A Note on Toroidal Compactifications of the Type I Superstring and Other Superstring Vacuum Configurations with 16 Supercharges

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ABSTRACT

We show that various disconnected components of the moduli space of superstring vacua with 16 supercharges admit a rationale in terms of BPS un-orientifolds, i.e. type I toroidal compactifications with constant non-vanishing but quantized vacuum expectation values of the NS×NS antisymmetric tensor. These include various heterotic vacua with reduced rank, known as CHL strings, and their dual type II (2,2) superstrings in $D \leq 6$. Type I vacua without open strings allow for an interpretation of several disconnected components with $N_V = 10 - D$. An adiabatic argument relates these unconventional type I superstrings to type II (4,0) superstrings without D-branes. The latter are connected by U-duality in $D \leq 6$ to type II (2,2) superstrings. We also comment on the relation between some of these vacua and compactifications of the putative M-theory on unorientable manifolds as well as F-theory vacua.
1 Introduction

The recent astonishing insights on the non-perturbative formulation of superstrings tend to favor a picture according to which many if not all consistent vacua emerge as different points on the moduli space of an underlying theory, commonly termed M-theory. Although the uniqueness of maximally extended supergravity strongly suggests that vacua with 32 supercharges in $D < 10$ be all continuously connected, the same property does not seem to be shared by vacuum configurations with 16 supercharges. In this note we show that the toroidal compactifications of the type I superstrings [1] give a remarkably simple geometric rationale for this phenomenon. We indeed showed that the moduli space of type I vacua with 16 supercharges, the maximum allowed for this kind of theories in perturbation theory, includes several disconnected components that can be discriminated according to the rank of the gauge group or equivalently the rank of the constant non-vanishing expectation value of the NS×NS antisymmetric tensor [1]. More explicitly, the Chan-Paton (CP) multiplicity, i.e. the number of D9-branes needed to soak up the $R \times R$ charge of the O-planes introduced by the worldsheet parity $\Omega$-projection [4, 5, 6], turns out to be

$$N_{CP} = 32 \times 2^{-b/2}$$  \hspace{1cm} (1)

where $b$ is the rank of the constant non-vanishing expectation value of the internal NS×NS antisymmetric tensor $B_{ij}$ and is always an even integer. In the unoriented closed-string spectrum of type I compactifications on $T^d$, the $d$ vectors from the mixed components $G_{\mu ij}$ of the metric in the NS×NS sector and the $d$ vectors from the mixed components $B_{\mu ij}$ of the antisymmetric tensor in the $R \times R$ sector combine to give $d$ graviphotons and $d$ vectors in as many vector multiplets. In the presence of generic Wilson lines in the Cartan subalgebra of the CP group, the total number of vector multiplets is $N_V = d + N_{CP}/2$ and the gauge group for un-conventional toroidal orientifolds with non-vanishing NS×NS antisymmetric tensor, or BPS un-orientifolds [1], turns out to be $U(1)^{d+N_V}$ [1]. Further rank reduction may be induced by Wilson lines that lie in $O(32)$ but not in $SO(32)$ [1] or by a modified $\Omega$-projection either in the closed-string sector [7, 8] or in the open-string sector [9, 1]. The latter was associated to open-string discrete Wilson lines in [9] but it is more appropriate to associate it to open-string discrete torsion, since it results from an ambiguity in defining the Möbius-strip $\Omega$-projections of the twisted sector [4] of some un-orientifolds, in full analogy with the ambiguity that leads to discrete torsion in some target-space orbifolds [10].

Notice that, contrary to naive expectation, a non-vanishing expectation value of the NS×NS antisymmetric tensor may be perfectly compatible with the $\Omega$-projection. Indeed, although the fluctuations of the NS×NS antisymmetric tensor are projected out of the perturbative unoriented closed-string spectrum, a properly quantized vacuum expectation

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1Throughout the paper, we use the term moduli space in a broad sense as the space locally parameterized by the massless scalar fields, often termed moduli, with only derivative couplings in the low-energy effective lagrangian. These are in one-to-one correspondence with truly marginal deformations of the conformal field theory underlying the compactification [2, 1]. Note that since there could be more than one disconnected component of the moduli space of a theory with 16 supercharges for any given number of vector multiplets, one must also consider automorphism symmetries. Such examples as well as global identifications, known as T- S- and U-dualities [3], will be addressed in Sections 3 and 4.

2Here and in most of the following, BPS does not stand for Bogomolny, Prasad and Sommerfield.
value, i.e. one for which $2B$ belongs to the integer cohomology of the compactification manifold, is consistent with the projection $B \to -B$ up to a generalized Peccei-Quinn (PQ) shift [1, 11].

Moreover, the systematic construction [4, 5] of rational open-string models strongly motivated the discovery of the BPS un-orientifolds [1] since it often implies the presence of a non-vanishing but quantized NS×NS antisymmetric tensor [8, 9, 12, 13]. In retrospect, one may thus ascribe the very existence of disconnected components in the moduli space of superstring vacua with 16 supercharges to the consistency of the systematic construction [5] of type I superstrings from left-right symmetric type II theories [4].

In the heterotic case, similar models with reduced rank, known as CHL strings [14], were found long after the appearance of [1]. The CHL models, originally constructed in terms of free world-sheet fermions, display world-sheet current algebras that are typically realized at higher level. Several CHL models admit an interpretation as compactifications on asymmetric orbifolds [15, 16, 17], that allows the identification of their moduli space [18]. The complete list of possible $N_V$ can be found in [18]. In view of the conjectured duality between heterotic and type I superstrings in $D = 10$ [19, 20], that has been tested further in toroidal compactifications with maximal rank [21], it is natural to conjecture that BPS unorientifolds be dual to CHL models with the same number of vector multiplets. For instance, in $D = 8$ BPS unorientifolds are expected to be dual to CHL models with $N_V = 18, 10$; in $D = 6$ to CHL models with $N_V = 20, 12, 8$; in $D = 4$ to CHL models with $N_V = 22, 14, 10, 8$. Recently some of the CHL models have been given a gauge theoretic interpretation as toroidal compactifications with non-commuting Wilson lines [22]. In order to sharpen the above duality conjectures, in Section 3 we offer the type I counterpart of the mechanism of rank reduction. It is remarkable that an open-string derived concept, such as the non-commutativity of the Wilson lines, can be given a closed-string interpretation in terms of a quantized NS×NS tensor. This mechanism may have far reaching consequences in other important applications of open-string theories [23].

Further support to the above conjectured duality can be given by matching the massless spectra of the two theories at points of enhanced gauge symmetry. To this end, in Section 3 we derive the genus-one partition function of the CHL model with $E(8)$ current algebra at level $k = 2$. A proper choice of Wilson lines breaks $E(8)$ to $SO(16)$ or to other non-abelian groups found in the BPS unorientifold setting [1]. By arguments similar to those in [21], the apparent discrepancy between heterotic symmetry enhancement at special values of the radii and the lack of a similar phenomenon in the type I description is to be interpreted as requiring D-brane states to become massless, an effect that is beyond the reach of type I perturbation theory.

