac)}(0) ; \ g|V| \ll 1, \ (9)
$$

where

$$
u(x) = \frac{\sin x}{x} e^{\text{exp}(ix)}, \ (10)$$

Equation (3) implies $k'' \approx k$ in the second order contribution to (9).

In the derivation of (9) we have taken into account that

$$
\alpha_{ek}^{(vac)}(0) = \alpha_{ek}^{(vac)}(-\Delta t) e^{\text{exp}(-i\omega_{ek} \Delta t)}, \ 
\alpha_{ek}^{*(vac)}(0) = \alpha_{ek}^{*(vac)}(-\Delta t) e^{\text{exp}(i\omega_{ek} \Delta t)}. \ (11)
$$

After $t = 0$, $\alpha_{ek}(t)$ evolves as a free-field mode

$$
\alpha_{ek}(t) = \alpha_{ek}(0) e^{\text{exp}(-i\omega_{ek} t)}. \ (12)
$$

Now, let us consider two narrow correlated beams called “ordinary” and “extraordinary”, with average frequencies $\omega_o$, $\omega_e$, and wave vectors $k_o$, $k_e$ respectively, fulfilling the matching conditions

$$
\omega_o + \omega_e = \omega_p ; \ k_o + k_e = k_p. \ (13)
$$

Both light beams contain frequencies within a range between $\omega_{j\text{min}}$ and $\omega_{j\text{max}}$ ($j = o, e$), wave vectors whose transverse components are limited by a small upper value, and orthogonal polarization vectors which are practically independent of the wave vectors, that is

$$
\omega_{j\text{min}} < \omega_{jk} < \omega_{j\text{max}}, \ |k^{tr}| \ll \frac{\omega_{j\text{min}}}{c}, \ 
\epsilon_{ek} \equiv \epsilon_e , \ \epsilon_{ok} \equiv \epsilon_o ; \ \epsilon_e \cdot \epsilon_o = 0. \ (14)
$$
We also substitute slowly varying fields $F_j^{(\pm)}(r, t)$ ($F_j^{(-)}(r, t)$) for the amplitudes $E_j^{(\pm)}(r, t)$ ($E_j^{(-)}(r, t)$), the relation between them being

$$
F_j^{(\pm)}(r, t) \equiv \exp(i\omega_j t)E_j^{(\pm)}(r, t)
$$

where $\omega_j$ is some appropriately chosen average frequency midway between $\omega_{\text{min}}$ and $\omega_{\text{max}}$ (see(14)). The square brackets in the summation symbol indicates that the sum is restricted to the set of $k$ pertaining to the $j$-beam.

It is easy to obtain the amplitude $F_j^{(\pm)}(r_B, t)$ in terms of the amplitude $F_j^{(\pm)}(r_A, t)$ at another point of the light beam [1]. We find

$$
F_j^{(\pm)}(r_B, t) = F_j^{(\pm)}(r_A, t - \frac{r_{AB}}{c})\exp(i\omega_j \frac{r_{AB}}{c}) \quad (j = o, e), \quad (16)
$$

where $r_{AB} = r_B - r_A$ and $r_{AB} = |r_{AB}|$.

From expressions (9), and (15) we obtain

$$
F_j^{(\pm)}(r, t) = \left[ i \sum_{[k]_j} \left( \frac{\hbar \omega_k}{2L^3} \right)^{\frac{1}{2}} \alpha_k(0)\exp(ik \cdot r)\exp[i(\omega_j - \omega_j k)t] \right] \epsilon_j \quad (j = o, e), \quad (17)
$$

and similarly for $F_j^{(\pm)}(r_B, t)$.

Here $F_j^{(\pm)}(r, t)$ and $F_j^{(\pm)}(r_B, t)$ are the incoming vacuum fields and $F_j^{(\pm)}(r_A, t)$ the outgoing extraordinary (ordinary) fields — see Fig. 1. We have

$$
F_j^{(\pm)}(r, t) = i \sum_{[k]_e} \left( \frac{\hbar \omega_{ek}}{2L^3} \right)^{\frac{1}{2}} \exp(ik \cdot r)\exp[(\omega_e - \omega_{ek})t]\alpha_{ek}^{(\pm)}(0), \quad (19)
$$

and similarly for $F_j^{(\pm)}(r_B, t)$. $G$ and $J$ are linear operators which are defined as:

$$
G F_j^{(\pm)}(r, t) = i \sum_{[k]_e} \left( \frac{\hbar \omega_{ek}}{2L^3} \right)^{\frac{1}{2}} \exp(ik \cdot r)\exp[i(\omega_e - \omega_{ek})t]\beta_k, \quad (20)
$$
\[
\beta_k = \sum_{[k']_e} f(k, k') u[\frac{\Delta t}{2} (\omega_p - \omega_{ek} - \omega_{ok'})] \alpha_{ok'}^\ast (0),
\]  

