Quantization of spontaneously broken gauge theory
based on the BFT–BFV Formalism

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Abstract

We quantize the spontaneously broken abelian U(1) Higgs model by using
the improved BFT and BFV formalisms. We have constructed the BFT physical
fields, and obtain the first class observables including the Hamiltonian in
terms of these fields. We have also explicitly shown that there are exact form
invariances between the second class and first class quantities. Then, accord-
ing to the BFV formalism, we have derived the corresponding Lagrangian
having U(1) gauge symmetry. We also discuss at the classical level how one
easily gets the first class Lagrangian from the symmetry-broken second class
Lagrangian.
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I. INTRODUCTION

Many of the fundamental theories of modern physics can be considered as descriptions of dynamical systems subjected to constraints. The foundations for Hamiltonian quantization of these constrained systems have been established by Dirac [1]. By requiring the strong implementation of second class constraints, however, this method implies Dirac brackets, whose non-canonical structure may pose serious problems on operator level. This makes it desirable to embed the second class theory into a first class one in which the commutator relations remain canonical.

An example is provided by the Higgs model with spontaneous symmetry breakdown [2] whose quantization is usually carried out in the so called “unitary” gauge. As is well known, in this gauge the model is a purely second class system characterized by two sets of the second class constraints [3,4]. The required strong implementation of these constraints leads to non-polynomial field dependent Dirac brackets. As mentioned above, one can circumvent the problems associated with this non-polynomial dependence by turning this system into a first class one with a usual Poisson bracket structure in an extended phase space and implementing the first class constraints on the physical states.

A systematic procedure for achieving this has been given by Batalin and Fradkin (BF) [5] in the canonical formalism, and applied to various models obtaining the Wess-Zumino (WZ) action [6]. In particular, this analysis explicitly carried out for the above Higgs model [4]. However, it is already proved that the construction of the first class Hamiltonian in the BF framework is non-trivial even in the abelian case because of the field dependence on the constraint algebra. In this case the only weakly involutive first class Hamiltonian is obtained after the fifth iteration, and thus it does not appear particularly to be suited for treating non-abelian cases.

A more systematic and transparent approach for this iterative procedure, called Batalin–Fradkin–Tyutin (BFT) formalism when combined with the BF one, has been developed by Batalin and Tyutin [7]. This procedure has been applied to several interesting models [8,9], where the iterative process is terminated after two steps. In general, it has been, however, still difficult to apply this BFT formalism to the nonabelian case [10]. On the other hand, we have recently improved the BFT formalism by introducing the novel concept of the BFT
physical fields constructed in the extended phase space [11] in order to construct the strongly involutive observables including the Hamiltonian. This modified version of the BFT method has been successively applied for only finding the first class Hamiltonian of several nontrivial nonabelian models [12–14]. In particular, the origin of the second class constraints of the Higgs model comes from the spontaneous symmetry breaking effects, while that of non-abelian Proca model [12,13] due to the existence of the explicitly symmetry broken mass term. Therefore, it is very interesting to analyze the Higgs model having the different origin, which is phenomenologically important.

In this letter we shall revisit the spontaneously broken abelian U(1) Higgs model by following the constructive procedure based on the improved BFT version [11]. In section 2, we convert the second class constraints into a first class ones, and construct in section 3 the BFT physical fields in the extended phase space corresponding to the original fields in the usual phase space, following the improved BFT formalism. We then systematically obtain all observables containing the first class Hamiltonian as functionals of the BFT physical fields showing the form invariances between the second class and first class quantities. In section 4, through the standard path integral quantization established by Batalin, Fradkin and Vilkovsky (BFV) [16,17], we derive the gauge invariant Lagrangian. In section 5, we suggest a novel path at the classical level how one can easily get this Lagrangian from the original second class one by simply replacing the original fields with the BFT ones through an additional relation without following usually complicated path integral quantization. This new method will be possible to analyze the realistic non-abelian Higgs models. We summarize in section 6.

