Neutrino Physics in a Muon Collider

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Abstract. The muon collider is expected to produce a high intensity neutrino beam which is an admixture of either $\nu_e + \bar{\nu}_e$ or $\bar{\nu}_\mu + \nu_\mu$ which can be directed to underground detectors far away from the source. It will not only allow a probe of the $\nu_e - \nu_\mu$ as well as $\nu_\mu - \nu_e$ oscillations in a range of mixing angle and $\Delta m^2$ not probed heretofore but it will also provide information about the mixing angle $\theta_{13}$, for a wide range of $\Delta m^2_{\nu_e\nu_e}$, from $10^{-4} \text{eV}^2$ to $10^{-1} \text{eV}^2$ which cannot be obtained from any other existing or proposed machine. One can also search for violations of Lorentz invariance and deviations from equivalence principle for neutrinos at a level which is three to four orders of magnitude more sensitive than possible at the moment. This will test for instance some unorthodox suggestions to understand both solar and atmospheric neutrinos using a single mass difference between the $\nu_e$ and $\nu_\mu$. It can also test various proposed models neutrino masses and mixings to understand existing neutrino data.

I. INTRODUCTION

Neutrino physics is now going through a very exciting period. For the first time in its history, there are several hints for a nonvanishing mass for at least two of three known neutrinos which look very promising and credible. They come (i) from the observations of the solar neutrinos in various experiments and their disagreement with the predictions of the standard solar model [1]: the earlier experiments from Homestake, Kamiokande, SAGE and GALLEX [2] and the most recent high statistics confirmation of these results by the super-Kamiokande experiment [3] and (ii) from the observations of the atmospheric neutrinos by several previous experiments [4,5] and the most recent confirmation of the earlier results by the super-Kamiokande [3] collaboration. Then there is the result from the Los Alamos liquid scintillation neutrino detector (LSND) which gives the first laboratory evidence for the oscillation of both $\bar{\nu}_\mu \to \bar{\nu}_e$ [6] as well as $\nu_\mu \to \nu_e$ type [7].

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Once the neutrinos have mass, they can mix with each other leading to a rich variety of new physical phenomena, which in turn may lead to insight into the kind of new physics responsible for such mixings and masses.

While at the moment detailed fits to all the above data considerably restrict the nature of the masses and mixings among the three neutrino species, they do not fix the complete neutrino mass texture. On top of this, there are ambiguities that open the neutrino oscillation interpretation of the solar neutrino data to question. Therefore it is crucial that other experiments are performed not only to confirm what is known but also to gain complete knowledge of this basic sector of the standard model.

Presently in planning and construction stage are several such experiments—MINOS and PALO VERDE to give two examples; several others which are beginning to provide this information are the CHORUS, NOMAD, CHOOZ. These are variously known as long base line and short base line experiments, which will either involve low energy electron neutrino beams from the reactors or high energy $\nu_\mu$ beams from the accelerators. None of them will have a high energy $\nu_e$ beams. While the disappearance of the $\nu_e$ in the reactor experiments one can get information about the mixing angles for a certain mass range, a large range of mixing angles and masses of theoretical interest remains unexplored at the moment. Specifically, information on the $\nu_e - \nu_\mu$ mixing angle at the moment is very poor. A similar remark also applies to the $\nu_\mu$ case where appearance experiments involving the $\nu_\mu$ beam provide knowledge of the mixing of $\nu_\mu$ with $\nu_e$ or $\nu_\tau$ (i.e. $\theta_{\mu e}$ or $\theta_{\mu \tau}$) for some mass range while leaving a considerable range of interest unexplored.

The goal of this article is to explore whether the neutrino beams from a muon collider can provide any useful information regarding the neutrino masses and mixings that are not already available (or will not be available once the above mentioned experiments are completed). In other words is there a neutrino physics justification for the muon collider?

The muon collider will be the first place where one can get an energetic beam of electron neutrinos. Therefore with a suitable long baseline experiment, one can probe the mixing angle $\theta_{e\mu}$ in a completely unexplored domain. Of course needless to say that the muon collider will also provide extensive information on $\theta_{e\tau}$ or $\theta_{\mu\tau}$ for very small mass difference squared as well and thus nicely complement the other experiments and may even extend the domain of the search.