(21)

and

\[
J F^{(+)(\text{vac})}_e(r, t) = i \sum_{[k]_e} \left( \frac{\hbar \omega_{ek}}{2L^3} \right)^{\frac{1}{2}} \exp(i(k \cdot r)) \exp[i(\omega_e - \omega_{ek}) t] \gamma_k,
\]

(22)

with

\[
\gamma_k = \sum_{[k']_o} \sum_{[k'']_o} f(k, k') f^\ast(k', k'') u[\frac{\Delta t}{2} (\omega_{ok'} + \omega_{ek''} - \omega_p)] \times
\]

\[
\times u[\frac{\Delta t}{2} (\omega_{ek''} - \omega_{ek})] \alpha_{ek''}^\ast (0).
\]

(23)

From (17) we see that the outgoing extraordinary beam, to order \(g^2\), consists of three parts: i) A zeropoint radiation with amplitude \(F^{(+)(\text{vac})}_e\), which passes through the crystal without any change; ii) a radiation produced by the nonlinear interaction (mediated by the crystal) between the laser beam, with amplitude \(V\), and the zeropoint radiation, with amplitude \(F^{(-)(\text{vac})}_o\), entering the crystal in the direction of the ordinary beam; iii) one part which just modifies a little (to order \(g^2\)) the amplitude \(F^{(+)(\text{vac})}_e\). The ordinary beam is constituted in a similar manner.

Now, let us consider the correlation properties of the fields, which are identical to the ones calculated in a previous work [2]:

a) Autocorrelations.

Taking the extraordinary field \(F^{(+)}_e(r, t) = F^{(+)}_e(r, t)e_e\) at a point \(r\) and times \(t\) and \(t'\), we have:

\[
\langle F^{(+)}_e(r, t) F^{(-)}_e(r, t') \rangle - \langle F^{(+)(\text{vac})}_e(r, t) F^{(-)(\text{vac})}_e(r, t') \rangle =
\]

\[
2g^2 |V|^2 \langle GF^{(-)(\text{vac})}_o(r, t) G^* F^{(+)(\text{vac})}_o(r, t') \rangle \equiv g^2 |V|^2 \mu_e(t' - t),
\]

\[
\langle F^{(+)}_e(r, t) F^{(+)}_e(r, t') \rangle = 0.
\]

(24)

Here “\(\langle \rangle\)” means an average using the Wigner function in the vacuum state as probability density. \(\mu_e(t - t')\) is a correlation function which goes to zero when \(|t' - t|\) is greater than the correlation time of the extraordinary beam, \(\tau_e\). Similar expressions hold for the ordinary field by exchanging the indices “\(e\)” and “\(o\)”.

7
b) Crosscorrelations.

Taking the extraordinary ($F_e^{(+)}(r, t) = F_e^{(+)}(r, t)\epsilon_e$) and ordinary ($F_o^{(+)}(r, t) = F_o^{(+)}(r, t)\epsilon_o$) fields at the center of the crystal $r = r' = 0$ and times $t$ and $t'$, we have:

$$\langle F_e^{(+)}(0, t)F_o^{(+)}(0, t') \rangle = gV\nu(t' - t).$$

$$\langle F_e^{(+)}(0, t)F_o^{(-)}(0, t') \rangle = \langle F_e^{(-)}(0, t)F_o^{(+)}(0, t') \rangle = 0. \quad (25)$$

Here $\nu(t' - t)$ is a function which vanishes when $|t' - t|$ is greater than the coherence time between the extraordinary and ordinary beams. From (25) it is possible to derive all crosscorrelations at different points $r \neq r'$ by using (16).

Finally, the quantum theory of detection in the Wigner representation gives us the following results for single and joint detection probabilities:

a) Single probability.

