Renormalization of gauge-invariant operators for the structure function $g_2(x, Q^2)$

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Abstract
We investigate the nucleon’s transverse spin-dependent structure function $g_2(x, Q^2)$ in the framework of the operator product expansion and the renormalization group. We construct the complete set of the twist-3 operators for the flavor singlet channel, and give the relations among them. We develop an efficient, covariant approach to derive the anomalous dimension matrix of the twist-3 singlet operators by computing the off-shell Green’s functions. As an application, we investigate the renormalization mixing for the lowest moment case, including the operators proportional to the equations of motion as well as the “alien” operators which are not gauge-invariant.

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Recently, the nucleon’s transverse spin-dependent structure function $g_2(x, Q^2)$ has been observed\cite{1} by measuring asymmetry in the deep inelastic scattering using the transversely polarized target. The structure function $g_2(x, Q^2)$ plays a distinguished role in spin physics because it contains information inaccessible by the more familiar spin structure function $g_1(x, Q^2)$\cite{2, 3}: $g_2(x, Q^2)$ is related to the nucleon’s transverse polarization and to the twist-3 operators describing the quark-gluon and three gluon correlations in the nucleon.

In the framework of the operator product expansion, not only the twist-2 operators but also the twist-3 operators contribute to $g_2(x, Q^2)$ in the leading order of $1/Q^2$\cite{2, 3}. The $Q^2$-evolution of $g_2(x, Q^2)$ is governed by the anomalous dimensions, which are determined by the renormalization of these operators. A characteristic feature of the higher twist operators is the occurrence of the complicated operator mixing under the renormalization: Many gauge-invariant operators, the number of which increases with spin (moment of the structure function), mix with each other\footnote{Recently, it has been proved that the twist-3 nonsinglet structure functions obey simple DGLAP evolution equation for $N_c \to \infty$\cite{11}.}. Furthermore, the operators which are proportional to the equations of motion (“EOM operators”), as well as the ones which are gauge-variant (“alien operators”), also participate in the mixing\cite{4}.

There have been a lot of works on the $Q^2$-evolution of $g_2(x, Q^2)$. Most of them discussed the flavor nonsinglet case\cite{5, 6}. Only a few works treated the singlet case\cite{7}: Bakhvostov, Kuraev and Lipatov derived evolution equations for the twist-3 quasi-partonic operators. Recently, Müller computed evolution kernel based on the nonlocal light-ray operator technique, and obtained the identical results. However, both of these two works employ a similar framework based on the renormalization of the nonlocal operators in the (light-like) axial gauge. Balitsky and Braun also treated the nonlocal operators although they employed the background field method. On the other hand, a covariant approach based on the local composite operators is missing. Furthermore, some subtle infrared problem...
occurring in the renormalization of the generic flavor singlet operators has been emphasized in Ref.[8]. Therefore, the computation of the anomalous dimensions in a covariant gauge in a fully consistent scheme is desirable and should provide a useful framework.

We develop a covariant framework to investigate the flavor singlet part of $g_2(x, Q^2)$ based on the operator product expansion and the renormalization group, by extending our recent work on the flavor nonsinglet part[5, 6]. The purpose of this letter is twofold. Firstly, we provide all necessary theoretical stuffs and convenient techniques: We list up all relevant twist-3 flavor singlet operators appearing in QCD. We discuss the relations satisfied by these operators[7]. In particular, we obtain a new operator identity, which relates the gluon bilinear operator with the trilinear ones. Based on these developments, we give a basis of independent operators for the renormalization. We describe a general framework to perform the renormalization of the twist-3 flavor singlet operators in a covariant gauge, by computing the off-shell Green’s functions. A convenient projection technique[6, 9] is also introduced to simplify the calculation. Secondly, we apply our framework to the lowest ($n = 3$) moment case, and demonstrate the consistency and the efficiency of our method: We investigate in detail the mixing structure of the higher twist singlet operators under the renormalization. The EOM operators as well as the alien operators should be included as independent operators. One advantage of our approach is that an infrared cut-off is provided by the off-shellness of the external momenta. This allows us to assess unambiguously the renormalization constant ; otherwise the calculation might be subtle and potentially dangerous[8]. We also clarify the connection of our approach with other works[7] based on the on-shell calculation without the EOM operators nor the alien operators. As a byproduct, we confirm the prediction of the $Q^2$-evolution obtained in previous works[2, 7], which will be subject to future experimental study.

