A Supersymmetric Resolution of Solar and Atmospheric Neutrino Puzzles

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Abstract

Renormalizable lepton number violating interactions that break R–parity can induce a Majorana mass for neutrinos. Based on this, we show that it is possible to obtain a phenomenologically viable neutrino mass matrix that can accommodate atmospheric neutrino data via $\nu_\mu$–$\nu_\tau$ mixing and the solar neutrino data via either the large or small angle MSW effect. We argue that such a mass matrix could result from an approximate discrete symmetry of the superpotential that forbids renormalizable baryon number violating couplings.
I. INTRODUCTION AND PHENOMENOLOGY

The deficit of solar neutrinos [1], the observed ratio of $\nu_\mu$ to $\nu_e$ events from atmospheric neutrinos [2], and the signal from the LSND experiment [3] all point toward non–trivial mixing effects amongst neutrinos which, in turn, implies that not all neutrinos can be mass-less. The flux of solar neutrinos may be accounted for by MSW oscillations provided [4] either $\Delta m^2 \sim 2 \times 10^{-5}$ eV$^2$ and the mixing angle is large, or $\Delta m^2 \sim (7 \pm 3) \times 10^{-6}$ eV$^2$ with $\sin^2 2\theta \sim 7 \times 10^{-3}$. The explanation of the atmospheric neutrino puzzle, on the other hand, appears to require $\Delta m^2 \sim 5 \times 10^{-3}$ eV$^2$ with large mixing between $\nu_\mu$ and another neutrino. Recently, the CHOOZ [5] collaboration has excluded the mixing of $\nu_e$ with another neutrino if $\Delta m^2 > 2 \times 10^{-3}$ eV$^2$ and $\sin^2 2\theta > 0.2$. Thus $\nu_\mu-\nu_\tau$ is the favoured explanation of the Super Kamiokande data [6]. Finally the LSND data [3] require a much larger mass difference $\Delta m^2 \sim 1$ eV$^2$ (and small mixing). The LSND effect will, in the near future, be independently probed by the KARMEN experiment [7].

Three distinct mass differences can only be accommodated in models with at least four neutrino flavours (one of them sterile), except when two of the phenomena “share”the same mass difference [8,9], although this interpretation is now disfavoured. We take a much less ambitious approach in this paper, and exclude the LSND observation from our consideration. We point out that supersymmetry (SUSY) models in which R–parity is explicitly broken by lepton number violating superpotential interactions offer a novel approach for the construction of phenomenologically viable neutrino mass matrices. We show that it is possible to obtain a neutrino mass matrix that is in accord with solar neutrino and atmospheric neutrino data, without any need to assume a hierarchy between different R–parity violating couplings. To achieve this, we are led to impose additional global (discrete) symmetries that also automatically forbid dimension four baryon number violating (B/) interactions.

Assuming the field content of the minimal supersymmetric model, the most general R–parity violating trilinear superpotential can be written as,

\[ f = f_1 + f_2 + f_3, \]  

(1.1)

with

\[ f_1 = \lambda_{ijk} L_i L_j E^C_k, \]  

(1.2)

\[ f_2 = \lambda'_{ijk} L_i Q_j D^C_k, \]  

(1.3)

and

\[ f_3 = \lambda''_{ijk} U_i^C D^C_j D^C_k. \]  

(1.4)

The superpotential interactions (1.2) and (1.3) provide us the desired source of lepton number violation while the simultaneous presence of those in (1.4) is dangerous since they can lead to proton decay at the weak rate. As mentioned above, we will show that the couplings in (1.4) can be forbidden as a result of a symmetry.

We begin our analysis with the observation [10,11] that $f_1$ and $f_2$ superpotential interactions lead to Majorana masses for neutrinos. For example, the $\lambda'$ couplings lead to a Majorana mass
for the $i^{th}$ neutrino via diagrams involving a quark–squark loop. Here, $i = e, \mu$ or $\tau$, $m_{j}$ ($m_{k}$) is the mass of the down–quarks in the $j^{th}$ ($k^{th}$) generation and $M_{SUSY}$ ($\sim A \sim \mu$, where $A$ and $\mu$ are the usual SUSY parameters) is a mass–scale that determines mixing between $\tilde{q}_{L}$ and $\tilde{q}_{R}$. Notice that (in the absence of a miraculous cancellation) a non–zero mass for $\nu_{i}$ results if $\lambda_{i}^{j}j \neq 0$, whereas for $j \neq k$, $\lambda'$ interactions induce a mass only [12] when $\lambda_{ij}^{k}j \neq 0$. We also remark that the $\lambda$ interactions can similarly induce neutrino masses.