Besides with the high flux neutrino beams which are supposed to result in a muon collider, one can also test the validity of several fundamental laws of physics such as Lorentz invariance, CPT theorem as well as equivalence principle and I will show that improvements by several orders of magnitude are possible with the muon colliders in these cases.

This article is organized as follows: in section 2, the implications of the solar, atmospheric and the LSND experiments for neutrino masses and mixings are briefly summarized; in section 3 a summary of the various popular
scenarios for neutrino masses and mixings are touched upon; in section 4, the neutrino mixings that can be probed by the muon collider is given using different experimental scenarios and in section 5, we discuss possible tests of the Lorentz invariance, equivalence principle and CPT theorem are considered.

II. INDICATIONS FOR NONZERO NEUTRINO MASS

II.a Solar neutrino deficit

We will assume the explanation of the solar neutrino deficit in terms of the oscillation between the \( \nu_e \) and \( \nu_x \) where \( x \) is another species of neutrino not necessarily of muon or tau type. The oscillation can be pure vacuum oscillation which requires a mass difference-squared \( \Delta m^2 \sim 10^{-10} \text{ eV}^2 \) and large mixing or it could be matter enhanced MSW [8] type in which case the neutrino mass differences and mixing angles fall into one of the following ranges [1],

\[
a) \text{Small} - \text{angle MSW, } \Delta m^2_{\text{ei}} \sim 5 \times 10^{-6} - 10^{-5} \text{eV}^2, \sin^2 2\theta_{\text{ei}} \sim 7 \times 10^{-3}, \\
b) \text{Large} - \text{angle MSW, } \Delta m^2_{\text{ei}} \sim 9 \times 10^{-6} \text{eV}^2, \sin^2 2\theta_{\text{ei}} \sim 0.6.
\]

If the solar neutrinos oscillate into sterile neutrinos, the MSW effect is different from the \( \nu_e \) to \( \nu_x \) case and the large angle solution is no more allowed. The above results are based on the approximation that only two of the neutrino species are involved in the oscillation.

II.b Atmospheric Neutrino Deficit

The atmospheric \( \nu_{\mu} \)'s and \( \nu_{\tau} \)'s arise from the decays of \( \pi \)'s and \( K \)'s and the subsequent decays of secondary muons produced in the final states of the \( \pi \) and \( K \) decays. In the underground experiments the \( \nu_{\mu} \) and \( \bar{\nu}_{\mu} \) produce muons and the \( \nu_e \) and \( \bar{\nu}_e \) lead to e\( ^{\pm} \). Observations of \( \mu^{\pm} \) and e\( ^{\pm} \) indicate a far lower value for \( \nu_{\mu} \) and \( \bar{\nu}_{\mu} \) than suggested by naïve counting arguments which imply that \( N(\nu_{\mu} + \bar{\nu}_{\mu}) = 2N(\nu_e + \bar{\nu}_e) \) [4]. The assumed oscillation in this case could a priori be between \( \nu_{\mu} \) to \( \nu_e \) or \( \nu_{\mu} \) to \( \nu_{\tau} \). However, a recent CHOOZ collaboration result rules out the Kamiokande allowed \( \nu_{\mu} \) to \( \nu_e \) mass-squared mixing region [9]. Thus we can assume that the oscillation of \( \nu_{\mu} \) to \( \nu_{\tau} \) provides the explanation of the atmospheric neutrino results. Fits to both the sub-GeV and multi-GeV Kamiokande data require that [5]

\[
\Delta m^2_{\mu\tau} \approx 0.025 \text{ to } 0.005 \text{ eV}^2, \sin^2 2\theta_{\mu\tau} \approx 0.6 \text{ to } 1.
\]

The most recent Super-Kamiokande data has confirmed the deficit in both the sub-GeV and the multi-GeV data. Also there is now evidence for zenith
angle dependence in the multi-Gev data which according to preliminary analysis [3] would indicate a similar mass range as above for maximal mixing angle.