II. BFT CONSTRUCTION OF FIRST CLASS CONSTRAINTS

Consider the abelian U(1) Higgs model in the unitary gauge [3,4],

\[ \mathcal{L}_u = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^2 (\rho + v)^2 A_\mu A^\mu + \frac{1}{2} \partial_\mu \rho \partial^\mu \rho + V(\rho), \]

where the subscript “\(u\)” stands for the unitary gauge, the Higgs potential is \( V(\rho) = \frac{1}{2} \mu^2 (\rho + v)^2 - \frac{3}{4} (\rho + v)^4 \), and the field strength tensor \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). The momenta canonically
conjugate to $A^0, A^i$ and $\rho$ are given by $\pi_0 = 0, \pi_i = F_{i0}$, and $\pi_\rho = \dot{\rho}$, respectively. We have thus one primary constraint

$$\Omega_1 = \pi_0 \approx 0. \quad (2)$$

The canonical Hamiltonian density associated with the Lagrangian (1) is given by

$$H_c = \frac{1}{2}\pi_0^2 + \frac{1}{2}\pi_\rho^2 + \frac{1}{4}F_{ij}F^{ij} - \frac{1}{2}g^2(\rho + v)^2 \left((A^0)^2 - (A^i)^2\right) - A^0 \partial_i \pi_i + \frac{1}{2}(\partial_\rho \rho)^2 - V(\rho). \quad (3)$$

Persistency in time of $\Omega_1$ leads to one further (secondary) constraint

$$\Omega_2 = \partial_i \pi^i + g^2(\rho + v)^2 A^0 \approx 0. \quad (4)$$

Then, the constraints $\Omega_i$ in Eqs. (2) and (4) consist of a second class system because we have the nonvanishing Poisson brackets

$$\Delta_{ij}(x, y) \equiv \{\Omega_i(x), \Omega_j(y)\} = -g^2(\rho + v)^2 \epsilon_{ij}\delta^3(x - y), \quad (5)$$

where $\epsilon_{12} = \epsilon^{12} = 1$.

We now convert this second class system defined by the commutation relations (5) to a first class one at the expense of introducing additional degrees of freedom. According to the BFT method [7], we first introduce two auxiliary fields $\Phi^i$ corresponding to $\Omega_i$ with the Poisson brackets

$$\{\Phi^i(x), \Phi^j(y)\} = \omega^{ij}(x, y), \quad (6)$$

where we are free to make a choice

$$\omega^{ij}(x, y) = \epsilon^{ij}\delta^3(x - y). \quad (7)$$

The first class constraints $\tilde{\Omega}_i$ are then constructed as a power series in the auxiliary fields:

$$\tilde{\Omega}_i = \sum_{n=0}^{\infty} \Omega_i^{(n)}; \quad \Omega_i^{(0)} = \Omega_i, \quad (8)$$

where $\Omega_i^{(n)}$ are homogeneous polynomials in the auxiliary fields $\Phi^j$ of degree $n$, to be determined by the requirement that the first class constraints $\tilde{\Omega}_i$ be strongly involutive:

$$\{\tilde{\Omega}_i(x), \tilde{\Omega}_j(y)\} = 0. \quad (9)$$

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Since $\Omega_i^{(1)}$ are linear in the auxiliary fields, we could make the ansatz

$$\Omega_i^{(1)} = \int d^3y X_{ij}(x,y)\Phi^j(y).$$

(10)

Then, substituting (10) into (9) leads to the following relation

$$\int d^3zd^3z'X_{ik}(x,z)\omega^{kl}(z,z')X_{jl}(z',y) = -\Delta_{ij}(x,y).$$

(11)

For the choice of (7), Eq. (11) has a solution

$$X_{ij}(x,y) = \begin{pmatrix}
g^2(\rho + v)^2 & 0 \\
0 & 1
\end{pmatrix} \delta^3(x - y).$$

(12)

Substituting (12) into (10) as well as (8), and iterating this procedure one finds the strongly involutive first class constraints to be given by

$$\tilde{\Omega}_1 = \Omega_1 + g^2(\rho + v)^2\Phi^1,$$

$$\tilde{\Omega}_2 = \Omega_2 + \Phi^2.$$  

(13)

This completes the construction of the first class constraints in the extended phase space.

