Cosmological Moduli Problem in Gauge-mediated Supersymmetry Breaking Theories

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Abstract

A generic class of string theories predicts the existence of light moduli fields, and they are expected to have masses $m_\phi$ comparable to the gravitino mass $m_{3/2}$ which is in a range of $10^{-2}\text{keV} - 1\text{GeV}$ in gauge-mediated supersymmetry breaking theories. Such light fields with weak interactions suppressed by the Planck scale can not avoid some stringent cosmological constraints, that is, they suffer from ‘cosmological moduli problems’. We show that all the gravitino mass region $10^{-2}\text{keV} \lesssim m_{3/2} \lesssim 1\text{GeV}$ is excluded by the constraints even if we incooperate a late-time mini-inflation (thermal inflation). However, a modification of the original thermal inflation model enables the region $10^{-2}\text{keV} \lesssim m_{3/2} \lesssim 500\text{keV}$ to survive the constraints. It is also stressed that the moduli can be dark matter in our universe for the mass region $10^{-2}\text{keV} \lesssim m_\phi \lesssim 100\text{keV}$.
I. INTRODUCTION

Supergravity theories, which describe the low energy dynamics of string theories, gener-
ically have a large number of flat directions in their field spaces [1]. We call the scalar
fields corresponding to these flat directions moduli fields, or moduli, simply. Moduli \( \phi \) are
expected to take their values of the order of the Planck scale \( M_{Pl} \simeq 1.2 \times 10^{19} \text{GeV} \)
in the very early universe, because it is the only scale appearing in supergravity actions. At
the later epoch in the universe’s evolution supersymmetry (SUSY) is spontaneously broken.
SUSY- breaking effects may lift the flat potential for moduli through some non-perturbative
dynamics and generate their masses \( m_\phi \) comparable to the gravitino mass : \( m_\phi \sim m_{3/2} \) [2].

Moduli fields are generally accompanied by different kinds of cosmological problems
depending on the values of their masses. These problems are divided into two classes dis-
criminated by if or not the moduli decay takes place in the early universe. That is, if it does
the radiation produced by the moduli decay may conflict with some cosmological observa-
tions, and even if it does not a tremendous amount of moduli energy density itself causes
disaster. Let us make a crude estimation of moduli lifetime and determine its dependence
on \( m_\phi \). From dimensional analysis the decay width of the moduli is roughly

\[
\Gamma_\phi \simeq N \frac{m_\phi^3}{M_{Pl}^2},
\]

where \( N \) denotes the number of decay channels. The lifetime is given by

\[
\tau_\phi = \Gamma_\phi^{-1} \simeq 1 \times 10^{20} \text{ sec} \, N^{-1} \left( \frac{m_\phi}{10 \text{MeV}} \right)^{-3}.
\]

In conventional hidden sector models, the gravitino masses \( m_{3/2} \) are about 1TeV, so
from eq.(2) we see that the moduli lifetime is much shorter than the age of the present
universe \( \sim 3 \times 10^{17} \text{sec} \) for \( m_\phi \sim m_{3/2} \). Therefore, in these models one must worry about
whether or not the radiation produced by the moduli decay spoils the scenario of big bang
nucleosynthesis. The reheating temperature derived from the width (1) is

\[
T_R \sim \sqrt{M_{Pl} \Gamma_\phi} \simeq 0.3 \text{MeV} \, N^{1/2} \left( \frac{m_\phi}{10 \text{TeV}} \right)^{3/2}.
\]

In order for the big bang nucleosynthesis to work, high enough reheating temperature is
needed \( (T_R \gtrsim 10 \text{MeV}) \), and this requirement constrains the moduli mass to be lower bounded
: \( m_\phi \gtrsim \mathcal{O}(10) \text{ TeV} \) [3].

However, this relatively large moduli mass \( m_\phi(\sim m_{3/2}) \gtrsim 10 \text{TeV} \) is realized only in a
specific class of hidden sector models. Thus, one is faced with a difficulty that the process of
nucleosynthesis does not proceed efficiently in a generic class of hidden sector models [6,2].

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1It might appear that a large amount of entropy production by the moduli decay predicts an
extremely smaller value of the baryon-to-entropy ratio than the one observed today even for \( m_\phi \gtrsim 10 \text{ TeV} \), but the Affleck-Dine mechanism for baryogenesis [4] naturally explains the present value
[5].
Lyth and Stewart proposed a mechanism so-called thermal inflation [7] which solves this problem by diluting extensively the cosmic energy density of the moduli and consequently decreasing the number of photons produced by the moduli decay sufficiently so as not to upset the nucleosynthesis.

Gauge-mediated SUSY-breaking models [8], to which we pay our attention especially in this paper, have the gravitino mass in a range of $10^{-2}$ keV-1 GeV. As can be seen from eq.(2), in the mass range $10^{-2}\text{keV} \lesssim m_\phi \lesssim 100\text{ MeV}$ the moduli do not decay sufficiently fast and are left in the present universe. Generically, moduli fields are expected to take their vacuum-expectation values (vev’s) of the order $M_{Pl}$, which is so large that the moduli energy density easily exceeds the critical density of the present universe. Although the moduli lifetime (2) tells us that a class of these models with $100\text{ MeV} \lesssim m_\phi \lesssim 1\text{ GeV}$ does not suffer from the moduli problem of their too much amount of energy density, a constraint from the cosmic $\gamma$-ray backgrounds is crucial in this mass region [9,10].

Our main task in this paper is to apply the thermal inflation mechanism to gauge-mediated SUSY-breaking models, and argue the possibility that these models could pass the above cosmological constraints. We assume $m_\phi \simeq m_{3/2}$ throughout this paper.

Our analysis consists of two parts. In the first part (sections 2 and 3) we adopt a thermal inflation model proposed by Lyth and Stewart [7]. In this model there is a pseudo Nambu-Goldstone boson (called R-axion) arising from a spontaneous breakdown of an accidental R symmetry. We show that the mass of R-axion is always much smaller than the flaton mass in the gauge-mediated SUSY-breaking models. Thus, the flaton decays mainly into these R-axions and as a consequence the dilution of the moduli’s energy density becomes much milder. We show that there is no parameter region surviving the cosmological constraints in this type of thermal inflation model.

We modify the above thermal inflation model, in the second part (section 4), by adding a small explicit breaking term of the R symmetry to suppress the R-axion decay of the flaton. We find that a parameter region for the moduli mass, $10^{-2}\text{keV} \lesssim m_\phi \lesssim 500\text{keV}$ [10], survives the cosmological constraints. It should be noted here that the Affleck-Dine baryogenesis does work for these gravitino mass region [11]. We also stress that the moduli themselves could be the dark matter in the present universe for the region, $10^{-2}\text{keV} \lesssim m_\phi \lesssim 100\text{ keV}$. This may be a very crucial observation, since there is no dark matter candidate beside the moduli themselves because of the substantial dilution of any relic particle abundance by the late-time thermal inflation.

The last section is devoted to discussion and conclusions.

II. THERMAL INFLATION

In this section we review a thermal inflation model which was proposed by Lyth and Stewart [7]. Inevitable ingredients of the model are flaton fields, which are characterized by

\[^2\text{This attempt has been first proposed in Ref. [11].}\]

\[^3\text{Peccei-Quinn axion with high values of the decay constant } F_a \simeq 10^{15} - 10^{16} \text{ GeV may be another candidate for the dark matter [12]. See also Ref. [13].}\]
their ‘almost flat’ potentials and ‘large’ vacuum-expectation values (vev’s). Here, ‘almost
flat’ means that the roots of the second rank derivatives of potentials at their minima,
namely the masses of corresponding flaton particles, are of the order of SUSY-breaking scale
and the flaton fields have ‘large’ vev’s which are much larger than the scale. For simplicity,
let us concentrate on the case in which only one flaton field exists, even though more than
one flaton cases are also valuable to analyze.