II. c Results from the LSND experiment

The LSND collaboration first reported seeing indications for $\bar{\nu}_\mu$ to $\bar{\nu}_e$ oscillation using the liquid scintillation detector at Los Alamos in 1996 [6]. Their results in conjunction with the negative results by the E776 group and the Bugey reactor data imply a mass difference squared between the $\nu_e$ and the $\nu_\mu$ lying between

$$0.27 \ eV^2 \leq \Delta m^2 \leq 10 \ eV^2$$

with a mixing angle $\theta_{\mu e} \sim 0.05 - 0.1$. The region for $\Delta m^2$ above 10 eV$^2$ has been ruled out both by the recent CCFR data and the NOMAD data [10]. In a recent paper [7], LSND group has reported preliminary evidence for the $\nu_\mu - \nu_e$ oscillation with mass difference squares and mixings in the similar range as above.

II. d Hot dark matter of the universe

There is increasing evidence that more than 90% of the mass in the universe must be detectable so far only by its gravitational effects. This dark matter is likely to be a mix of $\sim 20\%$ of particles which were relativistic at the time of freeze-out from equilibrium in the early universe (hot dark matter) and $\sim 70\%$ of particles which were non-relativistic (cold dark matter). Such a mixture gives a very good fit to all available cosmological data [11]. This interpretation is however by no means unique and it has been claimed that an equally good fit to the power spectrum can be obtained by a pure CDM model with a tilted spectrum [12].

If however, the mixed dark matter picture is adopted, a very plausible candidate for hot dark matter is one or more species of neutrinos with total mass of $\Sigma_i m_{\nu_i} = 93 h^2 F_H \Omega = 4.8$ eV, if $h = 0.5$ (the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$), $F_H = 0.2$ (the fraction of dark matter which is hot), and $\Omega = 1$ (the ratio of density of the universe to closure density).

It is usually assumed that the $\nu_e$ would supply the hot dark matter. However, if the atmospheric $\nu_\mu$ deficit is due to $\nu_\mu \rightarrow \nu_e$, the $\nu_e$ alone cannot be the hot dark matter, since the $\nu_\mu$ and $\nu_e$ need to be closer to each other in mass. It is interesting that instead of a single $\sim 4.8$ eV neutrino, sharing that $\sim 4.8$ eV between two or among three neutrino species provides a better fit to the universe structure and particularly a better understanding of the variation of matter density with distance scale [13].

II. e: Neutrinoless double beta decay constraints
Finally, let us note the very stringent constraints on neutrino masses now implied by the neutrinoless double beta decay searches. The Heidelberg-Moscow $^{36}\text{Ge}$ experiment [14] has provided the most stringent upper limits on the effective Majorana mass of the neutrino: $< m_\nu > \leq 0.47 \text{ eV}$ where $< m_\nu > = \Sigma_i U_{ei}^2 m_{\nu i}$. This is beginning to put very strong constraints on model building. For instance, it has recently been noted [15] that the CHOOZ and Bugey [16] results already imply that $| < m > | \leq 3 \times 10^{-2} \text{ eV}$. Thus any signal for neutrinoless double with $< m >$ above 0.1 eV would be evidence against a hierarchical neutrino mass pattern. Similarly, one can infer from the LSND data that one must have $< m > \geq 4 \times 10^{-3} \text{ eV}$ assuming that $\theta_{\tau}\nu$ is small (or at least it does not precisely cancel this contribution). Thus high precision double beta searches are extremely important to a complete understanding of the neutrino masses.

III. NEUTRINO MASS TEXTURES IMPLIED BY DATA

In order to discuss the implications of the above data for the neutrino mass pattern, we will assume that all the neutrinos are Majorana particles, since it is easier to understand the smallness of Majorana masses of neutrinos within the framework of grand unified theories. We will then proceed by assuming that the solar and the atmospheric neutrino data are the two core items that appear as the most secure indications of neutrino oscillation and study their significance for neutrino masses. We will then add the HDM and the LSND results and see their implications.