**III. CONSTRUCTION OF FIRST CLASS OBSERVABLES**

The construction of the first class Hamiltonian $\tilde{H}$ can be done along similar lines as in the previous case of the constraints. However, we shall follow here a somewhat different path [11] by using the novel property that any functional $\mathcal{K}(\tilde{F})$ of the first class fields $\tilde{F} = (\tilde{A}^\mu, \tilde{\pi}_\mu, \tilde{\rho}, \tilde{\pi}_\rho)$ corresponding to the original fields $F = (A^\mu, \pi_\mu, \rho, \pi_\rho)$ will also be first class. i. e.,

$$\tilde{\mathcal{K}}(F; \Phi) = \mathcal{K}(\tilde{F}).$$

(14)

This leads us to the identification $\tilde{H}_c = \mathcal{H}_c(\tilde{F})$. To do this, we should first construct first class “physical fields” $\tilde{F}$ in the extended phase space, which are obtained as a power series in
the auxiliary fields \( \Phi^i \) by requiring them to be strongly involutive: \( \{ \tilde{\Omega}_i, \tilde{\mathcal{F}} \} = 0 \). Expressions of the strongly involutive \( \tilde{\mathcal{F}} \) are given by

\[
\tilde{A}^\mu = (A^0 + \frac{1}{g^2(\rho + v)^2} \pi_\theta, A^i + \partial^i \theta),
\]

(15)

\[
\tilde{\pi}_\mu = (\pi_0 + g^2(\rho + v)^2 \theta, \pi_i),
\]

(16)

\[
\tilde{\rho} = \rho,
\]

(17)

\[
\tilde{\rho} = \pi_\rho + 2g^2(\rho + v)A^0 \theta.
\]

(18)

Here, for the later convenience, we have identified the auxiliary fields \( \Phi^i \) as a canonically conjugate pair \((\theta, \pi_\theta)\) by choosing

\[
\Phi^i = (\theta, \pi_\theta),
\]

which satisfy the symplectic structure (2.6) with the choice of Eq. (2.7). In order to understand the meaning of these BFT fields, let us now consider the Poisson brackets between the BFT fields in the expended phase space. From the relations of Eqs. (15–18), one can easily calculate the Poisson brackets of the abelian Higgs model as follows

\[
\{ \tilde{A}^0(x), \tilde{A}^i(y) \} = \frac{1}{g^2(\rho + v)^2} \partial^i \delta^3(x - y),
\]

\[
\{ \tilde{A}^0(x), \tilde{\pi}_\rho(y) \} = -\frac{2\tilde{A}^0}{(\rho + v)} \delta^3(x - y),
\]

\[
\{ \tilde{A}^i(x), \tilde{\pi}_j(y) \} = \delta^i_j \delta^3(x - y),
\]

\[
\{ \tilde{\rho}(x), \tilde{\pi}_\rho(y) \} = \delta^3(x - y).
\]

(19)

Note that these Poisson brackets in the extended phase space have the form invariance as compared with the Dirac brackets in the original phase space, and moreover, if we take the limit of \( \Phi^i = (\theta, \pi_\theta) \to 0 \), the brackets (19) are nothing but the usual Dirac brackets of the abelian Higgs model [3]. On the other hand, we observe that the first class constraints (13) can be written in terms of the BFT physical fields \( \mathcal{F} \) as

\[
\tilde{\Omega}_1 = \tilde{\pi}_0,
\]

\[
\tilde{\Omega}_2 = \partial^i \tilde{\pi}_i + g^2(\rho + v)^2 \tilde{A}^0,
\]