We follow the convention of Refs.[5, 6]. We list up the twist-3 flavor singlet operators which contribute to the moment $\int_0^1 dx x^{n-1} g_2(x, Q^2)$ ($n = 3, 5, 7, \cdots$). Firstly, there is a
set of operators which are bilinear in the quark fields. These operators can be obtained from
the nonsinglet operators, eqs.(1)-(4) of Ref.[6] by removing the flavor matrices \( \lambda_i \).
An independent base can be chosen as (see Refs.[6, 10]):

\[
R_{n,l}^{\mu_1 \cdots \mu_{n-1}} = \frac{1}{2n} (V_l - V_{n-1-l} + U_l + U_{n-1-l}) \quad (l = 1, \ldots, n-2),
\]

(1)

\[
R_{n,m}^{\mu_1 \cdots \mu_{n-1}} = i^{n-2} S m \bar{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} \psi - \text{traces},
\]

(2)

\[
R_{n,E}^{\mu_1 \cdots \mu_{n-1}} = i^{n-2} \frac{n-1}{2n} \left( \bar{\psi} (i \not{\mathcal{D}} - m) \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} \psi 
+ \bar{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} (i \not{\mathcal{D}} - m) \psi \right) - \text{traces},
\]

(3)

where

\[
V_l = i^n g S \bar{\psi} \gamma_5 D^{\mu_1} \cdots G^{\mu_l} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} \psi - \text{traces},
\]

\[
U_l = i^{n-3} g S \bar{\psi} \gamma_5 D^{\mu_1} \cdots \tilde{G}^{\mu_l} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} \psi - \text{traces}.
\]

In the above equations, (- traces) stands for the subtraction of the trace terms to make
the operators traceless and \( D^\mu \) is the covariant derivative. \( S \) means symmetrization over
\( \mu_i \) and \( g \) the QCD coupling constant. \( m \) represents the singlet component of the quark
mass matrix. The operators in eq.(1) contain the gluon field strength \( G_{\mu\nu} \) and the dual
tensor \( G_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} G^{\alpha\beta} \) explicitly.

Secondly, we have the following “new” operators (see e.g. Ref.[10]):

\[
T_{n,G}^{\sigma_1 \cdots \mu_{n-1}} = \bar{\psi}^{-1} S \mathcal{A} S \left( \tilde{G}^{\mu_1} D^{\mu_2} \cdots D^{\mu_{n-1}} G_{\nu}^\sigma \right) - \text{traces},
\]

(4)

\[
T_{n,l}^{\sigma_1 \cdots \mu_{n-1}} = \bar{\psi}^{-1} g S \left( G^{\mu_1} D^{\mu_2} \cdots G^{\mu_l} \cdots D^{\mu_{n-2}} G_{\nu}^\mu G_{\nu}^{\mu_{n-1}} \right) - \text{traces},
\]

(5)

\[
T_{n,B}^{\sigma_1 \cdots \mu_{n-1}} = \bar{\psi}^{-1} S \left\{ \tilde{G}^{\sigma_1} D^{\mu_2} \cdots D^{\mu_{n-2}} \right\}^a \left\{ -\frac{1}{\alpha} \partial^{\mu_{n-1}} (\partial' A_{\mu}) + g f^{abc} (\partial^{\mu_{n-1}} \chi^b) \chi^c \right\} - \text{traces},
\]

