Photon decay $\gamma \rightarrow \nu\bar{\nu}$ in an external magnetic field

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Abstract

The process of the photon decay into the neutrino - antineutrino pair in a magnetic field is investigated. The amplitude and the probability are analysed in the limits of relatively small and strong fields. The probability is suppressed by a factor $(G_F m_e^2)^2$ as compared with the probability of the pure electromagnetic process $\gamma \rightarrow e^-e^+$. However, the process with neutrinos could play a role of an additional channel of stellar energy-loss.

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Nowadays, it is generally recognized that astrophysical objects and processes inside them give us unique possibilities [1] for investigations of particle properties under extreme conditions of a high density and/or temperature of matter, and also of a strong magnetic field. A concept of the astrophysically strong magnetic field has been changed in the recent years and now the field is considered as the strong one if it is much greater than the known Schwinger value, $B \gg B_e$, $B_e = m_e^2/e \simeq 4.41 \cdot 10^{13} G$. Possible mechanisms are now discussed of a generation of such strong fields ($B \sim 10^{15} - 10^{17} G$) in astrophysical cataclysms like a supernova explosion or a coalescence of neutron stars [2, 3], and in the early Universe [4].

The strong magnetic field, like the medium, makes an active influence on particle properties. First, it could induce new interactions, for example, an interaction of the uncharged massless neutrinos with photons. Second, the field changes essentially the kinematics of particles, causing new channels to be opened, which are forbidden in vacuum by the momentum conservation: $\gamma \rightarrow e^-e^+$ [5], $\nu \rightarrow \nu \gamma$ [6, 7, 8, 9], $\nu \rightarrow \nu e^-e^+$ [10, 11].

In this paper, the process of the photon decay into the neutrino - antineutrino pair $\gamma \rightarrow \nu \bar{\nu}$ is investigated in a presence of an external magnetic field. The field plays a role of the active medium to change the photon dispersion in such a way that the states with the timelike photon 4-momentum $q (q^2 > 0)$ become possible. Let us note that plasma influences the photon properties similarly. The process $\gamma \rightarrow \nu \bar{\nu}$ in plasma is well studied and has an importance in astrophysics as one of the main sources of stellar cooling (see a detailed review in the book [1]).

It seems reasonable to say that both components of the active medium, a magnetic field and plasma, are presented in the most of astrophysical objects. A situation is also possible when the magnetic component dominates. For example, in a supernova explosion or in a coalescence of neutron stars a region could exist outside the neutrinosphere, of order of several hundred kilometres in size, where plasma is rather rarefied, and the magnetic field of the toroidal type could reach a value of $10^{14} - 10^{16} G$ [2].

The purpose of our work is to calculate the probability of the photon decay $\gamma \rightarrow \nu \bar{\nu}$ in a strong magnetic field in the frame of the standard model, and to evaluate a contribution of this process into the neutrino luminosity in the above-mentioned astrophysical cataclysm.

It is known that two eigenmodes of the real photon exist in a magnetic field [12] with polarization vectors
where \( \varphi_{\alpha \rho} = \frac{F_{\alpha \rho}}{B} \) is the dimensionless tensor of the external magnetic field, \( \tilde{\varphi}_{\alpha \rho} = \frac{1}{2} \varepsilon_{\alpha \rho \mu \nu} \varphi_{\mu \nu} \) is the dual tensor, \( q_\parallel^2 = (q \tilde{\varphi} q) = q_\alpha \tilde{\varphi}_{\alpha \rho} \tilde{\varphi}_{\rho \mu} q_\mu \), \( q_\perp^2 = (q \varphi q) \). For the relatively small “photon mass”, \( |q^2| \ll \omega^2 \), one can see that \( q_\parallel^2 \approx q_\perp^2 \approx \omega^2 \sin^2 \theta \), where \( \omega \) is the photon energy, \( \theta \) is the angle between the photon momentum \( \vec{q} \) and the magnetic field direction. The vectors (1) are the eigenvectors of the photon polarization tensor in a magnetic field. The photon decay \( \gamma \to \nu \bar{\nu} \), Fig. 1, is the crossed channel of the gamma radiation by a neutrino \( \nu \to \nu \gamma \). Due to the photon dispersion in a magnetic field [13], the necessary condition of the decay, \( q^2 > 0 \), is realized for the perpendicular (\( \perp \)) polarization in the region \( q_\parallel^2 > 4 m_e^2 \) and for the parallel (\( \parallel \)) polarization in the region \( q_\parallel^2 > (m_e + \sqrt{m_e^2 + 2 e B})^2 \).