(20)

and these constraints also have the form invariance with the second class constraints \( \Omega_i \) in Eqs. (2) and (4).
Correspondingly, we take the first class Hamiltonian density $\tilde{\mathcal{H}}_c$ to be given by the second class one (3), expressed in terms of the physical fields:

$$\tilde{\mathcal{H}}_c = \frac{1}{2} \tilde{\pi}_i^2 + \frac{1}{2} \tilde{\pi}_\rho^2 + \frac{1}{4} \tilde{F}_{ij} \tilde{F}^{ij} - \frac{1}{2} g^2 (\tilde{\rho} + v)^2 \left((\tilde{A}^0)^2 - (\tilde{A}^i)^2\right) - \tilde{A}^0 \partial_i \tilde{\pi}_i + \frac{1}{2} (\partial_i \tilde{\rho})^2 - V(\tilde{\rho}), \quad (21)$$

and, by construction, $\tilde{H}_c = \int d^3 x \, \tilde{\mathcal{H}}_c$ is automatically strongly involutive

$$\{\tilde{\Omega}_i, \tilde{H}_c\} = 0. \quad (22)$$

It seems appropriate to comment on our strongly involutive Hamiltonian (21) derived by using the improved BFT formalism. Making use of (15–18) and (21), we may rewrite $\tilde{\mathcal{H}}_c$ in the form

$$\tilde{\mathcal{H}}_c = \mathcal{H}_c + \Delta \mathcal{H} - \frac{\pi_\theta}{g^2 (\rho + v)^2} \tilde{\Omega}_2, \quad (23)$$

where $\Delta \mathcal{H}$ is given by

$$\Delta \mathcal{H} = 2g^2 (\rho + v) \theta A_0 \left(\pi_\rho + g^2 (\rho + v) \theta A_0\right) - g^2 (\rho + v)^2 \theta (A_i + \frac{1}{2} \partial_i \theta)$$

$$+ \frac{\pi_\theta^2}{2g^2 (\rho + v)^2}.$$ 

If we construct our strongly involutive Hamiltonian along similar BFT lines as in the case of the constraints, we obtain this Hamiltonian (23) after only second iterations (see the Appendix A), while the weakly involutive Hamiltonian in ref. [4] derived by using the BF formalism is obtained after the fifth iteration in spite of the abelian case and has rather complicated additional terms given in Eq. (2.29) of ref. [4]. Any Hamiltonian weakly equivalent to (21), however, describes the same physics since the observables of the first class formulation must be first class themselves, and thus these two Hamiltonians are equivalent to each other. Therefore, we can add to $\tilde{H}_c$ any terms freely proportional to the first class constraints. In particular, if we choose the simplest Hamiltonian density among infinite equivalent ones:

$$\tilde{\mathcal{H}}'_c = \mathcal{H}_c + \Delta \mathcal{H}, \quad (24)$$

then this naturally generates the Gauss law constraint $\tilde{\Omega}_2$

$$\{\tilde{\Omega}_1, \tilde{\mathcal{H}}'_c\} = \tilde{\Omega}_2, \quad \{\tilde{\Omega}_2, \tilde{\mathcal{H}}'_c\} = 0, \quad (25)$$
and it will be proved to be useful through the following discussion as well as the next section.

If we consider this Hamiltonian (24), the form-invariant Hamilton’s equations of motion for the physical BFT fields are found to be read

\[
\begin{align*}
\dot{\tilde{A}}^0 &= \partial^i \tilde{A}^i + \frac{2}{(\tilde{\rho} + v)} \tilde{A}^i \partial^i \tilde{\rho} - \frac{2}{(\tilde{\rho} + v)} \pi_\rho \tilde{A}^0, \\
\dot{\pi}_0 &= \tilde{\Omega}_2, \\
\dot{\tilde{A}}^i &= \tilde{\pi}_i + \partial^i \tilde{A}^0, \\
\dot{\tilde{\pi}}_i &= -\partial^j \tilde{F}_{ij} - g^2 (\tilde{\rho} + v)^2 \tilde{A}^i, \\
\dot{\tilde{\rho}} &= \tilde{\pi}_\rho, \\
\dot{\pi}_\rho &= -g^2 (\tilde{\rho} + v) \left( (\tilde{A}^0)^2 + (\tilde{A}^i)^2 \right) + \partial^2 \tilde{\rho} + V(\tilde{\rho}),
\end{align*}
\]