In order to guarantee the flatness of potential, we assume that the flaton’s potential pos-
sesses some exact (or at least approximate) global symmetries. If not, nonrenormalizable
higher order interactions would induce a tadpole term through the SUSY-breaking effects.
One familiar example of such global symmetries is $U(1)^R$ symmetry, but it should be explicitly broken to some discrete symmetry, say $Z_n$, by a constant term $C$ in the superpotential
which is required to cancel the cosmological constant. Thus, in this paper we postulate that
the superpotential for the flaton exhibits $Z_{n+3}$ ($n \geq 1$) symmetry and takes a form

$$W = C + \sum_{k=1}^{\infty} \frac{\lambda_k}{(n+3)k} \frac{X^{(n+3)k}}{M_*^{(n+3)k-3}},$$

where $X$ is the flaton chiral superfield and $\lambda_k$ ($\lambda_1 = 1$) coupling constants. $M_*$ denotes the cut-off scale of the models we consider. As will be verified later the vev of the superpotential
(4) is dominated by the constant term $C$, and hence we take

$$|C| \simeq M_G^2 m_{3/2},$$

to cancel the vacuum energy. Here, $M_G$ is the reduced Planck scale, $M_G = M_{Pl}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV.

Then, the effective potential of the flaton $X$ at low energy scale is represented as

$$V_{eff}(X) \simeq V_0 - m_0^2 |X|^2 + \frac{n}{n+3} \frac{C}{M_G^2 M_*^2} (X^{n+3} + X^{*n+3}) + \frac{1}{M_*^{2n}} |X|^{2n+4},$$

where we have used the same letter $X$ for the flaton complex scalar field as for the corre-
sponding superfield, hoping that readers should not be confused. Note that the dynamics of
the flaton field $X$ is governed by the leading term in the superpotential (4) for $X \ll M_*$ and
we have neglected higher order terms in eq.(6) since the vev of the flaton is much smaller
than the cut-off scale $M_*$. $V_0$ is determined by the requirement that the cosmological con-
stant vanishes. The quadratic term in eq.(6) is induced by SUSY-breaking effects [14]

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4Even if one assumes an approximate $U(1)^R$ symmetry instead of $Z_{n+3}$, one reaches the same conclusion as in this paper, since the higher power terms ($k = 2, \cdots, \infty$) in the superpotential (4)
are practically negligible as seen below.

5In the original paper [7] the cut-off scale $M_*$ is taken to be at the Planck scale. Here, we regard it as a free parameter to make a general analysis.

6$V_0$ is at the tree-level given by
and we assume the mass squared at the origin $X \simeq 0$ to be negative and of the order of SUSY-breaking scale. The vev of the flaton is given by

$$\langle X \rangle \equiv M \simeq \left( \frac{1}{n+2} \right)^{\frac{1}{2(n+1)}} (m_0 M^*_n)^{\frac{1}{n+1}},$$

(8)

where we have neglected $\mathcal{O}(m_{3/2}/m_0)$ terms. (Remember that the formulae given below are also valid only up to $\mathcal{O}(m_{3/2}/m_0)$. See the paragraph following eq.(18).) Then $V_0$ is given by

$$V_0 \simeq \frac{n+1}{n+2} m_0^2 M^2.$$  

(9)

Here, reflecting the $\mathbb{Z}_{n+3}$ symmetry the degenerate minima cause a potential domain wall problem. We will come back this point in section 4.

The superpotential (4) has an approximate $U(1)_R$ symmetry if we neglect the higher order terms and then the imaginary part of $X$ becomes a pseudo-Goldstone boson called R-axion. The R-axion receives a mass from the constant term $C$ which breaks the $U(1)_R$ symmetry explicitly [15]. If we parameterize $X$ as

$$X = \left( \frac{1}{\sqrt{2}} \chi + M \right) \exp \left( \frac{ia}{\sqrt{2}M} \right),$$

(10)

with $\chi$ representing the real flaton field and $a$ the R-axion, then the flaton and the R-axion mass squared are estimated as

$$m^2_\chi \simeq 2(n+1)m_0^2,$$

(11)

$$m^2_a \simeq \frac{n(n+3)}{\sqrt{n+2}} m_0 m_{3/2} \simeq \frac{n(n+3)}{\sqrt{2(n+1)(n+2)}} m_\chi m_{3/2}. $$

(12)

The thermal inflation occurs if the flaton field does not sit at the true minimum of the potential but at the origin in the early universe. To realize this initial condition for the thermal inflation the flaton must have interactions with other fields in the thermal bath of the universe\(^8\). From the finite temperature effects, the effective potential in the early universe takes a form as

$$V_0 \simeq |F|^2 - \frac{3}{M^2_P} |C|^2,$$

(7)

where $F$ is the SUSY-breaking $F$ term.

\(^7\)Here we choose the phase of the constant term in eq.(5) as $C \simeq -M^2_P m_{3/2}$ so that the R-axion mass squared is positive.

\(^8\)For example, a Yukawa interaction as $W = gX\bar{\xi}\bar{\xi}$ is sufficient. The fields $\xi$ and $\bar{\xi}$ receive a mass $m_\xi \simeq gM$ when the flaton sits at the true minimum, but for the flaton field values near the origin the fields $\xi$ and $\bar{\xi}$ become light and could be in the thermal bath if they couple to the particles in the standard model and the temperature $T$ is larger than the mass $m_\xi$. 

5
\[ V_{\text{eff}}(X) \simeq V_0 + (cT^2 - m_0^2)|X|^2 - \frac{n}{n+3} \frac{m_0^{3/2}}{M_n^n} (X^{n+3} + X^{*n+3}) + \frac{1}{M_n^{2n}} |X|^{2n+4}, \quad (13) \]

where \( c \) is a constant of \( \mathcal{O}(1) \) and \( T \) is the temperature of the universe. Then, at high temperature \( T \gtrsim T_c \sim m_0 \) the flaton field sits near the origin and produces the vacuum energy \( V_0 \).

If the energy of the universe is dominated by the radiation just before the thermal inflation, the vacuum energy \( V_0 \) becomes comparable to the radiation energy at the cosmic temperature \( T \sim V_0^{1/4} \). Thus, for \( T_c \lessgtr T \lessgtr T_* \), the vacuum energy of the flaton field dominates the energy of the universe and a mini-inflation, i.e. thermal inflation takes place [7]. On the other hand, if the moduli oscillations dominate the energy of the universe before the thermal inflation, the temperature at the beginning of thermal inflation is estimated as \( T_* \sim (V_0^2/(m_\phi M_G))^{1/6} \). Here \( m_\phi \) are the moduli masses.

In considering the history of the universe after the thermal inflation, the flaton decay is crucial since it is most responsible for the entropy production. Here we shall list the relevant decay channels and compute quantitatively the decay rate for each channel which is needed to trace the physics following the thermal inflation epoch.

The only possible renormalizable interaction of the flaton with SUSY standard model particles is

\[ W = \lambda X H \bar{H}, \quad (14) \]

where \( H \) and \( \bar{H} \) are Higgs chiral supermultiplets. When the flaton field develops the vev, \( \langle X \rangle = M \), the Higgs multiplets acquire a mass \( \lambda M \), and for this to be at most the electroweak scale the coupling constant \( \lambda \) has to be set so small, \( \lambda \lesssim \mu_H/M \). Here \( \mu_H \) is the SUSY-invariant mass for the Higgs multiplets. If the flaton mass is large enough, the flaton decays into a pair of Higgs fields\(^9\). The decay width is represented as

\[ \Gamma_{\chi \to 2h} \simeq C_h \frac{m_\chi^3}{16\pi M_h^2}, \quad (15) \]

where \( C_h \) is a constant parameter satisfying \( C_h \lesssim \mathcal{O}(1) \), and we have ignored the masses of the Higgs fields.