The R–parity violating couplings in $f_{1}$ and $f_{2}$ can also induce off–diagonal Majorana neutrino mass terms which violate both lepton number and flavour conservation. This is the origin of the neutrino mass matrix that we will analyse below.

The large number (9+27) of a priori unconstrained $\lambda$ and $\lambda'$ couplings makes it apparent that there is a lot of freedom in the neutrino mass matrix. We will, however, adopt the philosophy that all the new superpotential couplings that are allowed by the symmetries that we impose should be comparable in magnitude; i.e. there is no large hierarchy between these couplings. The dominant contribution to neutrino masses then comes from $\tilde{b}\tilde{b}$ loops, and the neutrino mass matrix takes the form,

$$m_{\nu} \sim \frac{3}{8\pi^{2}} \frac{m_{b}^{2}}{m_{q}^{2}} \lambda_{i}^{j}j^{k} \frac{1}{m_{q}^{2}} M_{SUSY} m_{j} m_{k}, \quad (1.5)$$

where we have introduced the short–hand notation $\lambda_{i}^{j}j \equiv \alpha_{i}$. The mass matrix in (1.6) is valid up to corrections which are suppressed by $m_{s}/m_{b}$ (from $\lambda_{23}^{i}$ and $\lambda_{32}^{i}$ couplings) and comparable corrections $\sim m_{s}^{2}/3m_{b}^{2}$ (from $\lambda_{33}^{i}$ couplings).

The interesting property of the matrix in (1.6) is that regardless of the precise values of $\alpha_{i}$, two of its eigenvalues are zero. Corrections of $O(m_{s}^{2}/3m_{b}^{2})$, in general, will cause these eigenvalues to shift from zero, and we may thus expect their mass difference (which should be comparable in magnitude to the individual masses) to have a similar ratio to the large mass difference. For $m_{s} = 200$ MeV and $m_{b} = 5$ GeV this is in remarkable agreement with the ratio $\Delta m_{\text{solar}}/\Delta m_{\text{atmos}}$ that we discussed in the beginning of this paper.

This cannot, however, be the complete story. In order to accommodate the Super Kamiokande data [6], the mixing of $\nu_{e}$ should be small. Then, to account for the 50% change in the “ratio of ratios” from SM expectation, we are required to have $\nu_{\mu}–\nu_{\tau}$ mixing close to its maximal value. We are, therefore, led to the phenomenological constraint,

$$(m_{\nu})_{12}, (m_{\nu})_{13} << (m_{\nu})_{22} \simeq (m_{\nu})_{33} \simeq (m_{\nu})_{23}.$$ 

Since our philosophy does not allow us to include a hierarchy between the various $\lambda'$ couplings, we will seek models where $\lambda_{33}^{i} = 0$ due to a symmetry that allows $\lambda_{23}^{i}$ and $\lambda_{32}^{i}$. In this case the magnitudes of $(m_{\nu})_{12}$ and $(m_{\nu})_{13}$ will be set by $m_{s}m_{s}$ (provided the appropriate $\lambda$'s do not vanish) and we will obtain a neutrino mass matrix of the form,

$$m_{\nu} \sim \frac{3}{8\pi^{2}} \lambda \cdot \lambda' m_{b}^{2} \frac{m_{s}^{2}}{m_{q}^{2}} M_{SUSY} \left( \begin{array}{ccc} m_{s}^{2} & m_{s} & m_{s} \\ m_{s} & m_{b} & m_{b} \\ m_{s} & m_{b} & 1 \\ 1 & 1 & 1 \end{array} \right), \quad (1.7)$$
which, as we have already discussed, readily accounts for the data provided
\[ \frac{3}{8\pi^2} \lambda' \cdot \lambda' \frac{m_\tilde{q}^2}{m_\tilde{q}^2} M_{\text{SUSY}} \sim (5 \times 10^{-3})^{1/2}/\text{eV}. \]