where \( V(\tilde{\rho})' = \frac{\partial}{\partial \tilde{\rho}} V(\tilde{\rho}) \). But, if one try to derive the equations of motion from the strongly involutive Hamiltonian (21), one can only obtain the weak relations since as an example the Hamilton’s equation of motion for \( \tilde{A}^i \) is reduced to be

\[
\dot{\tilde{A}}^i = \tilde{\pi}_i + \partial^i \tilde{A}^0 - \partial^2 \left( \frac{1}{g^2 (\tilde{\rho} + v)^2} \tilde{\Omega}_2 \right).
\]

As a result, these relations (26) together with Eqs. (20) and (25) confirm the form invariances between the second class quantities in the original phase space and the corresponding first class ones in the extended phase space.

### IV. CORRESPONDING FIRST CLASS LAGRANGIAN

In order to interpret the results presented at the previous sections from the Lagrangian point of view, let us apply the BFV quantization scheme [16,17] to the first class system described by Eqs. (20) and (24). We first introduce ghosts \( C^i \), antighosts \( \bar{P}^i \) and new auxiliary fields \( q^i \) with their canonically conjugate momenta \( p^i, C_i \) and \( p_i \) such that

\[
[C^i, \bar{P}^j] = [\bar{P}^i, C_j] = [q^i, p_j] = i\delta^i_j \delta^3(x - y),
\]

where the subscript \( i, j = 1, 2 \), due to having the two constraints in Eq. (20), and from now on we will use the commutators instead of the Poisson brackets. The nilpotent BRST charge \( Q \) and the fermionic gauge fixing function \( \Psi \) are then given by
\[ Q = \int d^3x \left( C^i \tilde{\Omega}_i + \mathcal{P}^i p_i \right), \]
\[ \Psi = \int d^3x \left( \mathcal{C}_i \chi^i + \mathcal{P}_i q_i \right), \]

where \( \chi^i \) are gauge fixing functions satisfying the condition \( \text{det}\{\chi^i, \tilde{\Omega}_j\} \neq 0 \) \([17,18]\). The total unitarizing Hamiltonian is then given by

\[ H_T = H_m + \frac{1}{i} [\Psi, Q]. \]

Since we have the involutive relations (25), the minimal Hamiltonian \( H_m \) is nothing but

\[ H_m = \tilde{\mathcal{H}}^t_c + \int d^3x \mathcal{P} \mathcal{C}. \]

The corresponding quantum theory is now defined by the extended phase space functional integral

\[ Z_I = \int D\mu e^{iS_I}; \]
\[ S_I = \int d^3x \left( \pi_\mu \dot{A}^\mu + \pi_\rho \dot{\rho} + \pi_\theta \dot{\theta} + \mathcal{P}_i \mathcal{C}^i + \mathcal{P}_i \mathcal{C}^i \right) - \mathcal{H}_T; \]
\[ D\mu = DA^\mu \pi_\mu \rho \pi_\rho \theta \pi_\theta \mathcal{P}_i \mathcal{C}_i \mathcal{P}_i \mathcal{C}_i. \]

According to the BFV formalism \([16,17]\), \( Z_I \) is independent of the choice of the gauge fixing functions \( \chi^i \). By choosing the proper \( \chi^i \), which do not include the ghosts, antighosts, and auxiliary fields and their conjugate momenta, and taking the limit of \( \beta \to 0 \) after rescaling the field variables as \( \chi^i \to \chi^i/\beta \), \( p_i \to \beta p_i \), and \( \mathcal{C}_i \to \beta \mathcal{C}_i \), one can integrate out all the ghost, antighosts and auxiliary variables in the partition function. As a result, one obtains

\[ Z_{II} = \int D\mu_{II} \prod_{i,j} \delta(\tilde{\Omega}_i) \delta(\chi^j) \text{det}[\chi, \tilde{\Omega}] e^{iS_{II}}, \]

where \( S_{II} \) is the action in the extended phase space

\[ S_{II} = \int d^4x \left( \pi_\mu \dot{A}^\mu + \pi_\rho \dot{\rho} + \pi_\theta \dot{\theta} - \mathcal{H}_c \right), \]

and \( D\mu_{II} \) is the measure containing all the fields and their conjugate momenta except for the ghosts, antighosts and auxiliary fields in the measure \( D\mu_I \). Note that this form of the partition function \( S_{II} \) coincides with the Faddeev-Popov formula \([19]\).