(6)

\[
T_{n,E}^{\sigma_1 \cdots \mu_{n-1}} = \bar{\psi}^{-1} S \left\{ \tilde{G}^{\sigma_1} D^{\mu_2} \cdots D^{\mu_{n-2}} \right\}^a \left\{ (D' G_{\nu}^{\mu_{n-1}})^a \right.
+ g \bar{\psi} F'_{\nu \mu_{n-1}} \psi + \left. \frac{1}{\alpha} \partial^{\mu_{n-1}} (\partial' A_{\mu}) - g f^{abc} (\partial^{\mu_{n-1}} \chi^b) \chi^c \right\} - \text{traces}.
\]

(7)
Here \( \mathcal{A} \) antisymmetrizes \( \sigma \) and \( \mu_1 \). The gluon field \( A_\mu \) and the covariant derivative \( D_\mu \) are in the adjoint representation. \( \chi \) is the ghost field. \( t^a \) is the color matrix as \([t^a, t^b] = i f^{abc} t^c\), \( \text{Tr} \left( t^a t^b \right) = \frac{1}{2} \delta^{ab} \), and \( \alpha \) is the gauge parameter. \( T_{n,l} \) is trilinear in the gluon field strength \( G_{\mu\nu} \) and the dual tensor, and thus represents the effect of the three gluon correlations. It satisfies the symmetry relation \( T_{n,l}^{\mu_1\cdots\mu_{n-1}} = T_{n,n-l}^{\mu_1\cdots\mu_{n-1}} \). \( T_{n,E} \) is the gluon EOM operator; it vanishes by the use of the equations of motion for the gluon, although it is in general a nonzero operator due to quantum effects. \( T_{n,B} \) is the BRST-exact alien operator\(^4\) which is the BRST variation of the operator: \( i g^{-1} S \{ \bar{G}^{\mu_1} D^\nu_2 \cdots D^\nu_{n-2} \} \alpha \partial^{\mu_{n-1}} \chi^\alpha - \text{traces} \).

We note that the gluon bilinear operator eq.(4) is related to the trilinear ones eq.(5) by

\[
T_{n,G}^{\mu_1\cdots\mu_{n-1}} = \frac{1}{n-1} \left\{ T_{n,E}^{\mu_1\cdots\mu_{n-1}} + T_{n,B}^{\mu_1\cdots\mu_{n-1}} + \sum_{l=1}^{n-2} n C_{n-l}^{n-2} (-1)^{l+1} R_{n,l}^{\mu_1\cdots\mu_{n-1}} \right\}, \tag{8}
\]

where \( C_r^n = n!/[r!(n-r)!] \). To derive this relation, we have used \([D_\mu, D_\nu] = -igG_{\mu\nu} \) and

\[
D_\sigma G_{\nu\alpha} + D_\nu G_{\sigma\alpha} + D_\alpha G_{\nu\sigma} = 0; \quad D_\sigma \tilde{G}_{\nu\alpha} + D_\nu \tilde{G}_{\sigma\alpha} + D_\alpha \tilde{G}_{\nu\sigma} = \varepsilon_{\nu\alpha\sigma\rho} D_\chi G^{\rho},
\]

where the first identity is the usual Bianchi identity while the second one is a consequence of the relation \( g_{\mu\nu} \varepsilon_{\alpha\beta\gamma\delta} = g_{\mu\alpha} \varepsilon_{\nu\beta\gamma\delta} + g_{\mu\beta} \varepsilon_{\nu\gamma\alpha\delta} + g_{\mu\gamma} \varepsilon_{\nu\beta\alpha\delta} + g_{\mu\delta} \varepsilon_{\nu\beta\alpha\gamma} \). The operator identity eq.(8) is new, and is one of the main results of this work. As a result of eq.(8), we can conveniently choose a set of independent operators as eqs.(5)-(7). For the \( n \)-th moment, these \( \frac{n+1}{2} \) independent operators will mix with each other, and with the \( n \) gauge-invariant operators bilinear in the quark fields discussed above, under the renormalization.