Precision tests of the duality between heterotic and type I compactifications with reduced rank would require a detailed matching of the spectrum of BPS$^3$ states or equivalently of the global identifications in the moduli spaces of the compactifications. Indeed, although supersymmetry alone does not fix uniquely the massless spectrum in these compactifications, it is still powerful enough to fix completely the low-energy effective lagrangian once the spectrum is fixed. In particular the moduli space parameterized by

\[N_V\]

Here, BPS stands for Bogomolny-Prasad-Sommerfield.
the scalars in the $N_V$ vector multiplets and the scalar in the supergravity multiplet is [24]

$$O(d, N_V) \quad O(d) \times O(N_V) \times R^+$$

up to global identifications. In $D = 4$ the factor $R^+$ is extended to $SL(2, R)/U(1)$ after the complexification of the scalar in the $\mathcal{N} = 4$ supergravity multiplet. For the component with maximal rank $N_V = 16 + d$ the heterotic string derivation of (2) has been given long time ago [25]. For the components with reduced rank, that admit both a geometric interpretation and a candidate BPS unorientifold dual, one can follow the arguments in [18]. In the type I setting a discussion of the truly marginal deformations and of some of the global identifications can be found in [1]. In Section 3 we expand on that discussion and give an algebraic argument along the lines of [26] in favour of the type I interpretation of (2). For the time being, let us notice that, contrary to the heterotic case, the scalar in the supergravity multiplet is not simply the string dilaton but rather a combination of the dilaton and the scalar parameterizing the volume of the compactification torus [27]. It is thus difficult to sort out the dilaton from the other moduli. As a result, T-duality becomes a non-perturbative symmetry as expected from D-brane considerations [6].

Below $D = 6$, type II compactifications on $K3 \times T^{d-4}$ give rise to another class of superstring vacua with 16 supercharges. These models are known as type II (2,2) models since in $D = 4$ the $\mathcal{N} = 4$ supersymmetry charges are evenly contributed from left and right movers [15, 16, 28]. In the absence of any truncation, the gauge group is generically $U(1)^{2d+16}$. Using the well-established type IIA - heterotic duality in $D = 6$ [3, 29] and mirror symmetry, i.e. T-duality, of the type II theories on $T^2$, one deduces that type IIA, type IIB and heterotic strings are related by S-T-U triality [3, 30]. As a result the non-perturbative S-duality is mapped to perturbative T-duality. Type II duals of some CHL models can be obtained after orbifold projections with a trivial action on the supercharges [15, 31, 32]. The detailed analyses [30, 32] show that there can be several models with the same $N_V$ that, however, are expected to display different global identifications. Aside from the two models with $N_V = 10$ in $D = 4$ [30], all the other instances have $N_V \leq d$. For the models with $N_V < d$ it seems difficult to find a perturbative type I description. The components with $N_V = d$ can be given a rationale in terms of type I superstrings without open strings [8, 7]. In these components the CP group is completely absent since the O-planes that correspond to unconventional $\Omega$-projections do not carry R×R charge. The number of vector multiplets that accompany the $d$ graviphotons is only $d$ in this case. They correspond to the combinations $\mathcal{G}_\mu - \tilde{B}_\mu$ orthogonal to the graviphotons. To the best of my knowledge there is no way to reduce the rank of the type I gauge group any further and to find perturbative type I duals of e.g. the models with $N_V = 4$ in $D = 4$ [15, 32, 30]. This should not necessarily sound as a breakdown of heterotic - type I duality, but rather as a motivation for exploring further these components of the moduli spaces.

In the type IIB superstring, the action of the world-sheet parity projection $\Omega$ and the right-moving fermionic parity $(-)^{F_R}$ are conjugate to one another, $\Omega = S^{-1}(-)^{F_R}S$, via an S-duality transformation:

$$S : \quad \lambda \to -1/\lambda ,$$

\text{(3)}

\footnote{It would be more appropriate to term this $S$ a U-duality transformation since it involves a mixing between the NS×NS dilaton $\phi$ with the R×R dilaton $\chi$.}
that inverts the (complexified) type IIB dilaton $\lambda = \chi + ie^{-\phi}$. Although in $D = 10$ self-duality of the type IIB superstring is not sufficient to prove that the two orbifolds of the type IIB with respect to $\Omega$ and $(-)^F_r$ are dual to one another – in fact one gets the type I superstring and the type IIA superstring respectively [3] – an adiabatic argument [3, 28, 15] suggests that one may recover equivalence after toroidal compactifications when the action on the fields is accompanied by a non trivial action on the geometry, e.g. a shift $\sigma_V$ of order two in the compactified directions. In Section 4, we will show that the resulting dual pairs consist in type I models without open strings and type II (4,0) models without D-branes. The latter correspond to asymmetric orbifolds that break all the supersymmetries in the right-moving sectors. Models of this kind have been considered both from the viewpoint of the fermionic construction [33, 34, 35] and from the viewpoint of the *freely-acting orbifolds* [16]. At generic points of the moduli space the gauge group is $U(1)^{d+d}$ that, at free fermionic points, gets perturbatively enhanced to $SU(2)^d \times U(1)^d$ or to another group whose structure constants satisfy the constraints for the existence of a cubic world-sheet supercurrent [35, 36, 33]. Notice that these enhanced symmetry points typically correspond to radii which are half the standard self-dual value $R = \sqrt{\alpha'}$. This modified Halpern-Frenkel-Kac (HFK) mechanism is consistent with the two following observations. First, the current algebra on the world-sheet that leads to the enhanced symmetry is realized at higher level, e.g. $k = 2$ in the simplest instance of $SU(2)^d$. Second, the T-duality group of asymmetric orbifolds is different from the T-duality group of standard toroidal or symmetric orbifold compactifications and the points of enhanced symmetry are the self-dual points of the modified T-duality group. Notice that, rather surprisingly, D-branes in type II (4,0) models do not carry $R \times R$ charge. Indeed, the process of breaking all the supersymmetries in the right-moving sector all the $R \times R$ states, together with their NS$\times R$ superpartners, become massive. A nice feature, however, is that the twisted sectors of these orbifolds do not give rise to massless particles in the decompactification limit, so that these vacua regain the *rigidity* properties of their parent type II (4,4) superstrings [16]. Using U-duality invariance in $D = 6$, some type II (4,0) models have been argued to be dual to type II (2,2) models [15, 16] and can thus be connected to heterotic CHL models [31, 30, 32].

In view of the mounting wave of interest in the putative M-theory [37], let us briefly comment on the relation of the above vacua to M-theory compactifications. According to common wisdom, M-theory is the strong coupling limit of the type IIA superstring in $D = 10$ [19] or alternatively of the $E(8) \times E(8)$ heterotic string in $D = 10$ [38]. Both equivalences follow from the identification of the vacuum expectation value of the dilaton with the length of an extra dimension. Although the former equivalence ascribes both the massless spectrum and low-energy effective lagrangian from a dimensional reduction *á la* Kaluza-Klein (KK), the latter rests on further assumptions such as residual supersymmetry, anomaly cancellation and a consistent string interpretation. For instance, relaxing the last assumption, the $\mathcal{N} = (1, 0)$ supergravity theory with gauge group $U(1)^{248} \times U(1)^{248}$ [39] could not be excluded. Still, it is remarkable how simply assuming the existence of a consistent 11D theory has provided so many non-perturbative connections in the web of superstring vacua [19]. In fact, it has been shown that the type I vacua without open strings are equivalent to M-theory compactifications on non-orientable manifolds without boundaries [7]. In particular, the type I superstring without open strings in $D = 9$ is
equivalent to M-theory on a Klein bottle. By the same token, M-theory compactification on the Möbius strip is expected to describe the strong-coupling limit of the CHL strings in $D = 8$ [7, 40] that are related to the BPS models with $b = 2$. Another class of compactifications of M-theory that are supposed to preserve 16 supercharges and have received some attention in view of heterotic - type IIA duality in $D = 6$ [19, 29, 3] consists of compactifications on $K3 \times S^1$ and orbifolds thereof [31, 32, 15, 41]. Prior to any truncation one simply gets type IIA on $K3$ with $20 \mathcal{N} = (1, 1)$ vector multiplets. Modding out by a symmetry that preserves the holomorphic 2-form on $K3$ and acts as a shift on the extra circle gives rise to other $\mathcal{N} = (1, 1)$ supersymmetric vacua with a gauge group of reduced rank. The complete classification [31, 32] include four possibilities with $N_V = 20, 12, 8, 4$ that can be accounted for by the BPS unorientifolds with $b = 0, 2, 4$ and the type I vacua without open strings. Once again we remark that the components of the moduli space of vacua with $N_V = 2$ [31, 32] do not seem to have an obvious perturbative type I interpretation.