Even if the above decay process would be kinematically forbidden, the flaton could decay into two photons through one-loop diagrams [10]\(^{11}\). The width of this decay channel is given by

\[ \text{Here, Higgs field denotes a Higgs boson or a Higgsino.} \]

\[ \text{For example, the coupling } C_h \text{ is } C_h = (\lambda M/m_\chi)^4 \text{ in the case the flaton decays into two Higgs bosons. And } C_h = (\lambda M/m_\chi)^2 \text{ for the case the flaton decays into two Higgsinos. Thus in this analysis we shall assume that the coupling } C_h \text{ is a free parameter of } C_h \lesssim \mathcal{O}(1), \text{ since } \lambda \lesssim \mu_H/M. \]

\[ \text{This decay process is induced by the Higgs fields loop diagram through the interaction (14) or by charged } \xi \text{ loop through the Yukawa interaction which is required in order that the flaton sits around the origin during the thermal inflation. If } \xi \text{ has a color charge, the decay into two gluons occurs and becomes a dominant decay for the flaton mass } m_\chi \geq \text{ a few GeV.} \]
\[ \Gamma_{\chi \to 2\gamma} \simeq \frac{1}{8\pi} \left( \frac{\alpha_{em}}{4\pi} \right)^2 \frac{m_\chi^3}{M^2}. \]

Through the above decay processes, the flaton energy is transferred to the radiation and reheats the universe.\(^{12}\)

One should note that the flaton can decays into two R-axions, if kinematically allowed. The decay rate is calculated as

\[ \Gamma_{\chi \to 2a} \simeq \frac{1}{64\pi} \frac{m_\chi^3}{M^2}. \]

Here, we have neglected the R-axion mass. The R-axions produced by this process successively decay into two photons similarly to the flaton decay and its decay rate is estimated as

\[ \Gamma_{a \to 2\gamma} \simeq \frac{1}{8\pi} \left( \frac{\alpha_{em}}{4\pi} \right)^2 \frac{m_a^3}{M^2}. \]

In gauge-mediated SUSY breaking theories the predicted gravitino mass range \((m_{3/2} \simeq 10^{-2}\text{keV}-1\text{GeV})\) indicates that the flaton decay into two R-axions is most likely allowed, since the flaton may obtain a mass of the order of the SUSY-breaking scale \((m_\chi \simeq 10\text{GeV}-1\text{TeV})\) and the R-axion has a mass \(m_a \sim \sqrt{m_{3/2}m_\chi}\) (see eqs.\((11)\) and \((12)\))\(^{13}\). Thus, in the following, we consider the case that the flaton can decay into standard model particles (Higgs fields or photons) and two R-axions. (For the case that the R-axion decay is forbidden, see section 4.)

Now we are ready to describe the history after the thermal inflation. Our aim here is to estimate the entropy produced by the flaton decay processes. At the temperature \(T \sim T_c\) the flaton field starts to roll down to the true minimum of the potential (6) and then oscillates around it. When the Hubble parameter becomes of the order of total width of the flaton, the flaton \(\chi\) decays into both standard model particles and R-axions. The energy of R-axions can not be transferred to the radiation at this time because it has only weak interaction suppressed by \(1/M\) with particles in the thermal bath. Then only the energy of the standard model particles is transferred to the radiation to reheat the temperature of the universe \(T_{R,SM}\) at the flaton decay epoch. The ratio of the entropy densities just before to after the flaton decay is estimated as

\[ \Delta_{SM} \simeq 1 + (1 - \epsilon_a)^4 \frac{V_0}{3(2\pi^2/45)g_s T_c^3 T_{R,SM}}, \]

where \(g_s\) is the effective number of degrees of freedom and \(\epsilon_a\) denotes the branching ratio of the flaton decay into two R-axions.

\(^{12}\)A possible decay into two pairs of bottom and antibottom quarks is strongly suppressed by phase volume effects.

\(^{13}\)In Ref. [7] hidden sector models of the SUSY breaking are considered where the gravitino has a mass \(m_{3/2} \simeq m_\phi \simeq 100\text{ GeV}-1\text{ TeV}\). In this case the R-axion has a mass of the order \(m_\phi\) (see eq.\((12)\)) and the flaton decay into R-axions may not be allowed.
Just after the flaton decay, the energy density of the R-axion per the entropy density is 
\[ \epsilon_a \left( \frac{V_0}{(2\pi^2/45) g^* T} \right) \frac{1}{\Delta_{SM}}. \]
The energy densities of the R-axion and radiation are both diluted by the expansion at the same rate \( R^{-4} \), where \( R \) is the scale factor of the universe. But after the R-axions become non-relativistic particles, their energy density \( \rho_a \) is diluted at \( R^{-3} \) and dominates over the energy of the radiation unless \( \epsilon_a \simeq 0 \).

The R-axion decay into two photons occurs at the Hubble parameter \( H \sim \Gamma_{a\rightarrow 2\gamma} \) and the universe is reheated again with the temperature \( T_R \). At this time, the R-axion decay increases the entropy by a factor
\[ \Delta_a \simeq 1 + \epsilon_a \left( \frac{4}{3} \frac{V_0}{(2\pi^2/45) g^* T^3 T R \Delta_{SM}} \right) \left( \frac{2m_a}{m_\chi} \right). \] (20)

Then, all the vacuum energy of the thermal inflaton (i.e. flaton) is eventually transferred to the radiation. The reheating temperature \( T_R \) is estimated from eq.(18) as
\[ T_R \simeq 1.1 \times 10^{-4} \frac{m^{3/2} M^{1/2}_{G}}{M}. \] (21)

And the thermal inflation increases the entropy of the universe by a factor
\[ \Delta = \Delta_{SM} \cdot \Delta_a \simeq 1 + (1 - \epsilon_a) \left( \frac{4}{3} \frac{V_0}{(2\pi^2/45) g^* T^3 T R_{,SM}} \right) + \epsilon_a \left( \frac{4}{3} \frac{V_0}{(2\pi^2/45) g^* T^3 T R} \right) \left( \frac{2m_a}{m_\chi} \right). \] (22)

As we have described the thermal inflation reproduces the entropy at the late-time of the universe’s evolution which dilutes substantially the cosmic energy densities of any relic particles such as moduli fields. The dilution factor is given by eq.(22) when the two R-axion decay of the flaton is allowed. The dilution factor in the other case will be given in section 4. We should note that the requirement \( T_R \gtrsim 10 \text{ MeV} \) in eq.(21) leads to the upper bound for \( M \) which justifies our ansatz in eq.(5).

III. MODULI PROBLEM IN GAUGE-MEDIATED SUSY BREAKING THEORIES

In this section, we consider the cosmological moduli problem in gauge-mediated SUSY breaking theories. The predicted gravitino mass \( m_{3/2} \approx 10^{-2} \text{ keV}–1 \text{ GeV} \) indicates that the string moduli fields may have lifetimes longer than the age of the universe. Then one clear problem arises that the energy densities of the moduli overclose the universe. Moreover, it has been pointed out recently that the contributions to the cosmic \( X(\gamma) \)-ray background from the moduli decays are very dangerous [9,10]. The only known possibility to solve these problems is the thermal inflation. Therefore, we examine whether the thermal inflation discussed in the previous section can solve these moduli problems.

A. The energy density of string moduli

First, we discuss the problem of the cosmic energy density of the moduli fields. To derive conservative cosmological constraints, we assume that at least one modulus field exists with a mass \( m_\phi \approx m_{3/2} \) and it has an initial value of the order of the gravity scale \( M_G \). (Here, we
have chosen vev of the modulus field at the origin). Generalization to the case that many moduli fields exist is straightforward.