After the determination of an independent operator’s basis, the renormalization is in principle straightforward: We follow the standard method to renormalize the local composite operators\(^4\). We multiply the operators discussed above by a light-like vector.
\[ \Delta_{\mu_1} \cdots \Delta_{\mu_{n-1}} R^\mu_1 \cdots \mu_{n-1} \equiv \Delta \cdot R^\mu_{n,d}, \quad \Delta_{\mu_1} \cdots \Delta_{\mu_{n-1}} T^\mu_1 \cdots \mu_{n-1} \equiv \Delta \cdot T^\mu_{n,d}. \]

We then embed the operators \( O_j = \Delta \cdot R^\mu_{n,d}, \quad \Delta \cdot T^\mu_{n,d} \), into the three-point function as,

\[
\langle 0 \left| T O_j(0) A_\mu(x) \bar{\psi}(y) \psi(z) \right| 0 \rangle, \quad \langle 0 \left| T O_j(0) A_\mu(x) A_\nu(y) A_\rho(z) \right| 0 \rangle,
\]

and compute the 1-loop corrections. We employ the Feynman gauge \((\alpha = 1)\) and use the dimensional regularization. To perform the renormalization in a consistent manner without subtle infrared singularities[8], we keep the quark and gluon external lines off-shell; in this case the EOM operators as well as the BRST-exact operators mix through renormalization as nonzero operators.

One serious problem in the calculation is the mixing of many gauge/variant as well as BRST/variant EOM operators. As explained in Refs.[5, 12], the gauge/variant quark EOM operators are given by replacing some of the covariant derivatives \( D^\mu \) by the ordinary derivatives \( \partial^\mu \) in the corresponding gauge/invariant operator \( R^\mu_1 \cdots \mu_{n-1}. \) In the present case, the BRST/variant EOM operators obtained similarly from the gluon EOM operator eq.(7) also participate in the mixing. However, the problem can be overcome by direct generalization of the method employed in Refs.[6, 9]. We introduce the vector \( \Omega^\mu_r \) (\( r = 1, 2, 3 \)) satisfying \( \Delta_\mu \Omega^\mu_r = 0 \) for each external gluon line, and contract the Green’s functions as \( \Omega^\mu_r \langle 0 \left| T O_j A_\mu(x) \bar{\psi}(y) \psi(z) \right| 0 \rangle, \quad \Omega^\mu_2 \Omega^\mu_3 \langle 0 \left| T O_j A_\mu A_\nu A_\rho \right| 0 \rangle . \) This brings us three merits: Firstly, the tree vertices of the gauge (BRST) invariant and the gauge (BRST) variant EOM operators coincide. Thus, essentially only one quark (gluon) EOM operator is now involved in the operator mixing. Secondly, the structure of the vertices for the twist-3 operators are simplified extremely and the computation becomes more tractable (see eqs.(14)-(17) below). Thirdly, the three-gluon vertex of the BRST-exact operator eq.(6) vanishes for \( \Omega^\mu_2 = \Omega^\mu_3 \) and \( \Omega_1 \cdot \Omega_2 = 0. \) Thus, we can exclude the BRST-exact operator from the gluon three-point functions and compute its mixing separately.
We now apply our framework to the lowest \((n = 3)\) moment. In this case, there exists no three-gluon operator \(\Delta \cdot T_{m,t}^\sigma\). From the one-loop calculation of the one-particle-irreducible (1PI) three-point function using the projection by \(\Omega^\sigma_\mu\) (see Fig.1), it turns out that the renormalization takes the following form:

\[
\begin{pmatrix}
\Delta \cdot R_{3,1}^\sigma \\
\Delta \cdot R_{3,m}^\sigma \\
\Delta \cdot T_{3,B}^\sigma \\
\Delta \cdot T_{3,E}^\sigma
\end{pmatrix}_{\text{bare}} = \begin{pmatrix}
Z_{11} & Z_{1m} & Z_{1E_F} & Z_{1E_G} \\
0 & Z_{mm} & 0 & 0 \\
0 & 0 & Z_{BB} & Z_{BE_F} & Z_{BE_G} \\
0 & 0 & 0 & Z_{EF_E_F} & Z_{EF_E_G}
\end{pmatrix} \begin{pmatrix}
\Delta \cdot R_{3,1}^\sigma \\
\Delta \cdot R_{3,m}^\sigma \\
\Delta \cdot T_{3,B}^\sigma \\
\Delta \cdot T_{3,E}^\sigma
\end{pmatrix},
\]

where the operators with (without) the suffix "bare" are the bare (renormalized) quantities. The renormalization constant matrix is triangular; this is consistent with the vanishing physical matrix elements of the EOM operators and of the BRST-exact operators (see below).

![Figure 1: 1PI diagrams for 3-point functions (full lines: quarks; wavy lines: gluons)](image)

In the MS scheme, we express the renormalization constants \(Z_{ij}\) as,

\[
Z_{ij} \equiv \delta_{ij} + \frac{g^2}{8\pi^2(4 - D)}X_{ij} \quad (i, j = 1, m, B, E_F, E_G)
\]

with \(D\) the space-time dimension. The relevant components of \(X_{ij}\) read:

\[
X_{11} = \frac{1}{3}C_F - 3C_G - \frac{2}{3}N_f; \quad X_{1m} = \frac{2}{9}C_F; \quad X_{1B} = -\frac{2}{9}N_f,
X_{1E_F} = \frac{1}{6}C_F; \quad X_{1E_G} = -\frac{2}{9}N_f; \quad X_{mm} = -6C_F,
\]

\(6\)
where $C_F = \frac{N_c^2 - 1}{2N_c}$, $C_G = N_c$ for $N_c$ color, and $N_f$ is the number of quark flavors. The physically relevant component is only $X_{11}$. However it is inevitable to identify other components to get a correct result for $X_{11}$.

Now let us discuss the relation between our approach and other ones based on the on-shell calculation. For this purpose, we note that eq.(9) implies the following results for the 1PI 3-point functions including 1-loop corrections (we consider the massless quark case):

$$\Gamma^{\sigma}_{gqg}(k_1, p, p') = Z_{11} \left\{ R_{3,1}^{(3)} \right\}_{gqg} + Z_{1E} \left\{ R_{3,E}^{(3)} \right\}_{gqg} + Z_{1E_0} \left\{ T_{3,E}^{\sigma} \right\}_{gqg},$$

$$\Gamma^{\sigma}_{gqg}(k_1, k_2, k_3) = Z_{1B} \left\{ T_{3,B}^{(3)} \right\}_{gqg} + Z_{1E_0} \left\{ T_{3,E}^{\sigma} \right\}_{gqg},$$

where $\Gamma^{\sigma}_{gqg}(k_1, p, p')$ and $\Gamma^{\sigma}_{gqg}(k_1, k_2, k_3)$ are the Fourier transform of the 1PI Green’s functions $\Omega_1^{\mu}(0) [\Delta \cdot R_{3,1}^{\sigma} A_{\mu}(\vec{p}) \psi(0)]_{1PI}$ and $\Omega_1^{\mu}(0) \Omega_2^{\nu}(0) [\Delta \cdot R_{3,1}^{\sigma} A_{\mu} A_{\nu} A_{\rho}]_{1PI}$ with $k_1, p, p' = -k_1 - p$, $k_2$, and $k_3 = -k_1 - k_2$ the incoming (off-shell) external momenta. The 3-point vertices appearing in eqs.(12) and (13) are given by,

$$\left\{ R_{3,1}^{(3)} \right\}_{gqg} = -\frac{1}{3} \left\{ T_{3,E}^{(3)} \right\}_{gqg} = -\frac{g}{6} k_1 \left[ \gamma^\sigma, \Omega_1 \right] \gamma_5 \Delta t^\mu,$$