Another interesting class of superstring vacuum configurations goes under the name of F-theory [42]. Almost by definition, F-theory on a manifold $\mathcal{F}$ that admits an elliptic fibration, i.e. looks locally like $\mathcal{F} = B \times T^2$, is the compactification of the type IIB superstring on the manifold $B$ with 24 7-branes. The complexified dilaton $\lambda$ is identified with the complex modulus of the elliptic fiber $T^2$ and its variation over the basis $B$ is governed by the arrangement of 7-branes. It is a remarkable fact that under two T-dualities the 16 dynamical D9-branes, present in the type I compactification on $T^2$, are mapped to as many D7-branes while the O7-planes may be regarded as bound states of two 7-branes each [43]. The arrangement of 7-branes can be chosen so that $\lambda$ is constant and small and the resulting configuration is T-dual to a perturbative type I description [43, 44]. F-theory vacua may thus be used as non-perturbative definitions of the dynamics of D-branes and O-planes in type I vacua [43, 44, 46]. Moreover, the conjectured duality between the heterotic and type I superstrings allows one to establish dualities between F-theory and heterotic compactifications. In particular, F-theory on an elliptic $K3$ in the orbifold limit $K3 \approx T^4/Z_2$ may be related to the heterotic string on $T^2$ [43]. Indeed the moduli spaces of elliptic $K3$ surfaces is $O(2,18)/O(2) \times O(18)$ and, up to global identifications, coincides with the trivial component of the moduli space of heterotic string compactifications on $T^2$ [25]. By fiberwise application of the above duality [28], one can identify F-theory compactifications on elliptic Calabi-Yau threefolds and fourfolds with heterotic and type I compactifications on Calabi-Yau twofolds and threefolds in the presence of penta-branes [42, 45, 8, 46]. It is beyond the purposes of the present investigation on vacua with 16 supercharges to pursue this viewpoint, that is the subject of active research in view of the possibility of extracting some non-perturbative information on vacua with $\mathcal{N} = 2$ (8 supercharges) and $\mathcal{N} = 1$ supersymmetry (4 supercharges) in $D = 4$. There are however threefolds and fourfolds of reduced holonomy [30, 40, 45]. In such cases one gets enhanced supersymmetry both in $D = 6$ and in $D = 4$. According to the classification in [30], in $D = 4$ one finds $N_V = 22, 14, 10, 6, 4$. Aside from the last case, the values of $N_V$ precisely match with those found in BPS unorientifolds [1] and in type I theories without open strings [7, 8].

One last remark on the role of a quantized NS×NS antisymmetric tensor background in type I compactifications concerns the case $\mathcal{N} = (1, 0)$ in $D = 6$. The first model of this
kind [5] had a CP group $Sp(8)^4$ that, though smaller than expected from a naïve analogy with heterotic vacua with perturbative embedding of the spin connection in the gauge group, is not a subgroup of $SO(32)$. It is by now clear that the oblique CP symmetry enhancement is due to the presence of D5-branes [6]. What may still seem puzzling is the effective reduction by a factor of two in the number of both D5-branes and D9-branes! In order to solve this puzzle, one has to recall that the model in question descends from the type IIB compactification on a $Z_2$-orbifold of $T^4$ at the $SO(8)$ enhanced symmetry point [5]. Since the rank of the $NS\times NS$ antisymmetric tensor that correspond to the $SO(8)$ current algebra at level $k = 1$ is $b = 2$, one could expect a reduction by a factor of two of the overall CP multiplicities, while the symplectic nature of the CP group does the rest. A similar analysis can be performed for the models in [5, 8, 9] that corresponds to rational points in the moduli space of orbifold compactifications [6, 7]. In particular for models with one tensor multiplet, that admit perturbative heterotic duals, it has been shown [11] that a quantized $NS\times NS$ antisymmetric tensor corresponds to a compactification with a generalized second Stiefel-Whitney class $\omega_2(V)$ that satisfies $\omega_2(V) = 2B$. Since $\omega_2(V)$ belongs to $H^2(S, Z_2)$, where $S$ is the $K3$ surface under consideration, and $B$ is defined modulo shifts in $H^2(S, Z)$, one has three inequivalent choices [11]. Upon performing T-duality on the two-cycle with non-vanishing $B$-flux one ends up with an F-theory compactification with vanishing $B$ and a mirror $K3$ surface $\tilde{S}$ [11].

Clearly, in the long run, one would like to address the issues raised by the presence of a quantized $NS\times NS$ antisymmetric tensor in type I compactifications with $N = 1$ supersymmetry in $D = 4$. A preliminary analysis has been performed for the type I descendandants of the type IIB superstring on the $Z$-orbifold [27, 48]. The 6D cases should be taken as a guide to explore further connections in $D = 4$ between BPS unorientifolds and other consistent superstring vacua. Relation to M-theory compactifications and F-theory vacua [43, 45, 49] may help understanding geometric features, e.g. the moduli space of vacua, that sometimes look obscure from a superstring perspective.

The plan of the paper is as follows. Section 2 is a primer on rational unorientifolds that motivated the discovery of the BPS unorientifolds. In Section 3 we discuss generalized toroidal compactifications of the type I superstrings and show how a quantized $NS\times NS$ antisymmetric tensor may be interpreted in terms of non-commuting open-string Wilson lines. We also discuss the local structure of the moduli space of BPS unorientifolds and their global identifications. Finally, we derive the one-loop partition function of the CHL model with $E(8)$ current algebra at level $k = 2$ for the sake of comparison with the BPS unorientifolds in $D = 8$. In Section 4 we discuss type II superstring vacua with 16 supercharges and argue that some type II $(4,0)$ models without D-branes are dual to unconventional type I vacuum configurations without open strings. Finally, Section 5 contains some speculations and our conclusions.

\footnote{For a $Spin(32)/Z_2$ vacuum gauge bundle $V$, $\omega_2(V)$ represents the obstruction to defining a vector structure, i.e. a consistent parallel transport for fields in the representations of $SO(32)$ that belong to the vector conjugacy class [47].}
2 Quantized $B$ from Rational Un-orientifolds

The building blocks of perturbative closed-string theories are conformal field theories on closed orientable Riemann surfaces [50, 64]. The building blocks of perturbative open-string theories are conformal field theories on closed, open and/or unorientable Riemann surfaces\(^6\). The discovery by Green and Schwarz of infinity and anomaly cancellations in the type I superstring with gauge group $SO(32)$ [52], that triggered an enormous interest in the field and led to the discovery of the heterotic string [53], also motivated the discovery of the $SO(8192)$ bosonic string [54]. The proposal of interpreting open-string theories as descendants of left-right symmetric closed-string theories [4] was developed in [55, 56] and brought to a consistent systematization in [5, 9]. For rational models, the crucial issue of CP symmetry breaking was achieved borrowing some interesting results of Cardy’s on boundary effects in two-dimensional critical models [57].