Taking \( m_\tilde{q} = M_{\text{SUSY}} \), and assuming all the non–vanishing \( \lambda' \) couplings have comparable magnitude, we have
\[ \lambda' \sim 10^{-4} \left( \frac{m_\tilde{q}}{200 \text{ GeV}} \right)^{1/2}. \] (1.8)

It should be recognized that the mass matrix in (1.7) may be modified if the \( \lambda \) couplings in (1.2) are also non–vanishing, since these interactions can induce a mass \( \sim \frac{1}{8\pi^2} \lambda \cdot \lambda \frac{m_i m_j}{m_i^2} M_{\text{SUSY}} \). But as long as \( m_\ell \simeq m_\tilde{q} \) and \( \lambda \lesssim \lambda' \), the qualitative features of the neutrino mass matrix (1.7) are not altered since \( 3m_\nu^2 / m_b \approx m_\tau^2 / m_b \).

Before turning to a discussion of symmetries of the superpotential that could yield a neutrino mass matrix with the form in (1.7), we briefly discuss other aspects of the phenomenology of the model:

1. Neutrino masses are too tiny to allow for their direct detection.
2. R–parity violating couplings \( \lambda \) or \( \lambda' \sim 10^{-4} \) will result in the lightest supersymmetric particle (LSP), assumed to be the lightest neutralino, decaying inside the detector [13], probably without an observable displaced vertex. We do not expect the production or decays of SUSY particles (other than the LSP) to be modified by these tiny couplings.
3. LSP pair production can potentially lead to observable signals at LEP2, even if other sparticles are kinematically inaccessible [10]. Some bounds have already been obtained by the ALEPH Collaboration [14]. The R–parity violating interactions will also modify SUSY signals at hadron colliders, but exactly how these alter will depend on the decay pattern of the LSP. While the canonical \( F_T \) signal will be reduced relative to the usual expectation, signals in the same sign dilepton and multilepton (plus multijet) channels should be readily observable, both at the Tevatron [15] and at the LHC [16].
4. The R–parity violating interactions in (1.2) and (1.3) induce lepton number violating sneutrino masses, \( \delta m^2 \simeq \frac{3}{16\pi^2} |\lambda|^2 m_\tilde{b}^2 \) or \( \frac{1}{16\pi^2} |\lambda|^2 m_\tilde{\tau}^2 \) (the \( m_\tilde{b}^2 \) term is only possible for \( \tilde{\nu}_\mu \) and \( \tilde{\nu}_\tau \), since \( \lambda'_{133} = 0 \)) that possibly also violate flavour. Unfortunately, since sneutrino decays are generically governed by their gauge couplings which are much larger than \( 10^{-4} \times m_b / M_W \), sneutrinos will decay much before they can even begin to oscillate.
5. Finally, because we are allowing interactions where more than one lepton flavour is not conserved, we have to worry that these interactions do not induce decays such as \( \mu \to e\gamma \) or \( K^0 \to \mu e \) whose branching ratios are strongly constrained [17]. The lowest dimensional effective operator that causes the decay \( \mu \to e\gamma \) has the form \( K \bar{e} \sigma_{\mu\nu} \mu F^\mu_\nu \), and necessarily flips the lepton chirality. If lepton number is violated only by \( \lambda' \) interactions, only left handed leptons couple via these, so that \( K \) must include a lepton mass to flip the chirality. Dimensional analysis then tells us that
$K \sim \frac{3\lambda^2}{8\pi^2 m_q^4} m_\mu$, where we have included a loop factor $\sim 8\pi^2$ and a color factor of 3.