$$\left\{ R_{3,E}^{(3)} \right\}_{gqg} = \frac{g}{3} (\dot{p}^\mu \gamma^\sigma \Omega_1 \gamma_5 \Delta - \dot{p} \Omega_1 \gamma_5 \gamma^\sigma \Delta) t^\mu,$$

$$\left\{ T_{3,B}^{(3)} \right\}_{gqg} = -g f^{abc} e^{\sigma \lambda \mu \nu} \Delta \Omega_2_{\lambda} \Omega_3_{\nu}(k_1 \cdot \Omega_1) k_1$$

$$+ \text{(cyclic perm’s)},$$

$$\left\{ T_{3,E}^{(3)} \right\}_{gqg} = -g f^{abc} e^{\sigma \lambda \mu \nu} \Delta \Omega_2_{\lambda} \Omega_3_{\nu}(k_1 \cdot \Omega_1) \dot{k_1} + k_1 \mu \Omega_1 \nu \Omega_2 \cdot \Omega_3 (\dot{k_3} - \dot{k_2})$$

$$+ \text{(cyclic perm’s)} - \left\{ T_{3,B}^{(3)} \right\}_{gqg},$$

where $\dot{p} = \Delta \cdot p$, etc., and “(cyclic perm’s)” denotes the terms due to the cyclic permutation of the labels for the external lines $(k_1, \Omega_1, a), (k_2, \Omega_2, b)$, and $(k_3, \Omega_3, c)$. We now take the on-shell limit of the above expressions. We can identify the vectors $\Omega_i^\mu$ to be the polarization vectors $\epsilon_i^\mu(k_i)$ of the corresponding gluons. Therefore the on-shell limit is realized by taking $\dot{p} = \dot{p}' = k_1^2 = k_1 \cdot \Omega_1 = 0$ for eq.(12), and $k_1^2 = k_2^2 = k_3^2 = k_1 \cdot \Omega_1 =$
\[ k_2 \cdot \Omega_2 = k_3 \cdot \Omega_3 = 0 \text{ for eq.(13). We see } \langle T_{3, B}^\sigma \rangle_{ggg}^{(3)} = 0 \text{ from eq.(16) and thus the BRST-exact operator does not contribute in this limit. On the other hand, we find that } \langle R_{3, E}^\sigma \rangle_{ggg}^{(3)}, \langle T_{3, E}^\sigma \rangle_{ggg}^{(3)}, \text{ and } \langle T_{3, B}^\sigma \rangle_{ggg}^{(3)} \text{ of (14), (15), and (17) do not vanish; the EOM operators do contribute to the 3-point functions eqs.(12) and (13) even in the on-shell limit.} \]

There exist, however, the one-particle-reducible (1PR) contributions of the same order as the 1PI ones (see Fig.2).

Figure 2: 1PR diagrams for 3-point functions (full lines: quarks; wavy lines: gluons)

We denote the 1PR contributions corresponding to eqs.(12) and (13) as \( \tilde{\Gamma}_{ggg}^\sigma (k_1, p, p') \) and \( \tilde{\Gamma}_{ggg}^\sigma (k_1, k_2, k_3) \). Here it should be noted that the on-shell limit of three massless particles corresponds to a very singular (collinear) configuration in the momentum space. Actually the on-shell limit of the 1PR diagram (Fig.2) becomes indefinite due to the collinear singularity. This technical difficulty can be avoided by calculating the “non-forward” matrix element of the composite operator [8]. Namely we calculate the diagram with the composite operators at nonzero momentum \((-q \equiv k_1 + p + p' \equiv k_1 + k_2 + k_3 \neq 0\)). Then we “define” \( \tilde{\Gamma}_{ggg}^\sigma \) and \( \tilde{\Gamma}_{ggg}^\sigma \) to be the \( q \to 0 \) limit of the matrix element after putting the external momenta on the mass shell.