Let us show by explicit calculation that the modulus energy density exceeds substantially the critical density of the present universe if the modulus is stable. When the expansion rate of the universe becomes of the order of the modulus mass, \( H \sim m_\phi \), the modulus field \( \phi \) starts to oscillate around the minimum of the potential with the initial amplitude \( \phi_0 \) of the order \( M_G \). At that time the energy density of the modulus coherent oscillation is

\[
\rho_\phi = \frac{1}{2} m_\phi^2 \phi_0^2,
\]

and the energy density of radiation is of the same order of \( \rho_\phi \). Thus, the temperature of the universe when the modulus starts to oscillate is estimated as

\[
T_\phi \simeq \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{M_G m_\phi},
\]

\[
\simeq 7.2 \times 10^8 \text{ GeV} \left( \frac{m_\phi}{1 \text{ GeV}} \right)^{1/2}.
\]

Here, we have assumed that the modulus coherent oscillation begins after reheating of the ordinary inflation completes, and that its reheating temperature is higher than \( T_\phi \). Then the energy density per the entropy density is given by

\[
\frac{\rho_\phi}{s} \simeq \frac{m_\phi^2 \phi_0^2 / 2}{(2\pi^2/45) g_* T_\phi^3},
\]

\[
\simeq 0.9 \times 10^8 \text{ GeV} \left( \frac{m_\phi}{1 \text{ GeV}} \right)^{1/2} \left( \frac{\phi_0}{M_G} \right)^2.
\]

This ratio takes a constant value until the present if no entropy is reproduced, because the densities of the energy and of the entropy are diluted at the same rate as \( R^{-3} \) as the scale factor \( R \) increases.

On the other hand, the critical density of the present universe is given by

\[
\frac{\rho_c}{s} = 3.6 \times 10^{-9} \ h^2 \text{ GeV},
\]

where \( h \) is the present Hubble parameter in units of 100 km/sec/Mpc. When we consider gauge mediated SUSY-breaking theories where the predicted modulus mass is \( m_\phi \simeq m_{3/2} \simeq 10^{-2} \text{ keV-1 GeV} \), one can see from eq.(27) that the energy density of the modulus coherent oscillation overcloses the universe if the modulus is stable until now. The thermal inflation increases the entropy of the universe by the factor \( \Delta \) as shown in eq.(22) and dilutes the energy density of the modulus given by eq.(26).

Now we adopt the thermal inflation model in the previous section and estimate the minimum value of the present energy density of the modulus field \( \phi \). The relevant dynamics

\[14\text{In some models of inflation, such as a chaotic or hybrid inflation, one may easily have the reheating temperature higher than }T_\phi \text{ in eq.(25) [16].}\]
is determined by two mass scales $m_0$, $M_*$ and the branching ratio $\epsilon_a$ for a given gravitino mass $m_{3/2} \simeq m_\phi$. Since $\Delta$ in eq.(22) takes its maximum at $\epsilon_a = 1$ as far as $m_a \ll m_{\chi}$, we put $\epsilon_a = 1$ to obtain the most efficient dilution factor $\Delta$. In the following analysis we take two free parameters $m_{\chi}$ and $T_R$ instead of $m_0$ and $M_*$, and search the minimum energy density of the modulus in the present universe.

The amount of the present energy density of modulus takes different forms depending on whether the modulus field begins to oscillate before the thermal inflation or after the end of that. For the moment we assume that the modulus coherent oscillation starts before the thermal inflation, i.e. $m_\phi > H_{TI}$. Here, $H_{TI} \simeq \sqrt{V_0}/(\sqrt{3}M_G)$ is the Hubble parameter during the thermal inflation. Then, from eq.(26) the present energy density of such a modulus (“big-bang” modulus) is

$$
\left( \frac{\rho_\phi}{s} \right)_{BB} \simeq \frac{m_\phi^2 \phi_0^2/2}{(2\pi^2/45)g_*T_\phi^3} \frac{1}{\Delta}.
$$

Furthermore, the modulus energy is reproduced after the thermal inflation. Because, during the thermal inflation, the modulus sits at the minimum of the potential which is shifted from its true vacuum by an amount of $\delta_\phi \sim (V_0/m_\phi^2M_G^2)\phi_0$ [7] and restarts to oscillate around the true minimum with an amplitude $\delta_\phi$ after the end of the thermal inflation. Then the present energy density of this “thermal-inflation” modulus is estimated as

$$
\left( \frac{\rho_\phi}{s} \right)_{TI} \simeq \frac{m_\phi^2(\delta_\phi)^2/2}{(2\pi^2/45)g_*T_\phi^3} \frac{1}{\Delta}.
$$

Therefore, the present total energy density of the modulus with mass $m_\phi > H_{TI}$ is [10]

$$
\left( \frac{\rho_\phi}{s} \right)_0 \simeq \max \left[ \left( \frac{\rho_\phi}{s} \right)_{BB}, \left( \frac{\rho_\phi}{s} \right)_{TI} \right] \simeq \left( \frac{\rho_\phi}{s} \right)_{BB},
$$

$$
\simeq 6.1 \times 10^8 \frac{(n+1)(n+2)^2}{n^2(n+3)^2} \left( \frac{T_c}{m_\chi} \right)^3 \left( \frac{\phi_0}{M_G} \right)^2 \frac{T_R^3}{M_G^{1/2}m_{3/2}^{3/2}}.
$$

Here, notice that it depends only on $T_R$ and not on $m_\chi$ since $T_c \sim m_\chi$. The reheating temperature should satisfy $T_R \gtrsim 10$ MeV to maintain the success of the big bang nucleosynthesis. Then if we choose the minimum value of $T_R \simeq 10$ MeV in eq.(31), we obtain the lowest value of the present modulus energy density for $T_c \simeq m_\chi$ and $\phi_0 \simeq M_G$,

$$
\left( \frac{\rho_\phi}{s} \right)_0 \gtrsim 4.0 \times 10^{-7} \text{ GeV} \left( \frac{n+1}{n^2(n+3)^2} \right)^{3/2} \left( \frac{m_{3/2}}{1 \text{ GeV}} \right)^{-3/2}.
$$

On the other hand, if the modulus mass is smaller than $H_{TI}$, then the modulus oscillation begins after the end of the thermal inflation with the amplitude $\phi_0 \sim M_G$. The entropy of the universe when the modulus starts to oscillate is estimated as

---

15When the flaton decay into Higgs fields is forbidden, we have $\epsilon_a \simeq 1$. When it is allowd, $\epsilon_a$ depends $C_h$ in eq.(15). However, we find that the dilution factor $\Delta$ takes the maximum value at $C_h \simeq 0$ which means $\epsilon_a \simeq 1$.  

10
\[ s \simeq \frac{2\pi^2}{45} g_s T_c^3 \frac{3M_G^2 m^2_\phi}{V_0}, \]  
where we have used the fact that the entropy is diluted by the expansion of the universe at the rate \( R^{-3} \). Then the present abundance of the modulus is

\[ \left( \frac{\rho_\phi}{s} \right)_0 \simeq \left( \frac{15}{4\pi^2 g_*} \right) \frac{\phi_0^2 V_0}{M_G^2 T_c^3} \frac{1}{n_1}, \]  

\[ \simeq \frac{1}{16} \frac{1}{\sqrt{n(n+3)}} \left( \frac{\phi_0}{M_G} \right)^2 \frac{m_\chi^{1/2} T_R}{m^{3/2}_{3/2}}. \]

In this case, the mass of flaton \( \chi \), (since \( m_\phi < H_{TI} \)) should satisfy

\[ m_\chi \gtrsim 3.5 \times 10^2 \frac{(n+2)^{1/2}(n+1)^{3/14}}{[n(n+3)]^{3/7}} M_G^{2/7} m^{1/7}_{3/2} T_R^{4/7}, \]  

and this gives the lower bound as

\[ \left( \frac{\rho_\phi}{s} \right)_0 \gtrsim 1.6 \text{ GeV} \frac{(n+2)^{1/2}(n+1)^{5/14}}{[n(n+3)]^{5/7}} \left( \frac{1 \text{ GeV}}{m^{3/2}_{3/2}} \right)^{3/7}, \]  

for the minimum value of the reheating temperature \( T_R \simeq 10 \text{ MeV} \) and \( \phi_0 \simeq M_G \). Here we have neglected the region where the gravitino mass is less than

\[ m_{3/2} \simeq 2.4 \times 10^6 \frac{1}{(n+1)^{1/10}[n(n+3)]^{5/5}} \frac{T_R^{8/5}}{M_G^{5/5}}. \]

since the vev of the flaton \( M \) exceeds the cutoff scale \( M_* \) and our effective treatment of the flaton potential breaks down.