However, due to the collinearity of the kinematics, \( j_\alpha \sim q_\alpha \sim p_\alpha \sim p'_\alpha \), the amplitude of the \( \parallel \) mode decay is suppressed, and for the \( \perp \) mode it takes the form

\[
M_\perp \simeq \frac{2 e G_F g_A}{\sqrt{2} \pi^2} \sqrt{x(1 - x)} [e^2 (q F F q)]^{1/2} J, \tag{2}
\]

where \( e > 0 \) is the elementary charge, \( g_A \) is the axial-vector constant in the neutral electron current in the effective \( \nu \nu e e \) Lagrangian, \( x = E/\omega \), \( 1 - x = E'/\omega \) are the relative energies of the neutrino and the anti-neutrino correspondingly. The neutrino current \( j_\alpha \) is substituted into the amplitude (2) in the form

\[
j_\alpha = \bar{\nu}(p) \gamma_\alpha (1 - \gamma_5) \nu(-p') \simeq 4 \sqrt{x(1 - x)} q_\alpha, \tag{3}
\]

where the collinearity of the kinematics is taken into account. The amplitude (2) describes the process of the photon decay into the electron-neutrino pair (\( g_A = +1/2 \)) and into the muon-neutrino and tau-neutrino pairs (\( g_A = -1/2 \)) as well.

The dimensionless formfactor \( J \) in the general case has a form of a double integral.
\[ J = 1 - i \quad m_e^2 \int_0^1 \left[ \frac{t}{4} \left( m_e^2 - q_{\|}^2 \frac{1 - u^2}{4} \right) + \frac{q_{\|}^2}{2} \cos \beta u t - \cos \beta t \sin \beta t \right] \]  

where \( \beta = eB \), and it can be calculated rather easily in two limiting cases.

i) If the field strength \( B \) appears to be the largest physical parameter, \( eB \gg q_{\|}^2 \), one obtains

\[ J \simeq \frac{1 - v^2}{2v} \left( \ln \frac{1 + v}{1 - v} - i\pi \right) + 1, \]  

where \( v = \sqrt{1 - 4m_e^2/q_{\|}^2} \).

ii) In the alternative case, when \( eB \ll q_{\|}^2 \),

\[ J \simeq 1. \]  

The total probability of the photon decay into all neutrino species is defined by the expression

\[ W_{\perp} = \frac{1}{16\pi\omega} \int_0^1 dx \quad |M_{\perp}|^2 = \frac{\alpha G_F^2}{16\pi^4\omega} e^2(qFq) |J|^2. \]  

Here we assume that all neutrino masses are much smaller than the field-induced “photon mass”. Let us note that the photon decay probability (7) with (5) contains at first sight the pole-type singularity at \( q_{\|}^2 \to 4m_e^2 \) \( (v \to 0) \). However, the solution of the equation of the photon dispersion in this limit shows that

\[ |q_{\|}^2 - 4m_e^2|_{\text{min}} = \omega \quad \Gamma_{\gamma\to e^- e^+}. \]  