The starting point for the construction of a rational \textit{un-orientifold} is a left-right symmetric rational conformal field theory (RCFT). Rationality is related to the presence of a chiral algebra $\mathcal{C}$ of symmetries on the world-sheet, e.g. a current algebra, that extends the Virasoro algebra generated by the modes $L_n$ of the energy-momentum tensor $T(z)$ and allows to encode the spectrum of the theory in a finite number of characters

$$\chi_h(q) = \text{Tr}_{\mathcal{H}_h} q^{L_0 - c/24}, \quad (4)$$

As usual $c$ is the central charge of the Virasoro algebra, $q = \exp(2\pi i \tau)$, with $\tau$ the one-loop modular parameter and $\mathcal{H}_h$ denotes the sector of the spectrum formed by the descendants (with respect to the chiral algebra $\mathcal{C}$) of the primary field with $L_0 = h$. The characters $\chi_h(q)$ provide a (unitary) representation of the modular group $SL(2, \mathbb{Z})$ generated by the transformations:

$$S: \quad \chi_h(-1/\tau) = S_{hk} \chi_k(\tau) \quad (5)$$

and

$$T: \quad \chi_h(\tau + 1) = e^{2\pi i (h - c/24)} \chi_h(\tau) \quad (6)$$

and enter the torus partition function in a modular invariant way

$$\mathcal{T} = \sum_{\bar{h}h} N_{\bar{h}h} \chi_h(q) \chi_{\bar{h}}(\bar{q}) \quad (7)$$

where $N_{\bar{h}h}$ are integers that satisfy $N_{oo} = 1$, where $o$ labels the identity primary field with $L_o = 0$.

In order to construct an open-string descendant of an oriented closed-string model based on a RCFT, that is invariant under left-right interchange, i.e. $N_{\bar{h}h} = N_{\bar{h}h}$, one starts by dividing $\mathcal{T}$ by a factor of two. The $\Omega$-projection introduces O-planes [6] that are accounted for by the Klein-bottle amplitude

$$\mathcal{K} = \frac{1}{2} \sum_{\bar{h}} N_{\bar{h}h} \sigma_h \chi_h(q\bar{q}) \quad (8)$$

\(^6\)Early calculations of multi-loop open-string scattering amplitude in terms of Green functions on surfaces with boundaries date back to the work of Alessandrini and Amati [51].
that completes the \textit{untwisted sector} of the un-orientifold [4]:

$$Z_u = \frac{1}{2} \text{Tr}_{\mathcal{H}_c}(1 + \Omega)q^{L_o-\frac{c}{2}}\tilde{q}^{\bar{L}_o-\frac{c}{2}} = \mathcal{T} + \mathcal{K}$$ \hspace{1cm} (9)

where as indicated the trace is taken over the closed-string states. The signs $\sigma_h$ in the $\Omega$-projection 8 are restricted by the \textit{crosscap constraint} [13] that e.g. requires $\sigma_i\sigma_j = \sigma_k$ if the fusion-rule coefficient $N_{ij}^k$, given by the Verlinde formula [58], is non-vanishing.

To the resulting unoriented closed-string spectrum one can – and in many cases must – add the unoriented open-string spectrum. Observing that $q\bar{q} = q_o^2$, with $q_o = \exp(-2\pi\tau_2)$, the most general parameterization of the \textit{twisted sector} of the un-orientifold [4] is

$$Z_t = \frac{1}{2} \text{Tr}_{\mathcal{H}_o}(1 + \Omega)q^{L_o-\frac{c}{2}}\bar{q}^{\bar{L}_o-\frac{c}{2}} = \mathcal{A} + \mathcal{M}$$ \hspace{1cm} (10)

where as indicated the trace is taken over the open-string states. More explicitly, (10) involves the annulus partition function:

$$\mathcal{A} = \frac{1}{2} \sum_{h,a,b} A_{ab}^h n^a n^b \hat{\chi}_h(\sqrt{q_o})$$ \hspace{1cm} (11)

where $n^a$ are the CP multiplicities and $A_{ab}^h$ are integer coefficients, and the M"obius strip $\Omega$-projection

$$\mathcal{M} = \frac{1}{2} \sum_{h,a} M_{aa}^h n^a \hat{\chi}_h(e^{i\pi}\sqrt{q_o})$$ \hspace{1cm} (12)

where $M_{aa}^h = A_{aa}^h (\text{mod} 2)$ and $\hat{\chi}_h$ form a proper basis of \textit{hatted} characters [5]

$$\hat{\chi}_h(i\tau_2 + 1/2) = e^{-i\pi(h-c/24)}\chi_h(i\tau_2 + 1/2)$$ \hspace{1cm} (13)

real functions of $\tau_2$, thanks to the overall phase-shift. Although in general the CP factors $n^a$ and the sectors of spectrum $\mathcal{H}_h$ cannot be put in one-to-one correspondence, for the charge-conjugation modular invariant ($N_{h,h} = C_{h,h} = \delta_{h,h'}$) one is allowed to associate to each sector $\mathcal{H}_h$ a CP factor $n^i$ and let $A_{ij}^k = N_{ij}^k$ in (11) or an automorphism thereof [5, 9]. Sewing of surfaces with holes and crosscaps implies some consistency conditions on the above parameterization. Most notably, the \textit{completeness conditions} $A_{ab}^e A_{ab}^f = N_{ij}^k A_{ab}^k$ [13].

After switching to the transverse closed-string channel via a modular $S$-transformation (5) the Klein-bottle amplitude (8) gives the crosscap-to-crosscap amplitude

$$\tilde{\mathcal{K}} = \sum_h \Gamma_h^2 \chi_h$$ \hspace{1cm} (14)

that, up to some sign ambiguity, allows one to extract the crosscap reflection coefficients $\Gamma_h$, determining the coupling of the $h$-sector of the closed-string spectrum to O-planes. Similarly, via a modular $S$-transformation (5), the annulus partition function 11 gives the boundary-to-boundary amplitude

$$\tilde{\mathcal{A}} = \sum_h (B^h)^2 \chi_h = \sum_{h,a} (B_{ha} n^a)^2 \chi_h$$ \hspace{1cm} (15)
where $B_h$ are the boundary reflection coefficients that determine the coupling of the $h$-sector of the closed-string spectrum to D-branes. Given the amplitudes (14) and (15), the consistency of the construction requires that the boundary-to-crosscap amplitude be of the form
\[ \tilde{M} = \sum_{h,a} \Gamma_h (B_h a^a) \hat{\chi}_h \]
(16)

The modular transformation between loop and tree channel of the Möbius strip amplitude is induced by $P = T^{1/2}ST^2ST^{1/2}$ that acts on hatted characters and satisfies $P^2 = C$ [5]. The required form of (16) puts severe constraints on the choices of integer coefficients $A_{ab}^k$ that appear in (11) and of the various signs in the $\Omega$-projection both in the untwisted and in the twisted sectors. For the charge-conjugation modular invariant, these are seen to be all fulfilled by the choice $A_{ij}^k = N_{ij}^k$ [5, 13].