To compare our result with the critical density of the present universe, we show the obtained lower limit for

\[ \Omega_{\phi} h^2 = \frac{\rho_\phi h^2}{\rho_c}. \]  

in Fig.1. In this analysis we have assumed that the modulus is stable and hence this figure shows the cosmic energy density of the stable modulus’ coherent oscillation. We find that it exceeds largely the critical density of the present universe. Notice that in this figure we take the case of \( n = 1 \).

So far, we have assumed that the modulus field \( \phi \) is stable. However, it is not valid, since the modulus field \( \phi \) may couple to the ordinary particles through some nonrenormalizable interactions. The most plausible candidate for the modulus is the dilaton in string theories

\[ m_\phi \gtrsim \frac{1}{2} T_R^2/M_G \simeq 4.5 \times 10^{-23} \text{ GeV} \left( T_R/10 \text{ MeV} \right)^2. \]
and it decays most likely into two photons: \( \phi \rightarrow 2\gamma \) \cite{9} through a coupling to two photons written as

\[
\mathcal{L}_{\text{int}} = \frac{b}{4M_G} \phi F_{\mu\nu} F^{\mu\nu}.
\]

Here, we have introduced a dimension-less parameter \( b \) which depends on the type of superstring theories and compactifications\cite{18}. In the present analysis, we take \( b \) as a free parameter of the order one representing the various compactifications in string theories. Then, the lifetime of the modulus is given by

\[
\tau_\phi \simeq \frac{64\pi M_G^2}{b^2 m_\phi^3} \simeq 7.6 \times 10^{23} \text{ sec} \left( \frac{1 \text{ MeV}}{m_\phi} \right)^3.
\]

Thus if \( m_\phi \gg 100 \text{ MeV} \) the modulus would decay within the age of the present universe and its energy would be diluted below the critical density of the universe. But for such a case another stringent constraint should be considered.

### B. Constraint from the cosmic \( X(\gamma) \)-ray backgrounds

Even if the lifetime of the modulus is longer than the age of the universe, the modulus particle decays into photons in the past universe. Thus, the produced radiation will contribute to the cosmic \( X(\gamma) \)-ray backgrounds and the observed backgrounds give a constraint on the mass and the lifetime of modulus \( \phi \) \cite{9,10}.

The photon number flux induced by the modulus decay is given by \cite{9}

\[
F_\gamma(E_\gamma) = \frac{E_\gamma}{4\pi} \int_0^{t_0} dt \frac{2n_\phi}{\tau_\phi} (1 + z) \delta (E_\gamma (1 + z) - m_\phi/2),
\]

\[
\simeq \frac{n_{\phi,0}}{2\pi \tau_\phi H_0} \left( \frac{2E_\gamma}{m_\phi} \right)^{3/2} \exp \left[ -\frac{2}{3\tau_\phi H_0} \left( \frac{2E_\gamma}{m_\phi} \right)^{3/2} \right],
\]

where \( t_0 \) is the age of the universe, \( z \) the red-shift, \( H_0 \) the present Hubble parameter and \( E_\gamma \) the energy of \( X(\gamma) \)-ray. And \( n_{\phi,0} \) denotes the present number density of the modulus if it would be stable. Here, we have assumed that the present total density parameter is \( \Omega_0 \simeq 1 \).

The detailed derivation of the photon flux in eq.(42) will be given in Appendix.

We may obtain a constraint on \( \Omega_\phi h^2 \left( \Omega_\phi = m_\phi n_{\phi,0}/\rho_c \right) \) by requiring that the maximum value of the flux in eq.(42) should not exceed the observed \( X(\gamma) \)-ray backgrounds \cite{19–21}.

The observational data are fitted by the following power-low spectra \cite{9}

---

\(^{17}\)The modulus decay into two neutrinos is suppressed since it has a chirality flip and vanishes for massless neutrinos \cite{9}. Similarly to the two photon decay the modulus may decay into two gluons. For such a case the lifetime of the modulus may be shorter than that in eq. (41) by a factor about 9.

\(^{18}\)For example, the dilaton has a coupling \( b = \sqrt{2} \) \cite{17} for a compactification of the M-theory \cite{18}.
\[ F_{\gamma,\text{obs}}(E_{\gamma}) \] cm\(^2\)-sr-sec \[ \approx \begin{cases} 8 \left( \frac{E_{\gamma}}{\text{keV}} \right)^{-0.4} & 0.1\text{keV} \lesssim E_{\gamma} \lesssim 25\text{keV} \\ 380 \left( \frac{E_{\gamma}}{\text{keV}} \right)^{-1.6} & 25\text{keV} \lesssim E_{\gamma} \lesssim 350\text{keV} \\ 2 \left( \frac{E_{\gamma}}{\text{keV}} \right)^{-0.7} & 350\text{keV} \lesssim E_{\gamma} \lesssim 1\text{MeV} \\ 1.6 \times 10^{-2} \left( \frac{E_{\gamma}}{\text{MeV}} \right)^{-1.8} & 1\text{MeV} \lesssim E_{\gamma} \lesssim 20\text{MeV} \\ 1.5 \times 10^{-3} \left( \frac{E_{\gamma}}{\text{MeV}} \right)^{-1} & 20\text{MeV} \lesssim E_{\gamma} \lesssim 10\text{GeV} \end{cases} \] (44)

The result is also shown in Fig.1. We find that all mass region \( 10^{-2} \text{keV} \lesssim m_{\phi} \lesssim 10 \text{ GeV} \) is excluded completely by the observed \( X(\gamma) \)-ray backgrounds.

In summary, we have shown that the thermal inflation could not dilute the energy density of the modulus sufficiently so as to lower it below the critical density of the present universe, if the modulus is stable. Moreover, the observed cosmic \( X(\gamma) \)-ray backgrounds put a more stringent bound on the modulus with mass \( 200 \text{ keV} \lesssim m_{\phi} \lesssim 10 \text{ GeV} \) even if modulus is unstable. Therefore, the cosmological string moduli problem is not solved by the thermal inflation for the moduli masses less than \( \mathcal{O}(10) \) GeV. This excludes the mass range predicted in gauge-mediated SUSY-breaking theories as long as \( m_{\phi} \simeq m_{3/2} \).

IV. THERMAL INFLATION WITHOUT THE R-AXION DECAY AND THE COSMOLOGICAL MODULI PROBLEM

A. Modified thermal inflation model

If one sees carefully our scenario of the thermal history described in the previous section, one might recognize that the reason why the thermal inflation mechanism has failed to solve the moduli problem could be attributed to the R-axions produced by the flaton decay. Indeed, the energy density of the relativistic R-axions decreases faster than that of non-relativistic particles, and as a consequence the R-axion decay into photons releases much less entropy than in the case that the decay mode of the flaton into R-axions is not open. Thus we might expect that some mass regions in gauge-mediated SUSY breaking theories could survive the cosmological constraints if we could improve the model to forbid energetically the flaton decay into R-axions.

Besides the stringent constraints discussed in the previous section, there is another difficulty, i.e. the domain wall problem, which we have ignored so far. The origin of this problem is the degenerate minima of the potential which possesses the discrete symmetry \( Z_{n+3} \). Thus, in order to eliminate the domain walls we have to add a small term to the potential which breaks the discrete symmetry explicitly.