: Including only solar and the atmospheric data:

Since we only have constraints on the mass difference squares from the solar and the atmospheric data, we can have a “staircase” pattern with $m_{\nu e} \ll m_{\nu\mu} \approx \sqrt{\Delta m^2_{\text{solar}}}$ and $m_{\nu\mu} \ll m_{\nu\tau} \approx \sqrt{\Delta m^2_{\text{atmos}}}$ or a degenerate pattern [17,18] where all masses are nearly equal with appropriate mass differences. The latter is mandatory if one wants to explain the HDM picture of the universe. As far as the mixing angles go, the $\theta_{\mu\tau}$ is always maximal (i.e. near $\pi/4$) whereas $\theta_{e\mu}$ is either maximal (for vacuum oscillation or large angle MSW) or few percent (for small angle MSW). Several theoretical suggestions are now given.

III.a Model A: Minimal SO(10) inspired

This model is based on a minimal SO(10) model where the Yukawa sector involves so few parameters that all neutrino masses and mixing angles can be predicted [19]. Using the type II see-saw formula in the minimal SUSY
SO(10) model it is possible to get solutions that can accommodate both the solar as well as the atmospheric neutrino puzzles. Specifically, it predicts: $m_{\nu_{\mu}} \simeq 1.7 \times 10^{-5} \text{ eV}$ and $m_{\nu_{e}} \simeq 5.9 \times 10^{-2} \text{ eV}$ and the neutrino mixing matrix given by:

$$
\begin{pmatrix}
-0.995 & 0.052 & 0.080 \\
0.070 & 0.792 & 0.603 \\
0.032 & -0.608 & 0.793 \\
\end{pmatrix}
$$

(4)

III.b Model B: Maximal mixing scheme

In this scheme, the mixing matrix has the form:

$$
U_{\nu} = \frac{1}{\sqrt{3}} \begin{pmatrix}
1 & 1 & 1 \\
1 & \omega & \omega^2 \\
1 & \omega^2 & \omega \\
\end{pmatrix}
$$

where $\omega = e^{2\pi i/3}$. This scheme becomes essential if the neutrino masses are degenerate and if the limits on the neutrino mass from neutrinoless double beta decay keeps going down since the leading term in the $<m_{\nu_{e}}>$ cancels for this choice of mixing angles. This mixing angle pattern can be shown to fit both the solar and atmospheric neutrino data if one assumes vacuum oscillation solution to the solar neutrino problem.

III.c Model C: Democratic mixing among neutrinos

This model for mixings is based on the idea that the neutrino mass matrix may satisfy an approximate permutation symmetry among the three generations and also can fit the solar and atmospheric neutrino data and has a mixing matrix of the following form:

$$
U_{\nu} = \begin{pmatrix}
\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
\end{pmatrix}
$$

(6)

This model also can support a degenerate mass pattern consistent with neutrinoless double beta decay. The difference between this and the maximal mixing pattern is that the $\theta_{\nu_{e}}$ values are very different.

There are several other schemes based on attractive theoretical assumptions that lead to three neutrino mixing patterns that can fit both solar and atmospheric data. We do not enter into discussions of those ideas due to lack of space but they are generally similar to the SO(10) case discussed above.

: Accommodating solar, atmospheric and the LSND data:
The LSND result have two important implications for our discussion: first, their result implies oscillation from $\bar{\nu}_\mu$ to $\bar{\nu}_e$ i.e. unlike the solar and atmospheric data, the final state is not a matter of speculation but observation; secondly the $\Delta m^2_{\nu\mu}$ that fits data is between 0.2 eV$^2$ to about 10 eV$^2$, which is very different from the ranges derived from simple interpretations of the data as noted above. Before the latest results from super-Kamiokande experiment came out, two interesting neutrino mass schemes were proposed which seemed in accord (though rather marginally) with the previous Kamiokande data. The basic idea in these papers was the following: three experiments are sensitive to three mass difference squares; however with three neutrinos there are only two possible $\Delta m^2$'s. Therefore a three neutrino scheme can only fit data if two of the experimentally determined $\Delta m^2$'s turn out to be equal. The two models described below essentially exploit these two possibilities.