The suppression factor $1/m_q^2$ in $K$ then implies $\Gamma(\mu \to e\gamma) \sim (\frac{3\lambda^2}{8\pi^2})^2 \frac{m_\mu^5}{m_q^4} \times \mathcal{O}(1)$, so that in order not to violate the experimental bound [17] $B(\mu \to e\gamma) < 5 \times 10^{-11}$, we must have $\lambda^4 \lesssim 10^{-9}$ for $m_q = 200$ GeV. We thus see that $\lambda'$ couplings $\sim 10^{-4}$ as required by the neutrino data are comfortably within this bound even if $\lambda_{133}'$ is comparable in magnitude to $\lambda_{133}'$ and $\lambda_{333}'$. Notice that there is no bound if $\lambda_{ijk} \neq 0$ only for those $\{jk\}$ values for which $\lambda_{2jk}'$ vanishes, as could well be the case for us. Bounds on violation of $\tau$ number are weaker than those from $\mu \to e\gamma$ decay and do not pose serious constraints for the magnitudes of couplings relevant to us. We will revisit this decay when we consider specific models below and allow for the possibility of non–vanishing $\lambda$–type couplings.

6. The superpotential $\lambda'$ couplings can potentially also mediate $K^0 \to \mu e$ decays via virtual squark exchange. Once again the amplitude will be proportional to an explicit fermion mass factor. This amplitude is non–vanishing only when both $\lambda_{2ik}'$ and $\lambda_{21k}'$ or $\lambda_{1j2}'$ and $\lambda_{j21}'$ are simultaneously non–vanishing. We will return to this model–dependent issue once we set up the framework that yields the mass matrix of the desired form.

II. MODELS

Our strategy is to seek a discrete symmetry of the superpotential of the model which allows the desired Yukawa interaction of the Standard Model as well as $\lambda$ or $\lambda'$ interactions that yield a neutrino matrix of the form in (1.7); i.e. allows couplings such as $\lambda_{133}'$ and $\lambda_{333}'$ but forbids $\lambda_{133}'$ couplings which would lead to unit entries instead of $m_s/m_b$ and $m^2_s/m_b^2$. We will also require that the symmetry forbid the $B$ superpotential interactions (1.4). We will, however, see that it is not possible to find an Abelian symmetry that does the job. We will give general arguments to show that any such symmetry has to be broken by the strange quark Yukawa couplings. For definiteness, we will work with $Z_N$ symmetries, but the generality of these arguments will be clear. We write the superpotential interactions in the mass basis [18] of quarks, not in the current basis.

A. $Z_2$ symmetry

In this case each superfield carries a multiplicative quantum number $+1$ or $-1$. We can choose the quantum number of $L_3$ to be $+1$ by convention. We will denote this as $L_3 = +1$. Then, since $\lambda_{333}' \neq 0$, $Q_3 D_3^C = +1$. It then follows that to get maximal $\nu_\mu - \bar{\nu}_\tau$ mixing, we must have $L_2 = +1$, and $L_1 = -1$. This already shows that our $Z_2$ symmetry must be broken in order to explain solar neutrinos by mixing between $\nu_e$ and $\nu_\mu$ or $\nu_\tau$. But let us proceed further to understand how this is broken. A non–vanishing bottom Yukawa coupling means $h' = +1$ so that a non–vanishing $\mu$ term implies $h = +1$, where $h'$ and $h$ are the Higgs fields whose interactions give a mass to $T_3 = -1/2$ and $T_3 = 1/2$ fermions, respectively. Finally, the large top quark mass implies $Q_3 U_3^C = +1$. Thus $Q_3$, $U_3^C$ and $D_3^C$ all have the same charge.
Now, let us forbid the unwanted $\lambda''$ couplings. We first note that $D_{1,2}^C = -1$ to forbid $U_3^C D_3^C D_{i,2}^C$ coupling, which together with the absence of $U_1^C D_1^C D_2^C$ term requires $U_i^C = -1$. Thus all the quark singlets have charge $-1$.

In order to get $(m_{\nu})_{12} \sim (m_{\nu})_{13} \propto m_b m_s$, we must have $L_1 Q_2 D_3^C$ and $L_2 Q_3 D_2^C$ interactions, because the $L_1 Q_3 D_2^C$ term is forbidden. This forces us to choose $Q_2 = +1$, so that both $m_s$ and $m_c$ are forbidden! Since the charm Yukawa coupling ($\sim 10^{-2}$) is two orders of magnitude larger than the $\lambda'$ couplings that we require, this is not very appealing.

