Bare Quark Matter Surfaces of Strange Stars and $e^+e^-$ Emission

V.V. UsOV
Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

We show that the Coulomb barrier at the quark surface of a hot strange star may be a powerful source of $e^+e^-$ pairs which are created in an extremely strong electric field of the barrier and flow away from the star. The luminosity in the outflowing pair plasma depends on the surface temperature $T_q$ and may be very high, up to $\sim 3 \times 10^{11}$ erg s$^{-1}$ at $T_q \sim 10^{12}$ K. The effect of pair creation by the Coulomb barrier may be a good observational signature of strange stars which can give an answer to the question of whether a compact object is a neutron or strange star.

With the discovery of pulsars and their identification with the neutron stars, many people thought that neutron star matter is the ground state of matter at high density. Later, this belief was challenged by Witten [1]. He proposed that strange matter made of quarks is the ground state at ultra-high density. If the Witten’s hypothesis is true, then neutron stars with a sufficiently high central density may transform themselves into strange stars [2]. In this case, at least some part of the compact objects known to astronomers as pulsars, powerful accreting X-ray sources, X-ray bursters, soft $\gamma$-ray repeaters, etc. might be strange stars, not neutron stars as it is usually assumed [2,3]. In principle, the question of whether compact objects are neutron or strange stars can be answered by comparing the core density of a neutron star to the strange matter transition density. Unfortunately, all the nuclear-physics calculations to date do not yield a definitive answer. As to available data on pulsars and other compact objects, they are not able to give us an answer either [4]. This is because the bulk properties of models of strange and neutron stars of masses that are typical for neutron stars, $1 \lesssim M/M_\odot \lesssim 1.8$, are relatively similar. The situation changes, however, as regards the possibility that strange quark matter with the density of $\sim 4 \times 10^{14}$ g cm$^{-3}$ may be, by hypothesis, the absolute ground state of the strong interaction (i.e., absolutely stable with respect to $^{26}$Fe) and can exist up to the surface of strange stars. This differs qualitatively from the case of the neutron star surface and opens a unique possibility to observe a superdense quark matter.

"Normal" matter (ions and electrons) may be at the quark surface of strange stars. The ions in the inner layer are supported against the gravitational attraction to the underlying strange star by a very strong electric field of the Coulomb barrier in the quark surface vicinity (see below). There is an upper limit to the amount of normal matter at the quark surface, $\Delta M \lesssim 10^{-5}M_\odot$ [5]. Such a massive envelope of normal matter with $\Delta M \sim 10^{-5}M_\odot$ completely obscures the quark surface. However, it was pointed out [6] that a strange star at the moment of its formation is very hot. The temperature in the interior of such a star may be as high as a few $\times 10^{14}$ K. The rate of mass ejection from an envelope of such a hot strange star is very high [7], and if any normal matter remains at the quark surface of a strange star at $t \gg 10$ s, its mass is many orders smaller than the maximum. Besides, high temperatures lead to a considerable reduction of the Coulomb barrier, which favors the tunneling of nuclei toward the strange star surface [8]. Therefore, it is natural to expect that the quark surface of a very young strange star is nearly (or completely) bare. Recently, it was argued [9] that the normal-matter atmosphere of such a star remains optically thin in spite of a high accretion of gas onto the star until the quark-surface temperature is higher than $\sim 3 \times 10^7$ K. Such an atmosphere does not obscure the quark surface and has no influence on the surface structure. In this Letter, some properties of the bare quark surfaces of strange stars are considered.