**III.d Model D: Cardall-Fuller scheme**

This scheme [25] assumes that $\Delta m^2_{\nu\mu} \simeq \Delta m^2_{\text{atm}}$ and that the $\nu_\mu - \nu_e$ oscillation observed at Los Alamos is an indirect oscillation [19] which proceeds as $\nu_e$ to $\nu_\tau$ to $\nu_\mu$. To accomodate the LSND results in this picture assumes the LSND $\Delta m^2$ to be around 0.3 eV$^2$. Since the solar neutrino puzzle requires that $\Delta m^2_{\nu\mu} \simeq 10^{-3}$ eV$^2$, this scenario implies that we must have $\Delta m^2_{\nu\tau}$ be $\approx 0.3$ eV$^2$, so that the LSND neutrino oscillation frequency is determined by $\nu_e-\nu_\tau$ mass difference. Secondly, for the amplitude of indirect oscillation to be compatible with observations, the $\nu_e-\nu_\tau$ mixing angle should be nonnegligible (say $\approx 0.1-0.2$). The main problem for this scenario comes from the atmospheric neutrino data, since the original analysis of the Kamiokande sub-GeV and the multi-GeV data by the Kamiokande group excludes $\Delta m^2 \geq 0.1$ eV$^2$ at 90% confidence level (c.l.). The analysis of the atmospheric neutrino data from Super-Kamiokande will therefore provide crucial test of this model. Preliminary analysis of the super-Kamiokande data (which has a clear evidence for zenith angle dependence) appears to contradict this scenario. Furthermore, if we want to fit the HDM picture into this model, one must have $m_\nu_\tau \simeq 1.6$ eV. While at its face this value may be in conflict with the neutrinoless double beta decay results [14], one can hide under the uncertainties of nuclear matrix element calculations which typically could be as much as a factor of 2-3. As the precision in $\beta\beta_0$ search improves further (say to the level of 0.1 eV), nuclear matrix element uncertainties cannot be invoked to save the model anymore.

**III.e Model E: Acker-Pakvasa scheme**

The second three neutrino mass texture [26] also uses indirect oscillation to explain the LSND data but makes the assumption that $\Delta m^2_{\text{solar}} \simeq \Delta m^2_{\text{atm}}$ and assumed that the atmospheric neutrino oscillation involves $\nu_\mu$ to $\nu_e$ oscillation, which looks implausible in view of the latest CHOOZ data. In any case,
they choose $\Delta m^2_{\mu\mu} \simeq 10^{-3}$ eV$^2$, $\Delta m^2_{ee} \simeq \Delta m^2_{\mu\tau} \simeq 1 - 2$ eV$^2$. It is easy to see that in this case the general three neutrino oscillation formula for $P_{ee}$ becomes energy independent if $L$ is chosen to correspond to the distance of the earth from the Sun. It was shown in Ref. [26] that if one reduces the $^8$B production in the center of the Sun, one can fit all solar neutrino observations despite the energy independence of the oscillations. It has been pointed out that already in the present data, there is evidence for energy dependence [27] disfavoring this scheme. Therefore this can also be tested by the Super-Kamiokande observations.

To complete this model, we give a typical mixing matrix that characterizes this model:

$$U_\mu = \begin{pmatrix} .700 & .700 & .14 \\ - .714 & .689 & .124 \\ .010 & -.187 & .982 \end{pmatrix} \quad (7)$$

Note the large value of $\Theta_{e\tau}$.

**III.f Model F: The case for a sterile neutrino**

The case for a sterile neutrino is made clear by noting the difficulty of fitting the solar, atmospheric and the LSND data with three neutrinos as exemplified by the models D and E. The main obstacle, as we saw, comes from the conflict between the LSND data and the MSW resolution of the solar neutrino data.