The following result is a general expression for calculating single probabilities per unit time in the Wigner representation:

$$P_1(r_1, t) \propto \langle I(r_1, t) - I_0(r_1) \rangle, \quad (26)$$

where $I(r_1, t) = |E^{(+)}(r_1, t)|^2 = |F^{(+)}(r_1, t)|^2$, and $I_0(r_1)$ is the intensity of the vacuum field at the position of the detector.

b) Joint probability.

It can be proved that in PDC experiments

$$P_{12}(r_1, t; r_2, t + \tau) \propto \langle \{I(r_1, t) - I_0(r_1)\}\{I(r_2, t + \tau) - I_0(r_2)\} \rangle. \quad (27)$$

By taking into account that the Wigner fields amplitudes are Gaussian processes, and neglecting fourth order terms in $g$, we have [3]:

$$P_{ab}(r_1, t; r_2, t + \tau) \propto \sum_\lambda \sum_{\lambda'} |(F^{(+)}_\lambda(r_1, t)F^{(+)}_{\lambda'}(r_2, t + \tau))|^2, \quad (28)$$

where $\lambda$ and $\lambda'$ are polarization indices.
3 Tests of Bell’s inequalities using polarization correlation

Most experiments to test Bell’s inequalities with nonlinear crystals performed hitherto have used type-I parametric down-conversion in which the two correlated beams have the same polarization. In [1, 2, 3], experiments of this kind were analyzed in the Wigner function formalism. However, more recent experiments, using type-II phase matching, provide a more direct way to generate “entangled-photon” states. Type-II experiments are themselves of two types. In the first, that is collinear type-II PDC, the crystal is oriented so that the ordinary and extraordinary radiation cones are mutually tangent in the direction of the pumping beam. To date, nearly all type-II experiments have used collinear phase matching [8]. On the other hand [9], in noncollinear type-II phase matching, the two cones intersect along two directions, and this gives rise to an entangled state in the polarization (see Fig.2). It has been claimed that such a source produces true entangled states, capable of violating Bell’s inequalities.

The experimental outline is shown in Fig.3. The two beams “1” and “2”, in which the ordinary and extraordinary cones intersect, are selected and sent to two polarizers \( P_1 \) and \( P_2 \) oriented at angles \( \phi_1 \) and \( \phi_2 \) with respect to the polarization of the extraordinary ray. Coincidence rates were measured as functions of angles \( \phi_1 \) and \( \phi_2 \). In [9] additional optical devices, that is half- and quarter-wave plates, were used in order to produce four different Bell states, but we shall confine our analysis to just one of these states, namely the one which uses no additional devices.

Let us see how the entangled state is represented in the Wigner formalism. The two beams, coming out of the crystal along the directions where the ordinary and extraordinary cones intersect, are given by

\[
\begin{align*}
F^{(+)}_1(0,t) &= F^{(+)}_e(0,t)i + F^{(+)}_o(0,t)j, \\
F^{(+)}_2(0,t) &= F^{(+)}_{e'}(0,t)i' + F^{(+)}_{o'}(0,t)j',
\end{align*}
\]

(29)

where \( i, i' \) represent the polarizations of the extraordinary beams and \( j, j' \) the polarizations of the ordinary beams. The essential point is that the extraordinary component, \( F^{(+)}_e \), of the first ray and the ordinary component, \( F^{(+)}_o \), of the second ray are conjugated, and therefore correlated. Similarly, \( F^{(+)}_o \) and \( F^{(+)}_{e'} \) are correlated, but \( F^{(+)}_e (F^{(+)}_o) \) is uncorrelated to \( F^{(+)}_{e'} (F^{(+)}_o) \).
When a polarizer oriented at angle $\phi_1$ to the horizontal is placed in front of the detector $D_1$, the field at $D_1$ (placed at $r_1$) at time $t$ is

$$\mathbf{F}^{(+)}(r_1, t) = [\mathbf{F}_1^{(+)}(r_1, t) \cdot (\cos \phi_1 \mathbf{i} + \sin \phi_1 \mathbf{j})](\cos \phi_1 \mathbf{i} + \sin \phi_1 \mathbf{j})$$

$$= \exp[i\omega (d/c)] [F_{e}^{(+)}(0, t - \frac{d}{c}) \cos \phi_1 + F_{o}^{(+)}(0, t - \frac{d}{c}) \sin \phi_1]$$

$$(\cos \phi_1 \mathbf{i} + \sin \phi_1 \mathbf{j})$$ (30)

Here we recall that the action of a polarizer is to project the electric field vector on the polarization direction.