At the bare surface of a strange star the density changes abruptly from $\sim 4 \times 10^{14}$ g cm$^{-3}$ to zero. The thickness of the quark surface is about 1 fm, which is a typical strong interaction length scale. There are some electrons in quark matter to neutralize the electric charge. The density of electrons is $\sim 10^6 - 10^4$ times smaller than the baryon density of the quark phase. The electrons, being bound to the quark matter by the electromagnetic interaction and not by the strong force, are able to move freely across the quark surface, but clearly cannot move to infinity because of the bulk electrostatic attraction to the quarks. The electron distribution extends up to $\sim 10^5$ fm above the quark surface [2]. Associated with this electron layer is a strong electric field, which is radially outwardly directed. This field prevents the electrons of the layer from escaping to infinity. Introducing a local electron chemical potential $\mu_e(r)$, we have $\mu_e(r) - eV(r) = \text{const} = 0$, where $V(r)$ is the electrostatic potential at the distance $r$ from the stellar center [8]. The last equality follows from the fact that at $r = \infty$ there are no electrons, $\mu_e(\infty) = 0$, and $V(\infty) = 0$. If $kT_e$ is much less than the Fermi energy of electrons $\varepsilon_F$, the temperature dependence of $\mu_e(r)$ is negligible, and
in this case we have \( \mu_e(r) \simeq \varepsilon_p \). Taking into account that \( \varepsilon_p \simeq 20 \text{ MeV} \) inside quark matter, the potential difference across the Coulomb barrier at the quark surface is \( \Delta V_c \simeq 2 \times 10^7 \text{ V} \). The thickness of the region with a strong electric field is \( \Delta r_g \simeq 10^3 \text{ fm} \simeq 10^{-10} \text{ cm} \) at \( kT_\theta \lesssim mc^2 \simeq 0.5 \text{ MeV} \) and decreases with increase of \( T_\theta \), where \( m \) is the electron mass [8]. The typical magnitude of the electric field in this region is \( \sim 5 \times 10^{17} \text{ V cm}^{-1} \) [2]. This field is a few ten times more than a critical field

\[
E_{cr} = m^2c^3/\hbar c = 1.3 \times 10^{16} \text{ V cm}^{-1},
\]

where \( \hbar \) is Plank’s constant and \( e \) is the electron charge.

It is well known that a vacuum with such a strong electric field is unstable, and \( e^+e^- \) pairs are created spontaneously. The pair creation rate per unit volume and per unit time in a stationary and homogeneous electric field, \( E = \text{const} \), is [10,11]

\[
W_\pm = \frac{m^4e^5}{4\pi^2\hbar^4} \left( \frac{E}{E_{cr}} \right)^2 \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left[ -\pi n \left( \frac{E_{cr}}{E} \right)^2 \right].
\]

At \( E \gg E_{cr} \), we have

\[
W_\pm \sim \frac{m^4e^5}{24\pi^2\hbar^4} \left( \frac{E}{E_{cr}} \right)^2 \simeq 1.7 \times 10^{30} \left( \frac{E}{E_{cr}} \right)^2 \text{ cm}^{-3} \text{s}^{-1},
\]

(2)

(3)

In such a strong electric field the value of \( W_\pm \) is very high, and, at first sight, the Coulomb barrier has to be a very powerful source of \( e^+e^- \) pairs, irrespective of the surface temperature. But this is not the case. The matter is quite different. Eqs. (2) and (3) are valid only if no real particles are present. The electrons in the quark-surface vicinity occupy some part of the energy levels and can significantly reduce the rate of pair creation.

Let us use Dirac’s hole picture for considering of pair creation by the Coulomb barrier. Figure 1 shows the space dependence of the potential energy \( eV(r) \) as well as the corresponding energy gap of the Dirac equation between \( mc^2 - eV(r) \) and \( -mc^2 - eV(r) \). Pair creation results from the tunneling of an electron from the ”Dirac sea” through the classically forbidden zone. The probability for such a tunnel process is described by a penetration factor given by [11]

\[
P \simeq \exp \left( -\frac{2}{\hbar} \int_{r_-}^{r_+} q(r)dr \right),
\]

where \( q(r) = \sqrt{m^2c^2 - (\varepsilon - eEr)^2/c^2} \) is the imaginary momentum and \( \varepsilon \) is the energy of the tunnelling electron, the electron rest energy \( mc^2 \) is included into \( \varepsilon \).