The general picture for the case of sterile neutrino is as follows [17, 28, 29]: the solar neutrino puzzle is explained by the $\nu_e - \nu_s$ oscillation; atmospheric neutrino data would be explained by the $\nu_\mu - \nu_s$ oscillation. The LSND data would set the overall scale for the masses of $\nu_\mu$ and $\nu_\tau$ (which are nearly degenerate) and if this scale is around 2 to 3 eV as is allowed by the data [6], then the $\nu_{\mu,\tau}$ would constitute the hot dark matter of the universe. The mass matrix in this case would be in the basis $(\nu_s, \nu_e, \nu_{\mu}, \nu_\tau)$,

$$M = \begin{pmatrix} \mu_1 & \mu_3 & 0 & 0 \\ \mu_3 & \mu_2 & \epsilon & 0 \\ 0 & \epsilon & m & \delta/2 \\ 0 & 0 & \delta/2 & m + \delta \end{pmatrix} \quad (8)$$

Solar neutrino data requires $\mu_2 \ll \mu_1 \simeq 10^{-3}$ eV and $\mu_3 \simeq 0.05\mu_1$. The $\epsilon$ term is responsible for the $\nu_e - \nu_\mu$ oscillation. Clearly the crucial test of the sterile neutrino scenario will come when SNO collaboration obtains their results for neutral current scattering of solar neutrinos. One would expect that $\Phi_{CC} = \Phi_{NC}$ if the $\nu_e$ oscillation to $\nu_s$ is responsible for the solar neutrino deficit. There should be no signal in $\beta\beta_{0v}$ search. Precision measurement of the energy distribution in charged current scattering of solar neutrinos at Super-Kamiokande can also shed light on this issue.
IV. MUON COLLIDER FOR STUDYING THE NEUTRINO MASSES AND MIXINGS

The muon collider is expected to produce a high luminosity beam of neutrinos which is an admixture of either $\nu_{\mu} + \bar{\nu}_e$ or their antiparticles. Thus in some sense this is a “controlled atmospheric neutrino” beam. The energy of the neutrinos is expected to range from 10 GeV to 100 GeV. In our discussion we will entertain the possibility of two kinds of long base line experiments [30] with beam directed to either Gran Sasso or Soudan mine with distances respectively of $\sim 10^4$ or 750 kilometers. Recall that the oscillation formula

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2 \frac{1.27 \Delta m^2 L}{E}$$

where $E$ is in GeV and $L$ is in kilometers. With $7.5 \times 10^{30} \mu^2$ per year, one can have of the order of $10^{20}$ neutrinos/year [30]. Geer has calculated the charge current event rate for such particles at a 10 kiloton detector located at Gran Sasso as well as at Soudan. He finds that one can expect thousands of charged current events per year.

To discuss its utility in studying neutrino masses and mixings, note that for a 10 GeV neutrino beam and a distance of $10^4$ kilometers, one could probe $\Delta m^2$ down to $10^{-5}$ eV$^2$ for maximal mixing if we take 10 events per year to get a signal and for $\Delta m^2 \geq 10^{-3}$ eV$^2$, one could probe mixings $\sin^2 2\theta$ down to $10^{-4}$ or so. Thus, one can not only explore $\theta_{e\mu}$ in a totally unexplored region of parameters but also considerably extend our knowledge of the $\theta_{e\mu}$ as well as $\theta_{\mu\tau}$ into a domain further than what MINOS or COSMOS can accomplish. As a comparison, note that at present, $\theta_{e\tau}$ has an upper bound of about .14 from the Fermilab experiment E531 for $\Delta m^2 \geq 10$ eV$^2$, which is a very weak bound compared to the other two mixing angles. The main reason being that there does not exist any accelerator source of high energy $\nu_e$’s and muon collider will be the first one to provide one such source.

IV. TESTING LORENTZ AND CPT INVARIANCE AND EQUIVALENCE PRINCIPLE

Lorentz invariance, CPT invariance (which is a consequence of Lorentz invariance and locality in Quantum Field theories) and the principle of general covariance are some of the fundamental pillars on which the present day theoretical physics rests. While few would doubt that there is any deviation from these principles, science has to be based on experimentally tested ideas. It is therefore important to look for ways to test the validity of these principles. In order to make the tests quantitative, a framework that has some parameters that characterize the departures from the exactness of these principles is
useful. Such frameworks have recently been discussed and I summarize them below and point out how a muon collider can be useful.