In the same way, we write for the field at the detector $D_2$ (placed at $r_2$) at time $t + \tau$

$$\mathbf{F}^{(+)}(r_2, t + \tau) = [\mathbf{F}_2^{(+)}(r_2, t + \tau) \cdot (\cos \phi_2 \mathbf{i}' + \sin \phi_2 \mathbf{j}')](\cos \phi_2 \mathbf{i}' + \sin \phi_2 \mathbf{j}')$$

$$= \exp[i\omega (d/c)] [F_{e}^{(+)}(0, t + \tau - \frac{d}{c}) \cos \phi_2 + F_{o}^{(+)}(0, t + \tau - \frac{d}{c}) \sin \phi_2]$$

$$(\cos \phi_2 \mathbf{i}' + \sin \phi_2 \mathbf{j}')$$ (31)

In order to calculate the joint probability we combine Eqs. (28), (30), and (31), and take into account the correlation properties of the fields given by (24) and (25). After some easy algebra and an integration of $P_{12}(\tau)$ over the detection window, we obtain the coincidence probability

$$P_{12} = K \sin^2(\phi_2 + \phi_1),$$ (32)

$K$ being a constant. This expression is similar to the one obtained in [9], and corresponds to 100% contrast. This type of correlation is usually claimed to violate a Bell inequality (but see the discussion section). In the actual experiment a violation of the inequality by 100 standard deviations is reported.

4 The detection problem in the Wigner representation

The detection probability in quantum optics is usually written in terms of the normally ordered expression
\[ P \propto \int_0^{\Delta t} \langle \hat{E}^{(-)}(t) \hat{E}^{(+)}(t) \rangle dt, \]  
(33)

where \( \hat{E}^{(+)}(t) \) is the Heisenberg operator of the electric field at the detector, \( \hat{E}^{(-)}(t) \) its hermitian conjugate and \( \Delta t \) is the detection time window. For simplicity we consider a point detector and, therefore, ignore the standard volume integral in (33).

When we pass to the Wigner representation, the normally ordered expression should be written in terms of a symmetrically ordered expression minus a commutator. Then we may replace (see Eq.(26)) the Heisenberg operators by random wave amplitudes and, after some rather trivial algebra, we get

\[ P \propto \int_0^{\Delta t} \langle I(t) - I_0 \rangle dt, \]  
(34)

where \( I(t) = |E(t)|^2 \) is the intensity of the field arriving at the detector and \( I_0 \) is a constant corresponding to the average intensity of the zeropoint, i.e. the intensity that would arrive at the detector if all light sources, such as lasers, were switched off.

If the Wigner function is positive definite, \( I(t) \) may be interpreted as a stochastic process, which makes possible a wavelike interpretation of the propagation of light. This is the case in all experiments involving parametric down conversion. However, there remains a problem for a wavelike interpretation of the detection process, because \( I(t) - I_0 \) is not positive-definite. This means we cannot assume that \( I(t) - I_0 \) is proportional to a detection probability. Nevertheless, it is easy to show that the average \( \langle I(t) - I_0 \rangle \) is nonnegative-definite. Now we consider the usual case where \( I(t) \) is a stationary stochastic process. It then follows (see Ref.[10], page 49) that it is ergodic. We could consider substituting time averages for ensemble averages, thereby obtaining

\[ P \propto \langle I(t) - I_0 \rangle = \lim_{T \to \infty} \frac{1}{T} \int_0^T [I(t) - I_0] dt. \]  
(35)

This shows that, for every member of the ensemble of stochastic wave amplitudes (except for a subensemble of zero measure), the time-averaged photodetection probability precisely equals the ensemble averaged one \( \langle I(t) - I_0 \rangle \).

In practice we do not have a time-average but something which is almost equivalent, namely an integration over the detection window \( \Delta t \). A typical
detection window lasts more than one nanosecond, so that the dimensionless quantity $\omega \Delta t$, where $\omega$ is the frequency of visible light, is of the order of $10^7$. It is true that taking the limit $T \to \infty$ is not the same as taking $T \approx 10^7$ (in dimensionless units), but the difference must be small. Since the right hand side of (35) is nonnegative-definite, this means that the finite-time average

$$\frac{1}{\Delta t} \int_0^{\Delta t} [I(t) - I_0] dt$$

(36)
takes negative values only with a very small probability. Now let us modify the standard detection probability of Eq.(34) so that