In order for a rational un-orientifold to be interpreted as a consistent open-string vacuum configuration, one has to impose the proper connection between spin and statistics [55] and cancellation of the tadpoles of the unphysical massless states [59, 5]. The latter require $\Gamma_h + B_h a^a = 0$, for all the massless states that have been eliminated by the $\Omega$-projection in the direct closed-string channel. In imposing these conditions it is crucial to fix the relative normalization of $K$, $A$ and $M$ by expressing them in terms of the modular parameter of the common double-cover [54]. In modern language, this amounts to R×R charge neutrality of the relevant configuration of D-branes and O-planes [6]. For non-supersymmetric configurations, tadpole cancellation for other massless physical fields, such as the dilaton, can be unambiguously imposed as a requirement for vacuum stability [54]. The simplest essentially rational closed-string theory one can un-orientifold is the type I superstring in $D = 10$. The result is the type I superstring with gauge group $SO(32)$ [4]. In $D = 10$ there are two more left-right symmetric theories: the tachyonic models proposed long time ago [60]. Their open-string descendants [5] play a crucial role in some proposed string dualities without supersymmetry [61] and hopefully [62] may provide a rationale for the largely unexplored $\mathcal{N} = (1,0)$ 10D supergravity with gauge group $U(1)^{496}$ [39].

In order to display the subtlety that allows for the existence of the BPS unorientifolds one has to consider generalized $\Omega$-projections of type II compactifications [5, 9] that are compatible with target-space Lorentz symmetry and with the diagonal part of the internal symmetries\(^7\). In particular, the free fermionic constructions [33] or the covariant bosonic lattices [36] allow for simple and elegant rational compactifications. The intrinsic consistency of type I descendants of certain left-right symmetric type II models forces one to allow for the introduction of a non-vanishing but quantized NS×NS antisymmetric tensor background. For instance, closed-string models that involve a simply-laced current algebra of rank $r$ at level $k = 1$ correspond to propagation on an $r$-dimensional torus with constant internal metric $G_{ij}$ identified with the Cartan matrix $C_{ij}$ of the underlying Lie algebra and, more importantly for our goals, with the NS×NS antisymmetric tensor satisfying $B_{ij} = C_{ij}$ for $i > j$ and $B_{ij} = -C_{ij}$ for $i < j$ [36]. The left-right symmetry of the theory that becomes apparent when the torus partition function of the parent closed-string theory is written as in (7) suggests the possibility of introducing a quantized

\(^7\)One may also envisage the possibility of un-orientifolds that break some of the internal accidental symmetries. Some instances are discussed in [9].
NS×NS antisymmetric tensor at generic points of the moduli spaces of toroidal or orbifold compactifications of their open-string descendants[1, 8].

3 Toroidal Compactifications Revisited

In order to analyze generalized toroidal compactifications of the type I superstring [1] let us start with a discussion of the conditions for left-right symmetry of the parent type IIB compactifications. Un-orientifolds of type IIA models simply follow from T-duality [6, 63, 21]. Since the spectrum of oscillator excitations is automatically invariant under $\Omega$, the only potential troubles come from the Narain lattice $\Gamma_{(d,d)}$ of generalized momenta $(P_L, P_R)$. Left-right symmetry implies that for any state $(P_L, P_R)$ there exists a specular state $(P_R, P_L)$, i.e. a state with $P'_L = P_R$ and $P'_R = P_L$. Given a generic value of the metric $G_{ij}$, this turns out to be a constraint on the NS×NS antisymmetric tensor $B_{ij}$ [1]. Indeed using the standard parameterization

\[ P_{L/R}^i(m, n) = G^{ij}(m_j + B_{jk}n^k) \pm \frac{n^i}{2} \]  

with integer $n^i$ and $m_j$ and $G^{ij}$ the inverse of $G_{ij}$, and imposing $P_L(m, n) = P_R(m', n')$ for generic $G_{ij}$, one immediately finds

\[ n'^i = -n^i \quad m_j - m'_j + 2B_{jk}n^k = 0 \]  

the second condition implies that $2B$ belongs to the integer cohomology of the torus $T^d$ and determines $n^k$ in terms of $m_i$ and $m'_i$. Given these constraints and since only states with $P_L = P_R$, i.e. Kaluza-Klein (KK) momentum states with $n^i = 0$, are fixed under $\Omega$, one can check that the Klein-bottle contribution $K$ does not depend on $B_{jk}$. This should not sound unexpected, given the inconsistency of pulling $B$ back to an unorientable surface.

The perturbative unoriented closed-string spectrum is trivially invariant under constant continuous PQ shifts of the internal R×R antisymmetric tensor

\[ B_{ij} \rightarrow B_{ij} + \Delta_{ij} \]  

and less trivially invariant under constant discrete $GL(d, Z)$ transformations of the metric and the quantized NS×NS antisymmetric tensor

\[ G_{ij} + B_{kl} \rightarrow (M^{-1})^k_i (G_{kl} + B_{kl}) M^l_j \]  

The latter allow to skew-diagonalize $B_{ij}$, i.e. bring it to a form $\tilde{B}_{ij}$ with only non-vanishing components equal to 0 or $\pm 1/2$ in the diagonal two-by-two blocks. For later use, notice that rank$(B)$ is invariant under $GL(d; Z)$ transformations.

The open-string spectrum presents some amusing features [1]. First of all, thanks to the possibility of adding Wilson lines $A^a_i$ in the Cartan subalgebra of $SO(32)$, the KK momenta are shifted by an amount $q_a A^a_i$ as expected. Rather unexpectedly, however,

\[ ^8 \text{In order to adhere to the recent literature [11] we have changed the normalization of } B_{ij} \text{ by a factor of two with respect to [1] and put } \alpha' = 2. \]
they can suffer a further shift due the presence of a quantized $B_{ij}$. In order to deduce this shift one has to start from the transverse closed-string channel where Wilson lines become phases in the boundary reflection coefficients. The presence of boundaries and/or crosscaps requires the constraint $P_L = -P_R$ on the closed-string states flowing along the tube. To this end, one has to introduce exactly $b = \text{rank}(\tilde{B})$ $Z_2$-projections

$$\mathcal{P}_i = \frac{1}{2} \left(1 + (-)^{(2\tilde{B}^a)_i}\right).$$

(21)

After imposing the cancellation of the unphysical tadpole in the $R \times R$ sector, the CP multiplicity is reduced by a factor $2^{b/2}$, as stated in the introduction and found some time ago [1]. For instance, in $D = 8$, one can take advantage of the discrete choices of $B$ and the continuous Wilson lines to connect the BPS unorientifolds with $G_{CP} = SO(16)$ to the ones with $G_{CP} = Sp(16)$ passing through points where the CP symmetry is broken to $U(1)^8$ or is partially enhanced to $U(8)$ [1].

The existence of non-perturbative D-brane states [6] breaks the continuous PQ shifts of the $R \times R$ antisymmetric tensor to discrete ones. In type II compactifications with maximal supersymmetry the (pseudo)scalars from the $R \times R$ sector transform according to a spinorial representation of the $O(d, d)$ T-duality subgroup of the full $E_{(d+1)}(d + 1)$ U-duality group [26]. Under the $GL(d)$ subgroup of perturbative type I dualities the spinorial representation decomposes into a sum of rank $p$ antisymmetric tensors. In particular $\tilde{B}_{ij}$ transforms as a two-index antisymmetric tensor as it should. Similarly $A^a_i$ transforms according to the $(d, n)$ representation of $GL(d) \times O(n)$, where $n$ is the rank of the surviving CP group. Discrete shifts of $A^a_i$ can be compensated by discrete shifts of the KK momenta $m_i$ and are thus symmetries of the spectrum [1].