One way to ameliorate this situation is to change the scale of the mass factor outside the matrix in (1.7) to $m_s$ so that we can find a symmetry that is only violated by $m_s$. This forces us to choose $Q_2 = +1$, so that both $m_s$ and $m_c$ are forbidden! Since the charm Yukawa coupling ($\sim 10^{-2}$) is two orders of magnitude larger than the $\lambda'$ couplings that we require, this is not very appealing.

Continuing this line of thought it is natural to ask whether it is possible to change the scale of neutrino masses will be as in (1.7).

**B. Z$_3$ symmetry**

While the $Z_2$ symmetry that we found indeed gave us the mass matrix (1.7) it suffered from the fact that the charm quark Yukawa coupling (which is forbidden by this symmetry) far exceeds the $\lambda'$ couplings that are phenomenologically required and allowed by the symmetry. The reader can check that requiring instead a $Z_3$ symmetry with additive charges (modulo 3),

\[
U_1^C = Q_1 = D_3^C = h = h' = L_2 = L_3 = E_3 = 0
\]
\[
D_1 = D_2 = 1
\]
\[
L_1 = E_1 = E_2 = 2
\]

1. top, charm, bottom and tau Yukawa couplings are allowed

2. we obtain a neutrino mass matrix whose order of magnitude of entries is given by (1.7); $(m_{\nu})_{22}$ gets an additional contribution from the $L_2 L_3 E_3$ coupling. Nevertheless, the situation is still problematic because the couplings $\lambda' \sim 10^{-4}$ are still an order of magnitude smaller than the strange quark Yukawa coupling. We are thus forced to seek a model where the scale of the neutrino mass matrix is $m_s^2/m_q$ rather than $m_b^2/m_q$ as in (1.7).

Toward this end, we consider a $Z_3$ symmetry with charges given by,

\[
U_i^C = D_3^C = Q_2 = Q_3 = h = h' = 0
\]
\[
L_3 = L_2 = E_2 = 1
\]
\[
E_3 = D_2^C = D_1^C = Q_1 = L_1 = E_1 = 2
\]

This symmetry
1. allows $m_t$, $m_b$, $m_c$ and $m_\tau$, but is explicitly broken by other quark masses;
2. forbids all renormalizable $B$ interactions;
3. forbids lepton number violating bilinears in the superpotential, which gives us an a posteriori justification for ignoring these;
4. forbids $L_1 Q_3 D_3^C$ couplings, and since $Q_2 D_3^C \neq Q_3 D_2^C$ also simultaneous presence of $L_1 Q_2 D_3$ and $L_1 Q_3 D_2$ (which would give $m_b m_s$ terms in the neutrino mass matrix); $L_1 L_3 E_3$ couplings are also forbidden;
5. allows $L_{2,3} Q_2 D_2^C$ but forbids $L_1 Q_2 D_2^C$ couplings in (1.3);
6. allows $L_1 Q_1 D_2^C$ and $L_{2,3} Q_2 D_1^C$ couplings which allows $\nu_e$ to mix with $\nu_{\mu,\tau}$ via entries proportional to $m_d m_s / m_q$.
7. Only the $L_2 L_3 E_2^C$ couplings in $f_1$ are allowed; this gives a correction to the $(m_\nu)_{33}$ entry.
8. Finally, notice that $L_1 D_2^C \neq L_2 D_1^C$ and $L_1 Q_2 \neq L_2 Q_1$. Thus the potentially dangerous $K^0 \rightarrow \mu e$ decays are absent in this framework.