One of economical modifications of the model which satisfies both of the above two requirements is, for instance, to add a linear term in the superpotential

\[ \delta W = \alpha X, \] (45)

\[ ^{19}\text{In hidden sector models for the SUSY breaking the flaton decay into R-axions may not be allowed energetically (see footnote 13). Even if it is allowed the R-axions decay just after the flaton decay, since the R-axion mass is of the order } m_{\phi}, \text{ which reheats the universe immediately. In any case sufficiently large entropy is produced and the moduli problem may be solved [7].} \]
which breaks the $\mathbb{Z}_{n+3}$ symmetry completely together with the constant term $C$. To collapse the domain walls before its energy dominates the universe, the dimensionful parameter $\alpha$ is required to be \cite{22}

$$|\alpha| \gtrsim \frac{m_{3/2}^2 m_\chi M}{M_{pl}^2}. \quad (46)$$

The above explicit breaking term (45) in the superpotential modifies the low energy potential of the flaton in eq.(6) as

$$V_{\text{eff}}(X) = V_0 - 2\frac{\alpha C}{M_*^2} (X + X^*) - m_0^2 |X|^2 + \frac{\alpha}{M_*} (X^{n+2} + X^{*n+2})$$

$$+ \frac{n}{n+3} \frac{C}{M_* M_n^2} (X^{n+3} + X^{*n+3}) + \frac{1}{M_*^{2n}} |X|^{2n+4}. \quad (47)$$

In the following, instead of $\alpha$ we use a dimensionless parameter $x$ defined by $\alpha = -x M^{n+2}/M_*^2$, for simplifying the expressions below \cite{20}. Then the vev of the flaton is given by

$$\langle X \rangle \equiv M \simeq \left[ \frac{1}{(n+2)(1-x)} \right]^{\frac{1}{2(n+1)}} \left( m_0 M_*^n \right)^{\frac{1}{n+1}}, \quad (49)$$

and $V_0$ is

$$V_0 \simeq \frac{n(1-x) + 1}{(n+2)(1-x)} m_0^2 M^2. \quad (50)$$

It should be noted that the explicit breaking term (45) does not affect much the dynamics of thermal inflation. In the early universe the potential of the flaton near the origin takes the form as (see eq.(13).)

$$V_{\text{eff}}(X) \simeq (cT^2 - m_0^2) \left| X + \frac{2\alpha m_{3/2}}{cT^2 - m_0^2} \right|^2 + V_0 - \frac{4\alpha^2 m_{3/2}^2}{cT^2 - m_0^2} + \cdots. \quad (51)$$

Although at the high temperature $T \gtrsim T_c \sim m_0$ the flaton does not sit at the origin, the position of the minimum is

$$\langle X \rangle \simeq -\frac{2\alpha m_{3/2}}{cT^2 - m_0^2} \simeq \frac{2x}{\sqrt{(n+2)(1-x)}} \frac{m_0 m_{3/2}}{cT^2 - m_0^2} M, \quad (52)$$

We can write down the relation of $x$ and $\alpha$ in terms of the parameters appearing in the super-potential as

$$\alpha = -\frac{x}{(n+2)(1-x)^{\frac{n+2}{n+1}}} m_0^{n+2} M_*^{\frac{n}{n+1}}, \quad (48)$$

up to the terms of $\mathcal{O}(m_{3/2}/m_0)$.
and the deviation from the origin is suppressed by a factor of $\mathcal{O}(m_{3/2}/m_0)$ compared to the vev $M$ for the true minimum, as far as $cT^2 - m_0^2 \gtrsim m_0^2$. Furthermore, the deviation of the vacuum energy of the flaton from $V_0$ is estimated as

$$\delta V_0 \simeq -\frac{4x^2}{n(1-x) + 1} \frac{m_{3/2}^2}{cT^2 - m_0^2} V_0,$$

which has a suppression factor of $\mathcal{O}(m_{3/2}^2/m_0^2)$ and is consequently negligible. Therefore, we find that if we take $x$ to be not so close to unity the dynamics of the thermal inflation is not modified much by adding the explicit breaking term (45).

Since the linear term (45) in the superpotential breaks also an approximate $U(1)_R$ symmetry, the term gives additional contributions not only to the flaton mass but also to the $R$-axion mass. Then their masses are given by

$$m_\chi^2 \simeq \frac{2(n+1) - nx}{1-x} m_0^2,$$

$$m_a^2 \simeq \frac{(n+2)x}{1-x} m_0^2.$$

Therefore, for the region

$$x_{\min} \equiv \frac{2(n+1)}{5n+8} < x (< 1),$$

the flaton decay into two $R$-axions is kinematically forbidden. In the following we consider the thermal inflation model with the explicit breaking term (45) satisfying eq.(56), and re-examine whether the model can solve the cosmological moduli problem in gauge-mediated SUSY breaking theories.

**B. Cosmological moduli problem with the modified thermal inflation**

First we argue the thermal history after the thermal inflation. At the end of the thermal inflation ($T \sim T_c$) the flaton field begins to oscillate around the true minimum of the potential (47), and when the Hubble parameter becomes comparable to the total width of

21If we take the value of $x$ to be very close to unity ($x < 1$ is always satisfied by the definition of $x$), the position of the minimum at the high temperature is far from the origin. Thus we consider the model with $x$ not so close to unity.

22Although this seems to be a highly restricted region, one can see by taking the original parametrization by $\alpha$ that it actually corresponds to a broad one, namely

$$(-\infty) < \alpha < -\frac{2(n+1)}{5n+8} \left(\frac{5n+8}{3(n+2)^2}\right)^{\frac{n+2}{2\pi(n+1)}} \frac{M^*}{\alpha \, m_0^{\frac{n+2}{2}}}.$$
the flaton, the flaton decays only into SUSY standard model particles since the decay into $R$-axions is not allowed. As discussed in section 2, the flaton decays dominantly into Higgs particles if kinematically allowed, or when the flaton mass is smaller than the threshold of the decay into two Higgs particles the flaton decays into two photons. Each width is represented in eqs.(15) and (16), respectively. In both cases, the flaton energy is transferred to radiation and reheats the universe immediately. Then the modified thermal inflation model increases the entropy by a factor given by putting $\epsilon_a = 0$ and denoting $T_{R,SM}$ by $T_R$ in eq.(19),

$$
\Delta \simeq 1 + \frac{4}{3} \frac{V_0}{(2\pi^2/45)g_*T_c^3 T_R},
$$

where the reheating temperature $T_R$ is obtained by the decay width of the flaton.

For the case that the flaton can decay into Higgs particles ($m_\chi > 130$ GeV) \(^{23}\), $T_R$ is represented as

$$
T_R \simeq 0.14 F(C_h) \frac{m_{3/2} M_1^{1/2}}{M},
$$

$$
\simeq 21 \text{ GeV} \frac{m_{3/2}}{100 \text{ GeV}} \left( \frac{10^{10}}{M} \right),
$$

where $F(C_h)$ is defined by

$$
F(C_h) \equiv \left[ C_h + 2 \left( \frac{\alpha_{em}}{4\pi} \right)^2 \right]^{1/2}.
$$

Thus in this case, as pointed out in Ref. [7], the reheating temperature $T_R$ can be taken to be high enough ($T_R \gtrsim 10$ MeV) to maintain the success of big bang nucleosynthesis. On the other hand for the case $m_\chi \leq 130$ GeV, the reheating temperature is estimated from eq.(16) as

$$
T_R \simeq 1.1 \times 10^{-4} \frac{m_{3/2} M_1^{1/2}}{M},
$$

$$
\simeq 17 \text{ MeV} \left( \frac{m_{3/2}}{100 \text{ GeV}} \right)^{3/2} \left( \frac{10^{10}}{M} \right).
$$

