The Statistics of the BATSE Spectral Features

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Abstract. The absence of a BATSE line detection during the mission’s first six years has led to a statistical analysis of the occurrence of lines in the BATSE database; this statistical analysis will still be relevant if lines are detected. We review our methodology, and present new simulations of line detectability as a function of the line parameters. We also discuss the calculation of the number of “trials” in the BATSE database, which is necessary for our line detection criteria.

INTRODUCTION

Whether spectral lines exist in the BATSE bursts is one of the most pressing issues in burst spectroscopy since no BATSE detections have been announced thus far [1,2]. The BATSE spectroscopy team has been attacking this issue on many fronts: searching for lines [3], analyzing the capabilities of BATSE’s spectroscopy detectors (SDs), checking that these detectors are functioning correctly [4], and studying the statistics of the detections and nondetections by BATSE and previous missions [5]. Here we describe our advances in these statistical studies.

LINE STATISTICS FORMALISM

The line statistics methodology is built on a hierarchy of probabilities based on the probability $p_i$ of detecting a line in a spectrum [6]. In this discussion we use Roman subscripts to indicate spectra and Greek subscripts to denote bursts. This detection probability $p_i$ is primarily a function of the signal-to-noise ratio (SNR) of the continuum and the angle between the detector normal and the burst. The probability is calculated through simulations.
The probability $p_\alpha$ of detecting a line in a burst is a weighted sum of the probabilities $p_i$ of detecting the line in each of the $N(N + 1)/2$ possible consecutive spectra formed from the $N$ spectra accumulated during the burst [7]. The weighting for each $p_i$ depends on a model of how the line may occur within the burst (e.g., whether lines are likely to persist a long or short time). These probabilities are specific to a given line type as parameterized by the energy centroid, line width and equivalent width.

Clearly, this methodology requires information not only about the line detections but also about all spectra in which lines were not detected. Such extensive data are available for BATSE and Ginga. Not coincidentally, Ginga reported the best documented line detections. We have assembled a database of the necessary BATSE information [7], and we are collaborating with the Ginga team in deriving the data for their detectors, as well as in calculating the detection probabilities for the Ginga burst detector. Fenimore et al. [8] calculated preliminary detection probabilities for a Ginga line detection.

The probabilities $p_\alpha$ of detecting a line in each of a given mission's bursts are then combined into the probability of the observed pattern of detections and nondetections for that mission. These probabilities are calculated under certain assumptions (if only that the detectors are modeled correctly), and the resulting probability is the likelihood for these assumptions.

These likelihoods are used for various measures of the consistency between BATSE and Ginga [5]. For example, we developed a Bayesian comparison of the consistency hypothesis (“lines exist and both BATSE and Ginga are modeled correctly”) to various alternative hypotheses that explain the apparent discrepancy (e.g., “BATSE is unable to detect lines”). We have also developed a number of standard “frequentist” (as opposed to Bayesian) consistency measures such as the probability that if there are two detections, both would be in the Ginga data. Using a number of approximations [7], we find that currently BATSE and Ginga are just marginally consistent; specifically, assuming that lines exist and BATSE and Ginga are understood correctly, the probability that Ginga would have the two line detections and BATSE none is of order a few percent. If lines exist, they are present in only a few percent of all bursts. These results are preliminary because of the many approximations used (e.g., for the Ginga detection probabilities).

**LINE SIMULATIONS**

Our previous detectability studies only treated line sets with the parameters of the reported Ginga detections [6]. However, the Ginga lines may have been drawn from a distribution of lines with different energy centroids, equivalent widths and intrinsic widths. In addition the computerized search is identifying emission line candidates [3]. Therefore, we ran a large number of simulations of both absorption and emission lines with different line and observational
parameters. The simulations used a photon model consisting of a continuum and a spectral line. The continuum was the canonical GRB functional form [9] with $\alpha = -1$, $\beta = -2$ and $E_0 = 300$ keV. We used a multiplicative factor for the absorption lines and an additive Gaussian line for the emission lines. These lines were characterized by the line centroid $E_{\text{cen}}$ and the equivalent width $\Delta E$. The intrinsic line width is assumed to be less than the instrumental resolution.