Now, let us perform the momentum integrations to obtain the configuration partition function. In general, if we choose the Faddeev-Popov type gauges in which the gauge fixing
conditions $\chi^i$ only depend on the configuration space variables, one can easily carry out the momentum integrations in the partition function (32).

The $\pi_0$ integration is trivially performed by exploiting the delta function $\delta(\tilde{\Omega}_1)$, and after exponentiating the remaining delta function $\delta(\tilde{\Omega}_2)$ in terms of a Fourier variable $\xi$ as $\delta(\tilde{\Omega}_2) = \int D\xi \exp(-i \int d^4x \xi \tilde{\Omega}_2)$ and transforming $A^0 \to A^0 + \xi$, the integration over the momenta $\pi_\theta, \pi_\rho$ and $\pi_i$ leads to

$$Z = \int D\pi_0 D\rho D\theta D\xi \prod_i \delta(\chi^i) \det[\chi, \tilde{\Omega}] \det(g(\rho + v)) e^{iS_{GI}},$$

where

$$S_{GI} = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}[\partial_\mu + ig(A_\mu + \partial_\mu \theta)](\rho + v)[\partial^\mu - ig(A^\mu + \partial^\mu \theta)](\rho + v) + V(\rho)\right),$$

and the $\xi$ field in the measure is an artifact which would be removed if we take the gauge fixing function $\chi^i$ explicitly. This action $S_{GI}$ is now gauge invariant under the transformation

$$A_\mu \to A_\mu + \partial_\mu \Lambda, \quad \theta \to \theta - \Lambda, \quad \rho \to \rho.$$  

This completes our analysis on the BFV quantization scheme.

It only remains to establish the equivalence between the above gauge invariant action and the well-known U(1) Higgs model. By defining the complex scalar field $\phi(x)$ as

$$\phi(x) = \frac{1}{\sqrt{2}} (\rho(x) + v) e^{-i\theta(x)}$$

with the BFT field $\theta$ playing the role of the Goldstone boson, and replacing the Jacobian factor $D\theta D\rho \det(g(\rho + v))$ in the measure part with $D\phi D\phi^*$, we can easily rewrite the partition function (34) with the action (35) as follows

$$Z_F = \int D\pi_0 D\rho D\phi \prod_i \delta(\chi^i) \det\{\chi, \tilde{\Omega}\} e^{iS_F};$$

$$S_F = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu \phi)^*(D^\mu \phi) + \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2\right),$$

where $D_\mu = \partial_\mu - igA_\mu$ is the covariant derivative. As a result, we have arrived at the well-known U(1) Higgs model, which describes the interaction of the abelian gauge fields $A^\mu$ with the complex scalar fields $\phi$, through the BFT–BFV construction.
Let us now consider in this section the similar economical method to simply obtain the first class Lagrangian (35) at the classical level. It consists in gauging the Lagrangian (1) by making the substitution $A^\mu \to \tilde{A}^\mu$ and $\rho \to \tilde{\rho}$. The spatial components $\tilde{A}^i$ among the vector potential components contain only the fields of the configuration space as in Eq. (15), and already take the usual form of the gauge transformation, i.e., $\tilde{A}^i \to A^i + \partial^i \theta$. However, since the time component $\tilde{A}^0$ contains the term of the momentum field $\pi_\theta$ as in Eq. (15), we should first replace this term with some ordinary field before carrying out the above substitution. In order to incorporate the $\tilde{A}^0$ field at this stage, we use an additional relation, which has not been recognized up to now.