Now let us estimate the cosmic energy density of coherent modulus oscillation in the present universe. The present energy density of the "big-bang" modulus is given by replacing $\Delta$ in eq.(29) by the expression of the entropy production (57), hence

$$
\left( \frac{\rho_{\phi}}{s} \right)_{BB} \simeq 3.8 \left( \frac{T_c}{m_\phi} \right)^3 \left( \frac{\phi_0}{M_G} \right) \frac{2 m_{1/2} m_\chi^3 M_1^{1/2} T_R}{V_0}.
$$

\(^{23}\)From the experimental lower bound on the mass for Higgs bosons [23], we take the Higgs mass to be 65 GeV in order to obtain as conservative constraints as possible.
On the other hand, the present energy density of the “thermal-inflation” modulus is estimated from eq.(30) as

\[
\left( \frac{\rho_\phi}{s} \right)_{TI} \simeq 0.38 \left( \frac{\phi_0}{M_G} \right)^2 \frac{V_0 T_R}{m_\phi^2 M_G^2}.
\]

(64)

Then we turn to estimating the lower bound of the total energy density of the modulus. The lower bound is given by [10]

\[
\left( \frac{\rho_\phi}{s} \right)_0 \simeq \max \left[ \left( \frac{\rho_\phi}{s} \right)_{BB}, \left( \frac{\rho_\phi}{s} \right)_{TI} \right] \geq \sqrt{\left( \frac{\rho_\phi}{s} \right)_{BB} \left( \frac{\rho_\phi}{s} \right)_{TI}},
\]

\[
\simeq 1.2 \left( \frac{\phi_0}{M_G} \right)^2 \left( \frac{T_c}{m_\chi} \right)^{3/2} \frac{m_\chi^{3/2} T_R}{m_\phi^{3/4} M_G^{3/4}}.
\]

(65)

Here, the equality is satisfied when \((\rho_\phi/s)_{BB} = (\rho_\phi/s)_{TI}\), i.e. when

\[
m_\chi \simeq \frac{2.5 \times 10^2}{C_{V_0}^{3/7}} \left( \frac{T_c}{m_\chi} \right)^{3/7} m_\phi^{5/14} M_G^{1/14} T_R^{4/7},
\]

(66)

where \(C_{V_0} = \frac{n(n+1)^{n-1}}{(n+2)[2(n+1)-n]}\) and we have used eq.(61) for an expression of \(T_R\), assuming \(m_\chi \leq 130\) GeV. Then, the lower bound of the moduli energy is estimated from eq.(65) as

\[
\left( \frac{\rho_\phi}{s} \right)_0 \simeq \frac{4.8 \times 10^3}{C_{V_0}^{3/7}} \left( \frac{\phi_0}{M_G} \right)^2 \left( \frac{T_c}{m_\chi} \right)^{15/7} \frac{T_R^{13/7}}{m_\phi^{3/4} M_G^{9/14}}.
\]

(67)

Thus the lowest reheating temperature \(T_R \simeq 10\) MeV and \(x = x_{\text{min}}\) give the minimum abundance as

\[
\Omega_\phi h^2 \gtrsim 3.9 \times 10^{-4} \left[ \frac{8(n+2)^2}{3n+8} \right]^{3/7} \left( \frac{1 \text{ GeV}}{m_\phi} \right)^{3/14},
\]

(68)

for \(\phi_0 \simeq M_G\) and \(T_c \simeq m_\chi\).

However, if we put \(T_R \simeq 10\) MeV in eq.(66) we observe that the assumption \(m_\chi < 130\) GeV holds only when

\[
m_\phi \lesssim m_c \equiv 5.2 \times 10^{-2} \text{ GeV} \left( \frac{C_{V_0}}{T_c} \right)^{6/5}.
\]

(69)

Indeed, for the modulus mass \(m_\phi > m_c\) we can obtain the minimum abundance lower than the r.h.s. of eq.(68) by making use of eq.(58) which is applicable for \(m_\chi > 130\) GeV instead of eq.(61) \(^{24}\). Therefore, for \(m_\phi > m_c\) the condition that the lower bound is saturated in eq.(65) is

\[\]

\(^{24}\)In this case \(m_3/2 > m_c\). Then, the flaton decay into Higgsinos should be forbidden, otherwise the gravitinos produced in successive decay of Higgsinos overclose the present universe.
\[ m_\chi \approx \frac{4.3}{[C_0 F(C_h)]^{2/7}} \left( \frac{T_c}{m_\chi} \right)^{3/7} m_\phi^{5/14} M_G^{1/14} T_R^{4/7}. \]  

The lowest possible values for the flaton mass and the reheating temperature satisfying eq.(70) are practically given by \( m_\chi \simeq 130 \text{ GeV} \) and \( T_R = 10 \text{ MeV} \), yielding the minimum abundance in eq.(65) as

\[ \left( \frac{\rho_\phi}{s} \right)_0 \gtrsim 2.9 \times 10^{-13} \left( \frac{\phi_0}{M_G} \right)^2 \left( \frac{T_c}{m_\chi} \right)^{3/2} \left( \frac{1 \text{ GeV}}{m_\phi} \right)^{3/4}. \]  

Comparing with the present critical density in eq.(28), \( \phi_0 \simeq M_G \) and \( T_c \simeq m_\chi \) leads to

\[ \Omega_\phi h^2 \gtrsim 8.1 \times 10^{-5} \left( \frac{1 \text{ GeV}}{m_\phi} \right)^{3/4}. \]  

In Fig.2 we show the lower bound for the energy density of the modulus predicted from eqs.(68) and (72). A remarkable consequence which distinguishes itself from the result in section 3 is that for all the moduli mass region \( 10^{-2} \text{ keV} \lesssim m_\phi \lesssim 10 \text{ GeV} \), the predicted lower bound can be taken to be below the critical density of the present universe.

So far, we have been considering the case \( m_\phi > H_{TI} \). Then let us discuss briefly what happens in the case \( m_\phi < H_{TI} \). From eqs.(34) and (57), the present abundance of such a modulus is given by

\[ \left( \frac{\rho_\phi}{s} \right)_0 \simeq \frac{1}{8} \left( \frac{\phi_0}{M_G} \right)^2 T_R, \]  

for any flaton mass. Here we have assumed that the modulus mass is larger than the decay width of the flaton (See the footnote 16.). Comparing with the case \( m_\phi > H_{TI} \), the abundance in eq.(73) is always greater than the minimum abundances in eqs.(68) and (72) for \( m_\phi \approx 10^{-2} \text{ keV}-1 \text{ GeV} \).

In Fig.2 we also show the constraint from the observed X(\( \gamma \))-ray backgrounds which is derived in section 3. Then we can see that it excludes the modulus mass region \( 500 \text{ keV} \lesssim m_\phi \lesssim 10 \text{ GeV} \). Therefore, we conclude that the cosmological problems are resolved in a class of gauge-mediated SUSY-breaking models with the gravitino mass \( 10^{-2} \text{ keV} \lesssim m_{3/2} \lesssim 500 \text{ keV} \), provided that the modified thermal inflation takes place and the flaton decay into R-axions is forbidden. We should comment here that the ansatz in eq.(5) is always satisfied for the parameter region we have analyzed.

\section*{V. DISCUSSION AND CONCLUSIONS}

We have found in the first part of our analysis that due to the presence of too light R-axions the original thermal inflation model \cite{7} could not resolve infamous cosmological moduli problems in gauge-mediated SUSY-breaking theories, and all the gravitino mass region, \( 10^{-2} \text{ keV} \lesssim m_{3/2} \lesssim 10 \text{ GeV} \), is excluded as long as \( m_\phi \approx m_{3/2} \) is fulfilled.