By the same line of arguments as in [26], one can determine the moduli space of the compactification, up to global identifications. Indeed the exact perturbative symmetries of the type I unorientifolds, i.e. the decoupling at zero-momentum of the moduli fields from all perturbative scattering amplitudes [1], allow one to put $G_{ij}, \tilde{B}_{ij}, A^a_i$ and the dilaton $\Phi$ in one-to-one correspondence with the generators of a solvable Lie algebra, that upon exponentiation produces the moduli space (2).

The global identifications that form the smallest group that combines the discrete shifts of $\tilde{B}_{ij}$ and $A^a_i$ with the $GL(d, Z) \times O(n; Z)$ transformations is the expected U-duality group $O(d, d + n; Z)$

[9]. Notice that the identification of the U-duality group relies on the assumption of $R \times R$-charge integrality for non-perturbative D-brane states. This assumption allows for discrete but otherwise arbitrary shifts of $\tilde{B}_{ij}$. Moreover the $O(d, d; Z)$ subgroup is not the perturbative T-duality group of the parent type II superstring. It is instead the subgroup of the full U-duality group $E_{(d+1)}(d + 1; Z)$ that involves mixing $G_{ij}/\sqrt{\text{det}(G_{ij})}$ with $\tilde{B}_{ij}$ and a certain combination of the dilaton $\Phi$ and $\text{det}(G_{ij})$. The scalar $\Phi_H$ parameterizing the $R^+$ factor of the type I moduli space follows from an educated use of heterotic - type I duality in $D = 10 - d$ dimensions and coincides with the combination of $\Phi$ and $\text{det}(G_{ij})$ that gives the heterotic dilaton [27]:

$$\Phi_H = \frac{6 - D}{4} \Phi_I - \frac{D - 2}{16} \ln(\text{det}G_I).$$

(22)

[9]This conclusion has been reached in collaboration with Carlo Angelantonj and Yassen Stanev.
One can interpret the observed rank reduction of the CP group in the BPS unorientifolds as associated to non-trivial holonomies for unconventional type I superstring propagation on $T^d$ with quantized NS×NS antisymmetric tensor much in the same way as in unconventional type I compactifications to $D = 6$ on $K3$ [11]. Indeed one can define the generalized second Stiefel-Whitney class of the vacuum gauge bundle $\mathcal{V}$, $\omega_2(\mathcal{V})$, via the relation

$$W(\gamma_1)W(\gamma_2) = \exp \left( i\pi \int_{\Sigma} \omega_2(\mathcal{V}) \right) W(\gamma_2)W(\gamma_1),$$  \hspace{1cm} (23)$$

where $W(\gamma_1)$ and $W(\gamma_2)$ are Wilson loops and, for the toroidal case under consideration, the two-cycle $\Sigma$ is the product of the one-cycles $\gamma_1$ and $\gamma_2$. Recall that for Spin(32)/$\mathbb{Z}_2$, the expected non-perturbative gauge group of the type I superstrings, $\omega_2(\mathcal{V})$ represents the obstruction to defining a vector structure [47]. The relation $B = \frac{1}{2}\omega_2$, up to a shift in integer cohomology, follows from duality between open- and closed-string viewpoints in the BPS unorientifolds. Indeed the phase-shift a closed-string experiences in the presence of a quantized flux of $B$

$$\exp \left( 2i\pi \int_{\Sigma} B \right)$$  \hspace{1cm} (24)$$

must exactly match the phase in (23) [11]. In a recent paper [22] the geometry of the CHL strings has been interpreted in terms of non-commuting Wilson lines. Another independent analysis can be found in [48]. Still, the type I description in terms of a quantized NS×NS antisymmetric background and the inextricable link between open and closed unoriented strings seem by far to provide the most economical explanation. It may also have far reaching consequences in phenomenologically viable compactifications, since $\mathcal{N} = 1$ supersymmetric heterotic models in $D = 4$ with adjoint Higgses often require higher level current algebras on the world-sheet. Higher level current algebras and their by-product in terms of adjoint Higgses should require a non-vanishing $\omega_2(\mathcal{V})$ in Calabi-Yau compactifications as well. Their type I duals are thus expected to correspond to $\mathcal{N} = 1$ supersymmetric unconventional type I compactifications to $D = 4$ such as those briefly discussed in [27, 48].

An alternative interpretation of the above phenomenon has been given in terms of D-branes for the case $D = 8$ [7, 40]. For completeness and for the sake of comparison let us briefly run the argument backwards. Performing a T-duality transformation along one of the two one-cycles of $T^2$ not only turns the type IB theory into the type IA theory but also trades the NS×NS antisymmetric tensor for an off-diagonal component of the metric. The latter is to be associated to an effective reduction by a factor of two in the volume of the elementary cell defining the torus. The same reduction by a factor two results in the number of fixed lines of the new $\Omega$-projection and the neutral configuration then requires only half the original number of D8-branes and gives rise to a CP group $SO(16)$ [40, 48]. By the same line of arguments as after eq.(21) one can continuously pass to $Sp(16)$ [48].

For a direct comparison with the heterotic string, let us consider the simplest CHL model that corresponds to breaking $E(8) \times E(8)$ at level $k = 1$ to the diagonal $E(8)$ subgroup at level $k = 2$ in $D = 8$. Although the analysis of the massless spectrum has been performed in [32], we derive the genus-one partition function in order to streamline the

\[\text{We adopt the terminology recently suggested by Clifford Johnson.}\]
striking similarity with the un-orientifold construction [4]. Denoting by $\tau$ the modulus of
the worldsheet torus and neglecting the invariant measure $|d\tau|^2/\tau_2^2$, the partition function
for the conventional toroidal compactification to $D = 8$ [25] reads

$$Z_T = (\bar{V}_8 - \bar{S}_8) \left( \sum_{W \in \Lambda} \frac{q^{W^2/2}}{\eta(q)^8} \right)^2 \left( \sum_{\sigma=(p,\bar{p}) \in \Gamma} \frac{q^{p^2/2}q^{\bar{p}^2/2}}{\tau_2^3|\eta(q)^8|^2} \right), \quad (25)$$

where $\Lambda$ is the root lattice of $E(8) \times E(8)$ and $\Gamma$ is the compactification lattice of gen-
eralized momenta in (17). For compactness, we have also introduced the four characters
$\{V_8, O_8, S_8, C_8\}$ of the $SO(8)$ current algebra at level one, that encode the contribution of
the worldsheet fermions and can be easily expressed in terms of $\theta$-functions [5].