It is known that the similar seeming singularity, but of the square-root-type, takes place in the process of the photon decay into the electron-positron pair \( \gamma \to e^- e^+ \) [5]. As was shown in Ref. [13], taking account of the photon dispersion in the process \( \gamma \to e^- e^+ \) leads to a finite value for the decay width, which is maximal at the point \( q_{\|}^2 = 4m_e^2 \)

\[ (\Gamma_{\gamma\to e^- e^+})_{\text{max}} = \frac{\sqrt{3}}{2} \left( \frac{2\alpha eB}{m_e^2} \right)^{2/3} \frac{m_e^2}{\omega}. \]
The probability of the decay $\gamma \to \nu \bar{\nu}$ is also finite, in view of Eqs. (8) and (9), and amounts up to the maximal value:

$$(W_\perp)_{\text{max}} = \frac{(G_F m^2_e)^2}{4\sqrt{3} \pi^2} \left(\frac{2\alpha eB}{m^2_e}\right)^{1/3} \frac{eB}{\omega}.$$  \hspace{1cm} (10)

In the limiting case of a rather weak field $eB \ll q_\parallel^2$, the probability (7) with (6) coincides with the result of Ref. [6], to the factor $3/4$. This discrepancy is due to the fact that the contribution of the $Z$ boson was not taken into account in Ref. [6].

It is evident that the probability (10) of the electroweak process $\gamma \to \nu \bar{\nu}$ is suppressed by a factor $(G_F m^2_e)^2$ as compared with the probability (9) of the pure electromagnetic process $\gamma \to e^-e^+$. However, the process with neutrinos could play a role of an additional channel of stellar energy-loss.

The energy-loss rate per unit volume of the photon gas due to the decay $\gamma \to \nu \bar{\nu}$ is defined by

$$Q = \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\omega/T} - 1} \omega W_\perp.$$  \hspace{1cm} (11)

Substitution of the probability (7) into Eq. (11) gives

$$Q = \frac{\alpha G^2_F}{8\pi^4} m^5_e (eB)^2 \mathcal{F}(T) \simeq 0.96 \cdot 10^{18} \text{ erg cm}^{-3} \text{ s} \left(\frac{B}{B_e}\right)^2 \mathcal{F}(T).$$  \hspace{1cm} (12)

The temperature function $\mathcal{F}(T)$ is rather complicated in the general case. It takes a simple form in the limiting cases:

i) In the strong magnetic field, $eB \gg T^2 \gg m^2_e$

$$\mathcal{F}(T) \simeq \frac{T}{2m_e} \left(\ln \frac{T}{m_e} \ln \frac{4T}{\Gamma_\gamma} - 0.187\right).$$  \hspace{1cm} (13)

Here the main contribution arises from the vicinity of the resonance $q_\parallel^2 \simeq 4m^2_e$. The width $\Gamma_\gamma$ should be taken from Eq. (9) at $\omega = 2m_e$.

ii) In another limit of the relatively weak magnetic field, $T^2 \gg eB, m^2_e$, one obtains

$$\mathcal{F}(T) \simeq \frac{4\zeta(5)}{\pi^2} \left(\frac{T}{m_e}\right)^5.$$  \hspace{1cm} (14)
In a case of the low temperatures $T \ll 2m_e$ the energy-loss rate is suppressed by the small factor $\exp(-2m_e/T)$.

In conclusion, we estimate the contribution of the photon decay $\gamma \to \nu\bar{\nu}$ into the neutrino luminosity in a supernova explosion, from a region of order of a hundred kilometres in size outside the neutrinosphere, where a rather strong magnetic field of the toroidal type could exist [14]

$$\frac{dE}{dt} \sim 10^{45} \frac{\text{erg}}{s} \left(\frac{B}{10^{15} G}\right)^2 \left(\frac{T}{2 \text{MeV}}\right)^5 \left(\frac{R}{100 \text{km}}\right)^3.$$  \hspace{1cm} (15)

It is obviously much smaller than the total neutrino luminosity from the neutrinosphere $\sim 10^{52} \text{erg/s}$. It is interesting, however, that the process $\gamma \to \nu\bar{\nu}$ could give an appreciable contribution, equal for all neutrino species, to the low-energy part of the neutrino spectrum.

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References


Fig. 1. The Feynman diagram for the process $\gamma \rightarrow \nu \bar{\nu}$. The double line corresponds to the exact propagator of an electron in a magnetic field.