**IV.a Lorentz invariance**

It was pointed out recently by Coleman and Glashow [31] that one way to parameterize a departure from Lorentz invariance for massless particles such is to write

\[ E_i = p(c + \delta c_i) \]  

(10)

Applying this to neutrinos, one gets for the energy difference between two eigenstates into which the weak eigenstate resolves as \( E_1 - E_2 = E(\delta c_1 - \delta c_2) \equiv E\delta v \). One can then write the oscillation probability of say \( \nu_e \) to \( \nu_\mu \) to be

\[ P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2 \frac{\delta v E L}{2} \]  

(11)

The energy dependence of the \( P(\nu_e \rightarrow \nu_\mu) \) in Eq. 11 is clearly very different from the case of mass oscillation where it goes like \( L/E \). Therefore longer the base line and higher the energy, the more precise the test of Lorentz invariance. Present limits on the \( \delta v \) from various oscillation experiments is \( \delta v \leq 10^{-21} \). In this case we must choose the neutrino energy from the muon collider to be as high as possible. Again taking \( E \approx 100 \text{ GeV} \) and \( L = 10^4 \text{ Km} \), this limit can be improved to \( \delta v \leq 10^{-26} \) which is an improvement of some five orders of magnitude.

**IV.b CPT for neutrinos**

A simple CPT violating combination of oscillation probabilities for the \( \nu_e - \nu_\mu \) system is given by \( P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) \) as is very easily checked. Note that as mentioned before the neutrino beam in a muon collider consists of \( \nu_\mu \) and \( \bar{\nu}_e \). Let us assume that \( N_\mu = N_{\bar{\nu}_e} \) (although the energy spectra in general will be different) for simplicity. Then without CPT violation but with \( \nu_e - \nu_\mu \) oscillation, one would expect in the detector \( N_{\nu_-} = N_{\nu_+} \). Thus any deviation from this equality would be a test of CPT violation. This result is independent of any specific underlying model for CPT violation.

**IV.c Violations of equivalence principle**

It has been pointed out by Halprin and Leung [32] that violations of equivalence principle can also lead to neutrino oscillation phenomena. To see this, let us parameterize the metric as

\[ \text{Metric} = g_{a\beta} + 2\gamma_i \phi \delta_{a\beta} \]  

(12)
where $\phi$ is the gravitational potential. The second term is absent in Einstein’s theory and characterizes the departure from the equivalence principle. The energy-momentum relation now looks as follows:

$$E^2(1 + 2\gamma_j\phi) = p^2(1 - 2\gamma_j\phi)$$  \hspace{1cm} (13)

This can be cast in the language discussed in connection with violation of Lorentz invariance identifying $\delta v = 2(\gamma_1 - \gamma_2)\phi$. Translating our earlier discussion then we can conclude that in a muon collider, one can probe $2\Delta\gamma\phi$ down to the level of $10^{-26}$ as before. Since this experiment will be done in the solar system, the value of $\phi \simeq 10^{-6}$, it will test for violation of equivalence principle down to the level of $10^{-20}$. Note the present long range force experiments test this principle down to the level of $10^{-12}$ or so.

More importantly, Halprin et al have made the unconventional suggestion that perhaps one could use this phenomenon to explain solar and atmospheric neutrino puzzle by setting $2\Delta\gamma\phi \simeq 10^{-21}$ using only $\nu_\mu - \nu_e$ oscillation. A muon collider could therefore provide a clean test of this hypothesis.

In conclusion, we find that the neutrino beams from the muon collider can provide extremely useful insight into the world of neutrino masses and mixings, specifically it can probe the $\nu_e - \nu_\tau$ mixing angle in a domain of parameters that is beyond the range of any proposed experiment. This will allow us to test several three neutrino mixing schemes such as the maximal mixing scheme and the SO(10) scheme. Muon collider can also extend the domain of validity of some of the fundamental laws governing the physical phenomena such as the equivalence principle, CPT theorem and Lorentz invariance.

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