In the next step, we have modified the original thermal inflation model so that the R-axion decay of flaton is kinematically suppressed, and as a result we have succeeded to make
a region $10^{-2} \, \text{keV} \lesssim m_{3/2} \lesssim 500 \, \text{keV}$ to survive the cosmological constraints. Moreover, it can be shown that a small window $1 \, \text{GeV} \lesssim m_{3/2} \lesssim 10 \, \text{GeV}$ appears, if we take $\phi_0 / M_G \simeq 0.01$--$0.1$ that is nevertheless a slightly improbable condition $^{25}$. One should note, however, that this window is located in the region $m_{3/2} \gtrsim \mathcal{O}(1) \, \text{MeV}$, in which it is far from easy to construct mechanisms to produce a sufficient number of baryons consistent with today’s observational data [11,9] if one adopts the original Affleck-Dine mechanism [4]. Here, it may be interesting that an enough number of baryons will be created even for $m_{3/2} \gtrsim \mathcal{O}(1) \, \text{MeV}$ if we adopt a variant type of Affleck-Dine baryogenesis discussed in Ref. [24], since gauge-mediated models with $m_{3/2} \sim 1 \, \text{GeV}$ are relatively easily constructed [25,26]

We would like to emphasize that the moduli could be the dark matter in the present universe with their masses being possibly in the region $10^{-2} \, \text{keV} \lesssim m_\phi \lesssim 100 \, \text{keV}$. Fig.3 exhibits the region of the parameters $m_\chi$ and $M_\ast$ in which the conditions $\rho_\phi \leq \rho_c$ and $T_R \gtrsim 10 \, \text{MeV}$ are satisfied for $m_\phi = 100 \, \text{keV}$. By noting that this region contains plausible values $m_\chi \sim 10^1$--$10^2 \, \text{GeV}$ and $M_\ast \sim 10^{18} \, \text{GeV}$ fortunately, one would be encouraged to expect that the above emphasis is by no means a nonsensical suggestion. The detailed study on the spectrum of the $X(\gamma)$-ray emitted from the cosmic moduli will be given in Ref. [27].

As concerns the moduli masses, it is another intriguing possibility that so-called “MeV-bump”, an excess in the $\gamma$-ray background spectrum around $\sim 1 \, \text{MeV}$ [20], might be a signal for the moduli decay [9]. Anyway, future analyses of $X(\gamma)$-ray backgrounds in these mass regions are expected to provide us with more detailed information about the moduli [28].

If the existence of light moduli fields is a generic prediction of string theories, it will probably present us with a plenty of subjects in low energy physics, typical examples of which are nothing but the cosmological moduli problems we have attempted to solve in this paper.

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APPENDIX A: PHOTON FLUX

Since the velocity dispersion of the moduli is negligible, two monochromatic photons with energy $m_\phi/2$ are produced in the moduli decay. Thus, the spectrum $S(E')$ of the photon per decay is written as

$$S(E') = 2\delta(E' - m_\phi/2). \quad (A1)$$

The number density $n_\phi(z)$ of the moduli at red-shift $z$ is given by

$$n_\phi(z) = n_{\phi,0}(1 + z)^3 \exp(-t/\tau_\phi). \quad (A2)$$

Then the present flux from the moduli is estimated as

$^{25}$This window will also appear if one adopts very small values for $b \simeq 0.01$--$0.1$ in eq.(40)
\[ F_\gamma(E_\gamma) = \frac{E_\gamma}{4\pi} \int_0^{t_0} dt' \frac{1}{\tau_\phi} n_\phi(z)(1+z)^{-3} \frac{dE'}{dE_\gamma} S(E') \]
\[ = \frac{E_\gamma}{4\pi} \int_0^{t_0} dt' \frac{1}{\tau_\phi} n_{\phi,0} \exp(-t'/\tau_\phi)(1+z)2\delta(E_\gamma(1+z) - m_\phi/2), \]  
(A3)

where \( t_0 \) is the present time, \( E_\gamma \) the present energy of the photon and the factor \( (1+z)^{-3} \) in the first equation represents the dilution due to the cosmic expansion. Here we have taken into account that the photon with energy \( E_\gamma \) had energy \( E' = (1+z)E_\gamma \) when it was produced at the decay time. The red-shift \( z \) is related to the cosmic time \( t \) by
\[ \frac{dt}{dz} = -H_0^{-1}(1+z)^{-5/2}[\Omega_0 + (1 - \Omega_0 - \Omega_\Lambda)/(1+z) + \Omega_\Lambda/(1+z)^3]^{-1/2}, \]  
(A4)

where \( H_0 \) is the present Hubble parameter, \( \Omega_0 \) the present (total) density parameter and \( \Omega_\Lambda \) the density parameter of the cosmological constant. Then photon flux is given by
\[ F_\gamma(E_\gamma) = \frac{n_{\phi,0}}{2\pi \tau_\phi H_0} \left( \frac{2E_\gamma}{m_\phi} \right)^{3/2} f(m_\phi/2E_\gamma) \]
\[ \times \exp \left[ -\int_{m_\phi/2E_\gamma}^{m_\phi/2E_\gamma} d(1+z) \frac{1}{H_0 \tau_\phi} (1+z)^{-5/2} f(1+z) \right], \]  
(A5)

where
\[ f(x) = [\Omega_0 + (1 - \Omega_0 - \Omega_\Lambda)/x + \Omega_\Lambda/x^3]^{-1/2}. \]  
(A6)

For \( \Omega_0 = 1 \) and \( \Omega_\Lambda = 0 \), eq.(A5) is simplified as
\[ F_\gamma(E_\gamma) = \frac{n_{\phi,0}}{2\pi \tau_\phi H_0} \left( \frac{2E_\gamma}{m_\phi} \right)^{3/2} \exp \left[ -\frac{2}{3H_0 \tau_\phi} \left( \frac{2E_\gamma}{m_\phi} \right)^{3/2} \right]. \]  
(A7)

The flux \( F_\gamma \) takes the maximum value \( F_{\gamma,max} \) at
\[ E_{\text{max}} = \begin{cases} \frac{m_\phi}{2} & \text{for } \tau_\phi > \frac{2}{3}H_0^{-1} \\ \frac{m_\phi}{2} \left( \frac{3\tau_\phi H_0}{2} \right)^{2/3} & \text{for } \tau_\phi < \frac{2}{3}H_0^{-1} \end{cases} \]  
(A8)
REFERENCES

FIG. 1. The lower bounds on the modulus densities $\Omega_\phi h^2$ in the original thermal inflation model for $n = 1$. The thick (thin) solid line represents the lower bound for the modulus density with $m_\phi > H_{TI}$ ($m_\phi < H_{TI}$). The upper bound from the present critical density is represented by the dashed lines with a kink at $m_\phi \sim 100$ MeV. The kink appears because the constraint becomes weaker for $m_\phi \gtrsim 100$ MeV due to the modulus decay. The experimental upper bound from the cosmic $X(\gamma)$-ray backgrounds ($b = 1$) is shown by dot-dashed line.
FIG. 2. The lower bounds on the modulus densities $\Omega_\phi h^2$ in the modified thermal inflation model for $n = 1$ where the flaton to two R-axions decay is forbidden. The thick solid line with a kink at $m_\phi \sim 10$ MeV represents the lower bound for the modulus abundance. The thin solid line for $m_\phi > 10$ MeV represents the lower bound for the case the flaton to Higgs fields decay is forbidden. The upper bound from the present critical density is represented by the dashed line. The experimental upper bound from the cosmic $X(\gamma)$-ray backgrounds ($b = 1$) is shown by dot-dashed line.
FIG. 3. The contours of $\Omega_\phi h^2$ in the modified thermal inflation model for $n = 1$ where the flaton to two R-axions decay is forbidden. We take the modulus mass $m_\phi = 10^{-4}$ GeV. The dashed line denotes $\Omega_\phi h^2 = 0.25$, and the dot-dashed line denotes $\Omega_\phi h^2 = 1$. In the shadow region $T_R < 10$ MeV in which the big bang nucleosynthesis is not operating well.