The neutrino mass matrix for this model has the form,

$$m_\nu \sim \frac{3}{8 \pi^2} \lambda' \cdot \lambda' \frac{m_s^2}{m_q^2} M_{SUSY} \begin{pmatrix} m_d/m_s & m_d/m_s & m_d/m_s \\ m_d/m_s & 1 & 1 \\ m_d/m_s & 1 & 1 + \delta \end{pmatrix}, \quad (2.1)$$

where $\delta \sim \frac{\lambda^2}{\lambda' \cdot \lambda' \frac{m_s^2}{m_q^2}} \sim \frac{1}{10} \frac{\lambda'}{\lambda' \cdot \lambda'}$ and so can easily be comparable to $m_d/m_s \sim 8 \text{ MeV}/200 \text{ MeV} \sim 1/25$. We also remind the reader that because the $\lambda' \cdot \lambda'$ factors that multiply each entry in the matrix (2.1) need not be identical, it is the form of the matrix and not the exact matrix elements listed that we should focus on. In the limit that the $m_d/m_s$ and $\delta$ terms are neglected, we have one massive neutrino state with $m_\nu \sim \frac{3}{8 \pi^2} \lambda' \cdot \lambda' \frac{m_s^2}{m_q^2} M_{SUSY}$ and two massless states. Furthermore, for equal $\lambda'$s this massive state is a maximal mixture of $\nu_\mu$ and $\nu_\tau$ as required by Super Kamiokande data [6]. Since we now have $m_s^2$, not $m_q^2$, setting the scale of the neutrino mass matrix, $\lambda'$ will now be larger by a factor $\sim m_b/m_s \sim 25$ compared to our earlier estimate; i.e. $\lambda' \sim 2.5 \times 10^{-3}$, somewhat larger than the strange quark Yukawa coupling. While the situation is not ideal, considering factors of $O(1)$ that we have ignored, it is probably not inconsistent.

The $m_d/m_s$ and $\delta$ terms in (2.1) result in a mass for the other states. The exact values of the masses are sensitive to the precise values of the various $\lambda'$ couplings, but the mass difference between them is suppressed (relative to the “large” mass) by a factor of $O(m_d/m_s$ or $\delta$), and so, is exactly in the right ballpark to account for the solar neutrino flux via the MSW effect. The mixing between $\nu_e$ and the other “massless” state due to the $m_d/m_s$ and $\delta$ terms is sensitive to model parameters but is generically large, so in general, we may expect solar neutrino to be accounted for by the large angle MSW solution. Its, however, not implausible that the mixing angle becomes suppressed due to cancellations between $(m_\nu)_{12}$...
and \((m_\nu)_{13}\) terms (which are expected to have very similar magnitude) when this mixing is estimated treating \(m_d/m_s\) and \(\delta\) to first order in perturbation theory. A cancellation precise to \(\sim 10\%\) can then account for solar neutrinos as a small angle MSW effect. As is well known, the two scenarios of small and large mixing MSW solutions can be distinguished experimentally by the energy spectrum distortion expected in the former, and the day night effect in the latter, both for \(^8\text{B}\) neutrinos.

The larger magnitude, \(\lambda' \sim 2.5 \times 10^{-3}\), of the superpotential couplings may cause renewed concern about the experimental bound on the decay rate for \(\mu \rightarrow e\gamma\). The reader can easily check that because of the \(Z_3\) symmetry (even though it is explicitly broken) there is no 1-loop diagram that mediates this decay. Moreover, \(\lambda\)–interactions conserve \(e\) and \(\mu\) number so that they cannot contribute. Finally, exactly as before, the oscillation length of sneutrinos is still larger than their decay length.

III. SUMMARY

In summary, we have proposed a novel mechanism involving R–parity violation for generating neutrino mass matrices consistent with solar and atmospheric neutrino data. The observed ratio of ratios of atmospheric neutrinos is due to maximal \(\nu_\mu - \nu_\tau\) mixing. We favour large angle MSW oscillations as the solution to the solar neutrino puzzle but do not exclude the small angle MSW solution. Future measurements will help discriminate between various scenarios and pin down the model parameters. We have argued that the structure of the neutrino mass matrix reflects an underlying approximate symmetry of the superpotential. Although our mechanism does not appear to have direct implications for low energy physics (other than the neutrino phenomenology already discussed), the new superpotential interactions will significantly alter SUSY signals at high energy colliders.

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The reader may worry that imposing symmetries on fields in the mass basis will restrict inter-generational mixing. We will see, however, that the symmetries we impose are approximate in that they are explicitly broken by some quark Yukawa interactions. Because of these explicit breaking terms, the Yukawa couplings can be exactly the same as in the Standard Model.