The contributions from Fig.2 are given by,

\[
\tilde{\Gamma}_{ggg}^\sigma (k_1, p, p') = -g \tilde{Q}_1 t^a \frac{1}{p' + q} \tilde{\Gamma}_{ggg}^\sigma (p, q) + \tilde{\Gamma}_{ggg}^\sigma (-p' - q, q) \frac{1}{p' + q} g \tilde{Q}_1 t^a
+ \Omega_{1\mu} \tilde{\Gamma}_{ggg}^{\sigma \mu} (k_1, q) \frac{1}{(k_1 + q) g} g \gamma_\nu t^a,
\]

(18)

\[ 8 \]
\[
\tilde{\Gamma}^\sigma_{ggg}(k_1, k_2, k_3) = \Omega_{1\mu} \Pi^\sigma_{g\mu}(k_1, q) \frac{-i}{(k_1 + q)^2} g_{\mu\nu} V_{\nu\alpha\beta}(-k_1 - q, k_2, k_3) \Omega_{2\alpha} \Omega_{3\beta} (19)
\]
\[\quad + \text{(cyclic perm's)}, \]

where \(V_{\mu\nu\rho\delta}(p_1, p_2, p_3) = (p_1 - p_2)_{\mu\alpha} g_{\nu\rho\mu\alpha 2} + \text{(cyclic perm's)}\) is the usual three-gluon coupling. \(\Pi^\sigma_{gg}(p, q)\) and \(\Pi^\sigma_{g\mu}(k, q)\delta^{\rho\sigma}\) correspond to \(\langle 0 | T \Delta \cdot R_{3,1}^\sigma(q) \psi(-p - q) \overline{\psi}(p) | 0 \rangle_{1\text{PI}}\) and \(\langle 0 | T \Delta \cdot R_{3,1}^\sigma(q) A^\alpha(k) A^\beta(-k - q) | 0 \rangle_{1\text{PI}}\) with the off-shell external momenta. Their explicit expressions (divergent term) read,

\[
\Pi^\sigma_{gg}(p, q) = Z_{1(q)} \frac{1}{3} \left( (\not{p} + \not{q}) (\not{p} + \not{q} + \not{g}) \gamma_5 \gamma^\rho \not{X} + \not{p} \gamma_5 \gamma^\rho \not{X} \not{g} \right), \tag{20}
\]

\[
\Pi^\sigma_{g\mu}(k, q) = Z_{2(q)} (-i) \varepsilon^{\alpha\lambda\mu\nu} \Delta^\lambda \left\{ k_\alpha g_\beta^\mu \left\{ (k + q)^2 \Delta^\nu - (k + q)(k^\nu + q^\nu) \right\} - (k + q)_\alpha g_\nu^\mu \left( k^2 \Delta^\mu - k k^\mu \right) \right\}. \tag{21}
\]

where

\[
Z_{1(q)} = Z_{1E_F} , \quad Z_{2(q)} = Z_{1E_G} .
\]

Substituting eqs.(20)-(21) into eqs.(18) and (19), we can go over to the on-shell limit with \(q\) being kept non-zero. After this procedure, we take the \(q \to 0\) limit leading to

\[
\tilde{\Gamma}^\sigma_{ggg} = -Z_{1E_F} \left< R_{3,1}^{\sigma_{gg}} \right>^{3}_{\text{ggg}} - Z_{1E_G} \left< T_{3,1}^{\sigma_{gg}} \right>^{3}_{\text{ggg}} \quad \text{and} \quad \tilde{\Gamma}^\sigma_{ggg} = -Z_{1E_G} \left< T_{3,1}^{\sigma_{gg}} \right>^{3}_{\text{ggg}} .
\]