An orbifold projection that preserve all the 16 supercharges corresponds to an order-
two shift $\sigma_V$ of the coordinates of $T^2$ along a vector $V = (v, \bar{v})$, accompanied by the exchange $\tilde{\Omega}$ of the two $E(8)$ factors. Only states with the same $E(8)$ weight in (25) are
fixed under $\tilde{\Omega}$, so that the untwisted sector reads:

$$Z_{++} = \frac{1}{2} Z_T \quad (26)$$

and the $\tilde{\Omega}$-projection yields

$$Z_{+-} = \frac{1}{2} (\bar{V}_8 - \bar{S}_8) \left( \sum_{W \in \Lambda} \frac{(q^2)^{W^2/2}}{\eta(q^2)^8} \right) \left( \sum_{\sigma=(p,\bar{p}) \in \Gamma} e^{2\pi i V \cdot p} \frac{q^{p^2/2}q^{\bar{p}^2/2}}{\tau_2^3|\eta(q)^8|^2} \right). \quad (27)$$

Performing an modular S-transformation, i.e. $\tau \rightarrow -1/\tau$, on (27) and a Poisson resum-
amon on $(p,\bar{p})$ yields the twisted sector:

$$Z_{--} = \frac{1}{2} (\bar{V}_8 - \bar{S}_8) \left( \sum_{W \in \Lambda} \frac{(-q)^{W^2/2}}{\eta(-q)^8} \right) \left( \sum_{\sigma=(p,\bar{p}) \in \Gamma} \frac{q^{(p+\bar{v})^2/2}q^{(\bar{p}+\bar{v})^2/2}}{\tau_2^3|\eta(q)^8|^2} \right) \quad (28)$$

Finally performing a modular T-transformation, i.e. $\tau \rightarrow \tau + 1$, yields the projection of
the twisted sector:

$$Z_{-+} = -\frac{e^{2\pi i V \cdot V}}{2} (\bar{V}_8 - \bar{S}_8) \left( \sum_{W \in \Lambda} \frac{e^{iW \cdot W}}{\eta(e^{i\pi} \sqrt{q})} \right) \left( \sum_{\sigma=(p,\bar{p}) \in \Gamma} \frac{q^{(p+\bar{v})^2/2}q^{(\bar{p}+\bar{v})^2/2}}{\tau_2^3|\eta(q)^8|^2} \right) \quad (29)$$

Notice the striking similarity of the $\tilde{\Omega}$-projection with the $\Omega$-projection [4, 5] described in
Section 2. Modular invariance requires $V \cdot P$ to be half-integer and $V \cdot V$ to be integer. Generically, the twisted sector does not contribute massless states while the untwisted sector contributes the supergravity multiplet (including two graviphotons), the vector multiplets for the diagonal $E(8)$ and two generically abelian vector multiplets. Adding
Wilson lines symmetrically in the two $E(8)$'s, i.e. $A_1 = A_2 = (1, 0)^2$, allows one to break
the diagonal $E(8)$ to $SO(16)$ [32]. By varying the Wilson lines one can continuously pass
to $Sp(16)$ or to $U(8)$ thus reproducing symmetry enhancements observed in BPS un-
orientifolds in $D = 8$ [1].
There are two classes of type II string vacua with 16 supercharges. The first class corresponds to type II (2,2) strings, that have half of the supersymmetries carried by the left-movers and half by the right-movers, and is available, as stated in the introduction, for $D \leq 6$. In fact models in this class include compactifications on $K3 \times T^{d-4}$ or freely acting orbifolds thereof [16, 15]. The full perturbative spectrum is encoded in the genus-one partition function, that may be easily derived in the abelian orbifold limits of $K3$, following the standard procedure for symmetric orbifolds, or at rational points where the compactification corresponds to a tensor product of $\mathcal{N} = 2$ superconformal minimal models, following the procedure devised by Gepner [64].

The second class, that corresponds to the type II (4,0) strings, includes asymmetric orbifolds of tori. In the notation introduced above, the one-loop partition function for the toroidal compactification of the type IIB superstrings reads

$$Z_T = (\bar{V}_8 - \bar{S}_8)(V_8 - S_8) \sum_{P=(p,\bar{p}) \in \Gamma} \frac{q^{p^2/2} \bar{q}^{\bar{p}^2/2}}{\tau^3_2 |\eta(q)^8|^2}$$  (30)

In order to be explicit but at the same time to keep the discussion as simple as possible, let us restrict our attention to a $\mathbb{Z}_2$-orbifold that acts as $(-)^F \times \sigma_V$, where $\sigma_V$ is a shift of order two in the lattice $\Gamma$, and thus preserves all the 16 left-moving supercharges. The $\mathbb{Z}_2$-projection of the untwisted sector yields:

$$Z_{++} = \frac{1}{2} Z_T$$  (31)

and

$$Z_{+-} = \frac{1}{2} (\bar{V}_8 - \bar{S}_8)(V_8 + S_8) \sum_{P=(p,\bar{p}) \in \Gamma} e^{2\pi i V \cdot P} \frac{q^{p^2/2} \bar{q}^{\bar{p}^2/2}}{\tau^3_2 |\eta(q)^8|^2}$$  (32)

Due to the change of sign in $S_8$, in the NS×R sector and, more interestingly, in the R×R sector only massive states with $V \cdot P$ an odd integer survive the projection, so that these type II (4,0) strings are in fact type II superstrings without D-branes! For the purpose of discussing gauge symmetry enhancement we also derive the twisted sector

$$Z_{-+} = \frac{1}{2} (\bar{V}_8 - \bar{S}_8)(O_8 - C_8) \sum_{P=(p,\bar{p}) \in \Gamma} q^{(p+\bar{v})^2/2} \bar{q}^{(\bar{p}+\bar{v})^2/2} \frac{1}{\tau^3_2 |\eta(q)^8|^2}$$  ,  (33)

and its projection

$$Z_{--} = -e^{2\pi i V \cdot V} \frac{1}{2} (\bar{V}_8 - \bar{S}_8)(O_8 + C_8) \sum_{P=(p,\bar{p}) \in \Gamma} e^{2\pi i V \cdot P} q^{(p+\bar{v})^2/2} \bar{q}^{(\bar{p}+\bar{v})^2/2} \frac{1}{\tau^3_2 |\eta(q)^8|^2}$$  .  (34)

As for the CHL model discussed in Section 3, a sensible projection requires $V \cdot V$ to be integer and $V \cdot P$ to be half-integer. In the twisted sector, one finds massless states from the tachyonic $O_8$ factor when $\bar{p} = 0$ and $p^2 = 1$, while the co-spinor $C_8$ contributes massless states only in the anti-decompactification limit $R_i \to 0$. Maximal supersymmetry,
together with its 32 supercharges, is restored in this limit, as well as in the decompacti-
fication limit \( R_i \to \infty \), where the spinor \( S_8 \) contributes massless states in the untwisted 
sector. Since in order to map the limit \( R_i \to 0 \) into the limit \( R_i \to \infty \) one has to perform 
a T-duality that flips the chirality of the spacetime spinors, i.e. \( C_8 \leftrightarrow S_8 \), one retrieves 
the original type IIB (4,4) model in both limits. In fact the self-dual point is the point 
where the generalized HFK mechanism takes place. Similar conclusions would have been 
reached had we started from the type IIA (4,4) superstring. This means that type IIA and 
type IIB (4,4) strings have disconnected moduli spaces because at least perturbatively – 
and the absence of D-brane for any finite volume drastically reduces the possible sources 
of non-perturbative effects – one cannot obtain one 10D theory in any limit point in the 
moduli space of the other! Moreover T-duality is part of the gauge group of these theories 
much in the same way as in more familiar heterotic compactifications [25].