The resulting model photon spectrum was convolved with the response matrix for a given burst angle $\theta$, and 200 realizations were created by adding Gaussian noise appropriate to the count spectrum and a representative background spectrum. These realizations were then fitted with continuum and continuum+line models, and the significance of the line evaluated through the value of $\Delta \chi^2$ for 3 line parameters (centroid, intrinsic width and equivalent width). The strength of the continuum was measured by a normed SNR calculated over the 25–35 keV band; the SNR was varied by changing the accumulation time. The detection probabilities are the fraction of these 200 simulations whose significance exceeds a specific threshold, here $p(\Delta \chi^2)$ of $10^{-4}$ and $10^{-5}$. Simulations were done for different values of the line centroid $E_{\text{cen}}$, the equivalent width $\Delta E$, the burst angle $\theta$, and the low energy cutoff $E_{\text{low}}$. Figures 1 and 2 give examples of the dependencies on $E_{\text{cen}}$ and $\Delta E$. In these figures the y axes are the normed SNR at which there is a 50% probability that the line would satisfy the significance threshold.

**THE NUMBER OF TRIALS**

The significance criterion must be set so that there is a small probability of a false positive for the entire database. The F-test or the maximum likelihood ratio test provide the probability that a given line feature is a fluctuation, but they do not include the number of “trials,” possibilities of obtaining that fluctuation. For example, a line-like fluctuation could occur at a variety of energies with different apparent line widths. Thus, while a given fluctuation may be rare, if the number of trials is very large, then it might be probable that such a fluctuation will occur somewhere within the burst database.

The issue is how to calculate the number of trials. The Bayesian formalism for a feature’s significance provides a conceptual framework. The Bayesian “odds ratio,” which compares the probabilities for the continuum+line and continuum only models, includes a factor that can be identified as the inverse of the number of trials contributed by the line parameters [10]. This “Occam’s Razor” factor suggests that the number of trials a parameter contributes is $\Delta A/\sqrt{2\pi\sigma_A}$, where $\Delta A$ is the range of possible values and $\sigma_A$ is the parameter’s uncertainty. This expression is strictly valid only in the absence of any correlations among the parameters (e.g., when the covariance matrices are diagonal). This dependence on the parameter uncertainty is reasonable because parameter values separated by less than the uncertainty are
essentially indistinguishable; also the uncertainty decreases and the number of trials increases as the spectrum’s SNR increases, as expected.

Because the F-test or maximum likelihood ratio test are frequentist tests and we derive the number of trials from a Bayesian expression, it may be simpler to adopt the Bayesian methodology completely. Nonetheless, because the parameter uncertainty varies across the spectrum, between spectra and from burst to burst, accounting for the number of trials results in a complicated multiple integral over the various parameters.

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FIGURE 1. Detectability as a function of the centroid energy $E_{\text{cen}}$. The upper curves of each pair are for a significance threshold of $10^{-5}$, and the lower for $10^{-4}$. The $y$ axes present the normed SNR at which there is a 50% probability that the line would satisfy the significance threshold. The curves are identified by letters: A—$\Delta E = 2.5$ keV, $\theta = 30^\circ$, $E_{\text{low}} = 10$ keV; B—$\Delta E = 5$ keV, $\theta = 30^\circ$, $E_{\text{low}} = 10$ keV; C—$\Delta E = 7.5$ keV, $\theta = 30^\circ$, $E_{\text{low}} = 10$ keV; and D—$\Delta E = 5$ keV, $\theta = 10^\circ$, $E_{\text{low}} = 10$ keV. The dip in the absorption line curves and the peak in the emission line curves around 30 keV results from the line moving through the 25–35 keV band in which the SNR is measured.
REFERENCES


FIGURE 2. Detectability as a function of equivalent width. For every parameter set there are 2 lines: the upper for a significance of $10^{-5}$, and the lower for $10^{-4}$. The curves are identified by letters: A—$E_{\text{cen}} = 20$ keV, $\theta = 20^\circ$, $E_{\text{low}} = 10$ keV; B—$E_{\text{cen}} = 60$ keV, $\theta = 30^\circ$, $E_{\text{low}} = 30$ keV; C—$E_{\text{cen}} = 40$ keV, $\theta = 30^\circ$, $E_{\text{low}} = 20$ keV; D—$E_{\text{cen}} = 30$ keV, $\theta = 30^\circ$, $E_{\text{low}} = 10$ keV; and E—$E_{\text{cen}} = 40$ keV, $\theta = 0^\circ$, $E_{\text{low}} = 10$ keV.