$$P \propto \left\{ \int_0^{\Delta t} [I(t) - I_0] dt \right\}_+,$$

(37)

where the notation $\{ \}_+$ indicates that we replace the contents of the brackets by zero if their value is negative. Then (37) will predict rather more photocounts than the standard quantum detection theory, but it seems not unreasonable to assume that these additional counts correspond to a part of the dark rate at the detector. As a matter of fact, the quantum theory of detection, leading to (33), involves first order perturbation theory. Therefore we may assume that quantum theory also predicts some dark background in photocounters when higher-order processes are taken into account, because the detector may be activated by vacuum fluctuations. A detailed study of such a background would have to enter fully into the electronic band structure of the photodetector material[11]; a treatment based on the photoelectric effect for single atoms is clearly inadequate.

5 Discussion

In the preceding sections we have outlined a theory of both the propagation and detection of light which is consistently local realist in the sense defined, for example, by Clauser and Shimony[12]. In fact, eq.(27) has the standard form introduced by Bell in his definition of local hidden variables models, in particular

$$P_{12} = \langle P_1(\lambda, \phi_1)P_2(\lambda, \phi_2) \rangle = \int \rho(\lambda)P_1(\lambda, \phi_1)P_2(\lambda, \phi_2)d\lambda,$$

(38)
where the amplitudes $\alpha_{j,k}$ involved in $I(r,t)$, via $E(r,t)$ (see eq.(6) and the line after eq.(26)) play the role of the hidden variables $\lambda$. On the other hand, our theory is also in almost perfect one-to-one correspondence with the standard Hilbert-space theory, the only difference being the modification in the detection probability which we have proposed in Eq.(37). Apart from a small dark rate, which is, in any case, a feature of all real experimental situations, this theory gives singles and coincidence rates agreeing with the standard theory. Therefore we have here an apparent contradiction. On the one hand, a Bell inequality is violated according to Ref.[9], which implies that no local hidden variables model exists for the experiment. On the other hand we have an explicit local hidden variables model, namely the quantum model in the Wigner representation with the modification of (37). What is the resolution of this apparent contradiction? The reason is that the Bell inequality which is actually violated in the experiment is not a genuine Bell inequality derived from the assumptions of realism and locality alone (like inequality (4) of Clauser and Horne [13]), but a homogeneous inequality involving additional assumptions (like inequality (11) of Clauser and Horne). One of these assumptions, rather than local realism, is what is violated in our LHV model.

The present authors have been insisting for many years now [14] that these auxiliary assumptions are not only unreasonably restrictive but also incorrect. We believe that the results reported in the present article vindicate our point of view.

As is well known a genuine Bell inequality cannot be tested in experiments with visible light, due to the low efficiency of the detectors presently available. The conventional wisdom is that this is just a technical problem that will be solved in the near future. However, the existence of an LHV model for the quoted experiment [9] which does not rest upon the low efficiency of the detectors, but on the existence of some unavoidable amount of dark rate (see Eq.(37)) shows that any future reliable test of LHV theories should involve detectors having both high efficiency and low dark rate.

Now, we turn to the interpretation of entanglement in the Wigner representation. We saw (see Eqs.(25) and (29)) that “photon entanglement” is just correlation between two light beams. Then, what is the difference between “classical” correlation and entanglement? In quantum optics what is usually called “classical” is light having a (Glauber) P-representation which is positive definite, and “classical correlation” usually means a correlation between
the P-distribution functions of the two light beams. Here we see that “local realist theories” is a class much bigger than standard “classical theories”. In particular, the quantum model following from the Wigner representation interpreted as a (positive) probability distribution allows for correlations much stronger than “classical correlations”. As in our functions $F_{e}^{(+)}$ and $F_{o}^{(+)}$ of Eq. (25) they involve correlation of the zeropoint part of the electromagnetic field, and this is “entanglement”.

Of course, the obvious objection to these arguments is that they cannot be extended to cases where the Wigner function is negative. Elsewhere we have conjectured that this never happens in actual experiments. We refer to our publication [5] for details.

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References


Figure captions

Fig. 1. The process of parametric down conversion.

Fig. 2. Polarization entanglement in noncollinear type-II down conversion.

Fig. 3. Tests of Bell’s inequalities using noncollinear type-II down conversion.

Figures have not yet been included, as the authors have to master the appropriate technology first. We hope it will not take very long!