The last remarks make the type II (4,0) strings a rather interesting arena to test 
various duality conjectures. In particular, the gauge group is generically abelian, in fact 
\( U(1)^{(d+d)} \) in the model discussed above, and gets enhanced only at points where \( \tilde{p} = 0 \) 
and \( \tilde{p}^2 = 1 \). This typically correspond to free fermionic points. The enhanced gauge 
symmetry is determined by the structure constants that appear in the cubic supercurrent 
[33, 35, 36]. Some type II (4,0) models admit U-dual type II (2,2) models [15, 16]. In 
particular two classes of models in \( D = 4 \) with \( N_V = 4 \) and \( N_V = 6 \) seem to fall into this 
category [15, 16]. For the former it seems a rather difficult task to construct a perturbative 
type I dual while for the latter, as promised in the Introduction, one can construct type I 
models that display exactly the same massless spectrum at generic points of the moduli 
space. They correspond to toroidal compactifications of the type I superstring with an 
unconventional Klein bottle projection that does not allow for the introduction of D9-

planes and their open-string excitations [8, 7, 40]. Indeed considering for simplicity tori 
with diagonal metric one may take advantage of some arbitrariness in the \( \Omega \)-projection 
and put

\[
\mathcal{K} = \frac{1}{2} \left( \frac{V_8 - S_8}{D/2 \eta (q{\bar{q}})^8} \right) \prod_{i=1,d} e^{i\pi \delta_i m_i} \left( q{\bar{q}} \right)^{m_i^2} R_i^2.
\]

where \( \delta_i = 0, \pm 1 \) are not fixed by the crosscap constraint [13]. The resulting crosscap-to-
crosscap amplitudes reads

\[
\mathcal{K} = \frac{2D/2}{2} \left( \frac{V_8 - S_8}{D/2 \eta (q{\bar{q}})^8} \right) \prod_{i=1,d} (q{\bar{q}})^{(2n_i + \delta_i)^2} R_i^2/2.
\]

When at least one \( \delta_i \neq 0 \), no massless closed-string states flow in the transverse channel. 
The O9-planes do not carry \( R \times R \) charge and there is no room for introducing D9-branes 
and open-strings [7, 8].

As argued in the Introduction self-duality of the type IIB superstring in \( D = 10 \) and 
application of the adiabatic argument [28] to the type IIB superstring on tori with shifts 
\( \sigma_V \) allow one to relate type II (4,0) models to type I superstrings without open strings. 
Indeed, it is easy to check that on the massless spectrum \( \Omega = S^{-1}(-)F_R S \). Although the 
argument does not work in \( D = 10 \) – the \( (-)^F_R \)-projection gives the type IIA superstring 
that is not S-dual to the type I superstring obtained via the \( \Omega \)-projection – the argument 
seems to work in lower dimension, when combined with a non-trivial \( Z_2 \)-action on the
compactification manifold, in the case under consideration an order two shift $\sigma_V$ on a torus $T^d$. Moreover the different inequivalent choices of $\sigma_V$ with $v = \bar{v}$ are in one to one correspondence with the different inequivalent choices of $\delta_i = v_i/2$ in (35). This allows one to deduce a precise map between the two classes of models and conclude that they give rise to several disconnected components of the moduli spaces of compactifications with the same $N_V = d$ but with different global identifications. In particular, the surviving T-dualities are those that commute with the shift $\sigma_V$.

Although the type II (4,0) strings and type I strings without open strings have generically the same low-energy limit and share the same moduli space, identifying the ones with the others requires a precise map between the BPS states in the two theories. In particular, one should study the behavior of the type I theory at images of the points of enhanced symmetry in the type II (4,0) strings. The analysis of [21] suggests that the type I string coupling diverge at these points and the relevant D-brane states become massless. It is not clear however whether the $R \times R$ charges of these states are quantized in units of the fundamental $R \times R$ charge of conventional D-branes.

5 Final Comments

Superstring vacua with 16 supercharges provide us with a plethora of models for which tests of various duality conjectures seem both feasible and interesting. The advantage of this class of theories is that supersymmetry alone does not fix uniquely the massless spectrum, but it is still powerful enough to fix completely the low-energy lagrangian once the spectrum is fixed. In particular, up to global identifications, the moduli space parameterized by the scalars in the vector multiplets is $O(d, d + n)/O(d) \times O(d + n)$, where $n + d = N_V$ is the total number of vector multiplets. For unconventional type I compactifications we have shown that $n$ can take the values $n = 16 \times 2^{-b/2}$, with $b = 0, 2, 4, 6$, and $n = 0$.

The state of the art suggests that the existence of several disconnected components can find a very simple rationale in terms of BPS unorientifolds, i.e. type I toroidal compactifications with a quantized NS×NS antisymmetric tensor [1]. Other components, that include type I theories with the minimum number of vector multiplets, i.e. $n = 0$ or $N_V = d = 10 - D$, admit at least two different realizations. The first in terms of type II superstrings without D-branes, i.e. type II (4,0) models, the second in terms of type I superstrings without open strings. An adiabatic argument suggests that the two description are dual to one another. We have not succeeded in finding a way to reduce the number of vectors in the unoriented closed-string spectrum of the type I superstring to reproduce the disconnected components with $N_V < 6$ in $D = 4$ and $N_V < 4$ in $D = 6$, but we hope to investigate this issue in the future.

For type I superstrings other disconnected components of the moduli space of vacua can be found by adding open-string discrete torsion and/or Wilson lines that do not lie in $SO(32)$ [9, 1]. In view of the conjectured heterotic - type I duality this seems to lead to inconsistencies at the non-perturbative level, because some non perturbative D-

\[^{11}\text{Here, BPS stands for Bogomolny-Prasad-Sommerfield.}\]
brane states are expected to carry spinorial charges of $SO(32)$ and thus would not admit a sensible action of $O(32)$ Wilson lines. However, it may well be possible that in these component of the moduli space the type I superstrings do not admit perturbative heterotic duals but rather type II $(4,0)$ superstring duals. Once again, establishing connections and checking various dualities for these unconventional models seems to be more than compelling.

The recent insights in the duality realm of super Yang-Mills theories have taught us that gauge symmetry is not a good order parameter for discriminating between different theories. It simply represent a redundancy in the description of a system. Indeed, in different regimes the same exact theory can display different gauge symmetries [65]12. Rolling among topologically different superstring vacua with 8 supercharges, such as Calabi-Yau compactifications of the type II superstrings or their heterotic and type I duals, is the superstring counterpart of the above field-theory phenomenon. For superstring vacua with 16 supercharges a mechanism analogous to black hole condensation in conifold transitions or to shrinking of instantons in transitions with tensionless strings has not been proposed so far. Rolling among topologically different superstring vacua with 16 supercharges seems forbidden or at least strongly suppressed.

As a final remark, the present development in our understanding of superstrings at the non-perturbative level makes the problem of vacuum selection sharper and more compelling [66]. Superstring vacua with 16 supercharges include vacua that do not admit a natural geometric interpretation such as unconventional un-orientifolds and asymmetric orbifolds that are clearly calling for a thorough analysis. The existence of an underlying theory of membranes and/or pentabranes bringing into play new consistent non-perturbative vacuum configurations, corresponding to the wrapping branes of complicated topology around cycles of the compactification manifold, can only make the problem more severe at first glance.

6 Note Added

While this work was being typed, I was informed by A. Sagnotti that E. Witten [67] was also considering issues related to the quantized NS×NS antisymmetric tensor introduced in [1].

7 Acknowledgements

I would like to acknowledge useful discussions with C. Angelantonj, D. Polyakov, G. Pradisi, K. Ray, Ya. Stanev, and A. Sagnotti, whom I especially thank for carefully reading the manuscript. I would like to thank the organizers of the “Abdus Salam Memorial Meeting” for their kind hospitality at the Abdus Salam ICTP while this work has been brought to completion.

12Clearly, in the regime in which a gauge symmetry seems to have disappeared, the global symmetry corresponding to gauge transformations with constant parameters is trivially represented because the new composite degrees of freedom are expected to be singlets of the original symmetry!
References


[3] See *e.g.* A. Sen, hep-th/9802501, for a comprehensive review.


[38] P. Hořava and E. Witten, hep-th/9510209.


[67] E. Witten, hep-th/9712028.