Therefore, the nonzero terms coming from the EOM operators exactly cancel out in the sum of the 1PI and the 1PR contributions for the on-shell external momenta:

\[
\Gamma^\sigma_{ggg} + \tilde{\Gamma}^\sigma_{ggg} = Z_{11} \left< R_{3,1}^{\sigma_{gg}} \right>^{3}_{\text{ggg}} , \quad \tilde{\Gamma}^\sigma_{ggg} + \tilde{\Gamma}^\sigma_{ggg} = 0 . \tag{22}
\]

The results eq.(22) correspond to the on-shell calculations of the evolution kernel in the literature[7], where neither the EOM operators nor the alien operators appear. The non-zero contributions from the EOM operators in the on-shell limit and their cancellation in the sums eq.(22) are novel features in the 3-point functions, whose consideration is indispensable for the twist-3 operators. This phenomenon is consistent with the well-known theorem that the physical matrix elements of the EOM operators vanish[4], because
both 1PI and 1PR diagrams contribute to the physical matrix elements. We emphasize that, in our demonstration, a calculation of the “physical” matrix element of the composite operators, in general, stays with some subtleties and dangers due to infrared singularities. One of the consistent method to avoid this problem from the beginning might be that all diagrams (1PI as well as 1PR) are calculated with the composite operators at nonzero momentum\cite{8}. In this case, however, we must consider additional composite operators which differ from the previous ones by the total derivative and this fact brings in another complexity\footnote{The operator $R_{3,1}^\sigma$ is unique even if $q \neq 0$}. Therefore the off-shell Green’s functions are much more tractable and easy to calculate especially for the general moment $n$.

Finally, we mention the $Q^2$-evolution obtained from eqs.(9)-(11): If we neglect the quark mass operator $\Delta \cdot R_{3,m}^\sigma$, only one operator $\Delta \cdot R_{3,1}^\sigma$ contributes to the $Q^2$-evolution of the physical matrix elements. The relevant renormalization constant is $Z_{11}$, which gives

$$
\int_0^1 dx x^2 g_2^{tw,3}(x, Q^2) = \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right)^{(3C_F-C_A/[3+2N_f/3]/b)} \int_0^1 dx x^2 g_2^{tw,3}(x, \mu^2). \tag{23}
$$

Here $b = \frac{11}{3} N_c - \frac{2}{3} N_f$, and $\alpha_s(Q^2)$ is the running coupling constant. $g_2^{tw,3}(x, \mu^2)$ is the twist-3 contribution of $g_2(x, \mu^2)$ after subtracting out the Wandzura-Wilczek piece\cite{3}, and its moment is equal to the nucleon matrix element of $\Delta \cdot R_{3,1}^\sigma(\mu^2)$ up to irrelevant factor. The result eq.(23) coincides with that of Refs.\cite{2, 7}, though our approach is quite different from those works; this fact confirms the theoretical prediction eq.(23), and also demonstrates the efficiency of our method. We note that the term $\frac{2}{3} N_f$ would be absent from the exponent of eq.(23) if we consider the non-singlet case\cite{5, 6}. For $N_c = 3$ and $N_f = 4$, the exponent of eq.(23) is $\frac{104}{96}$, while the corresponding exponent for the non-singlet case is $\frac{77}{96}$. Thus the singlet channel obeys rather stronger $Q^2$-evolution compared to the non-singlet one.

In the present study, we have developed a manifestly covariant approach to investigate the flavor singlet part of $g_2(x, Q^2)$. We derived the operator identities which relate the
two-particle operators with the three-particle ones. We have chosen the three-particle operators as an independent operator’s basis. To identify the renormalization constants correctly, the off-shell Green’s functions are considered. We have shown that the EOM operators as well as the BRST-exact operators play important roles. As an application, we investigated in detail the renormalization for the lowest \((n = 3)\) moment. We made contact with other methods\cite{7}, and also confirmed the prediction of the \(Q^2\)-evolution obtained by them. The off-shell Green’s functions are free from the infrared singularity coming from the collinear configuration as well as any other unphysical singularities. We believe that calculating the off-shell Green’s functions is the safest and the most straightforward method to obtain the anomalous dimensions.

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