From the useful property (14) and the definition of $\pi_i$, we observe the following relation:

$$\tilde{\pi}_i = \partial_i \tilde{A}_0 - \partial_0 \tilde{A}_i$$

$$= \partial_i \left( A_0 + \frac{1}{g^2(\rho + v)^2} \pi_\theta \right) - \partial_0 (A_i + \partial_i \theta).$$

(39)

On the other hand, another form of $\tilde{\pi}_i$ is already given in Eq. (16) as follows

$$\tilde{\pi}_i = \pi_i = \partial_i A_0 - \partial_0 A_i.$$ (40)

Comparing this with Eq. (39), we see that the following additional relation should be maintained for the consistency all the times

$$\partial^0 \theta = \frac{1}{g^2(\rho + v)^2} \pi_\theta,$$ (41)

which make it possible to directly replace the second term of $\tilde{A}^0$ with $\partial^0 \theta$. By making use of this relation, we can now rewrite the $\tilde{A}^0$ as the usual form of the gauge transformation as follows

$$\tilde{A}^0 = A^0 + \partial^0 \theta.$$ (42)

Note that the Hamilton’s equations (26) can be also used to confirm the relation (41). Therefore, gauging the original Lagrangian (1), i.e., making the substitution

$$\tilde{A}^\mu \to A^\mu + \partial^\mu \theta, \quad \tilde{\rho} \to \rho,$$ (43)
we have directly arrived at the same first class Lagrangian (35) at the classical level,

$$\mathcal{L}(\tilde{A}^\mu, \tilde{\rho}) = \tilde{\mathcal{L}}(A^\mu, \theta, \rho) = \mathcal{L}_{\text{GI}},$$

(44)

which is already obtained through the standard path integral procedure in the previous section.

VI. SUMMARY

In this letter, we have quantized the spontaneously broken abelian U(1) Higgs model, which is a phenomenologically interesting and simple toy model, through the BFT–BFV quantization procedure.

First, according to the improved version [11] of the BFT formalism, we have constructed the BFT physical fields, and proved that the Poisson brackets between these BFT fields naturally contain the structure of the Dirac brackets [3] in the original phase space for the abelian U(1) Higgs model as like in Eq. (19), while maintaining the form invariance in the extended phase space.

Second, we have shown that the strongly involutive first class Hamiltonian (21) is directly obtained by replacing the second class fields with the first class BFT ones.

Third, after directly obtaining the above Hamiltonian and by choosing the simplest involutive Hamiltonian $\mathcal{H}'_c$ among equivalent infinite ones, we have shown that this $\mathcal{H}'_c$ in Eq. (24) naturally generates the Gauss law constraint, and also gives the strong Hamilton’s equations of motion (26). Moreover, we have also shown that there are the exact form invariances between the second class and first class quantities in Eqs. (20, 21) and (26), which give us the deep physical meaning of the BFT fields when we embed a second class system into first class by using the BFT construction.

Fourth, we have carried out the BFV quantization procedure in order to interpret the results of the Hamiltonian embedding of the abelian U(1) Higgs model from the Lagrangian point of view, and constructed the gauge invariant Lagrangian corresponding to the first class Hamiltonian.

Fifth, by using the additional relation (41), we have newly shown that one can directly obtain the first class Lagrangian from the second class one by just replacing the original
fields with the BFT ones at the classical level, similar to the case of the Hamiltonian. In particular, this kind of the BFT Lagrangian construction will be powerful for the analysis of the non-abelian cases, where the non-abelian extension of the $\tilde{A}^0$ remains to be weakly involutive to the usual gauge transformation.

In conclusion, we have shown that the improved version [11] of the BFT formalism is more economical than the previous BFT versions [8–10] including the BF one [4] by explicitly analyzing the abelian U(1) Higgs model. We hope that this powerful BFT formalism with the additional relation, which we first used here, will improve the transparency of the analysis in the nonabelian cases which are realistic and phenomenological models related to the spontaneously broken symmetry.

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