At \( T_\theta = 0 \), we have \( \mu_e = \varepsilon_p \), and the tunnelling of an electron from the ”Dirac sea” may be possible only into the states with energies below the pair creation threshold, \( \varepsilon \leq \varepsilon_p - 2mc^2 \) (see Fig. 1.). However, at zero temperature all electronic states with energies \( \varepsilon \leq \varepsilon_p \) are occupied. Hence, at \( T_\theta = 0 \) such a tunnelling of electrons is kinematically forbidden, and there is no \( e^+e^- \) pair creation by the Coulomb barrier as it was adopted by tacit consent in all studies of strange-quark matter (e.g., [2,8]).

At finite temperatures, \( T > 0 \), a quantum-mechanical state with an energy \( \varepsilon \) cannot definitely be said to be occupied or empty. Instead an occupation probability function \( f(\varepsilon) \) may be used. For electrons in thermodynamical equilibrium, Fermi-Dirac statistics requires \( f(\varepsilon) \) to be of the form (e.g., Ref. [11])

\[
f(\varepsilon) = \left[ 1 + \exp \left( \frac{\varepsilon - \mu_e}{kT} \right) \right]^{-1},
\]

(5)

i.e. all electronic states are not completely occupied, \( f(\varepsilon) < 1 \). In this case, the tunnelling of electrons from the ”Dirac sea” into the empty states is kinematically allowed. Therefore, the Coulomb barrier at the quark surface may be a source of \( e^+e^- \) pairs if the surface temperature is non-zero.

The thickness of the classically forbidden zone is (see Fig. 1.)

\[
\Delta r_{forb} = r_+ - r_- = \frac{2\hbar}{mc^2} \left( \frac{E_{cr}}{E} \right)^2 \simeq 7.7 \times 10^{-11} \frac{E_{cr}}{E} \text{ cm}.
\]

(6)

In our case, \( E \gg E_{cr} \), we have \( \Delta r_{forb} \ll \Delta r_g \) and, therefore, for the tunnel process the electric field of the Coulomb barrier may be considered as a homogeneous one. For such a field, Eq. (4) yields

\[
P \simeq \exp \left[ -\pi (E_{cr}/E) \right],
\]

(7)

i.e. the penetration factor \( P \) is exactly the exponential factor in Eq. (2) for the one-pair term, \( n = 1 \). At \( E \gg E_{cr} \), from Eq. (7) we have \( P \simeq 1 \), and the electrostatic barrier for the tunnel process completely disappears. In this case, the rate of pair production when electrons are created into the empty states is extremely high, and all the empty states below the pair creation threshold (see Fig. 1.) are occupied, \( f(\varepsilon) = 1 \), by creating electrons very fast. Then, the rate of pair creation by the Coulomb barrier is determined by the process of thermalization of electrons which favors the empty-state production below the pair creation threshold.

Positrons are created mainly at the outer boundary of the Coulomb barrier, i.e. at \( r \simeq r_{\text{max}} \), and then, they are ejected from the barrier by the electric field. Outflow of positrons results in a small decrease of the potential difference across the Coulomb barrier, which, in turn, results in outflow of electrons from the electron layer. The flux of \( e^+e^- \) pairs from the quark surface is

\[
\dot{N}_\pm \simeq 4\pi R^2 \Delta \Theta \Delta n_{e^{-1}th},
\]

(8)

where \( R \) is the radius of the star, \( \Delta \Theta \) is the density of electronic empty states with energies below the pair creation threshold at thermodynamical equilibrium and \( \Theta_{e^{-1}th} \) is the characteristic time of thermalization of electrons.
At \(kT_s < \varepsilon_{\nu}\), we have [12]

\[
\Delta n_e \simeq \frac{3kT_s}{\varepsilon_{\nu}} \exp\left(\frac{2mc^2}{kT_s}\right)n_e,
\]  

(9)

where \(n_e\) is the density of electrons.

In the electron layer, the spectrum of electrons is thermalized due to electron-electron collisions, and the thermalization time is of the order of \(\nu_{ee}^{-1}\), where [13]

\[
\nu_{ee} \simeq \frac{3}{2\pi} \sqrt{\frac{\alpha}{\pi}} \frac{(kT_s)^2}{\hbar\varepsilon_{\nu}} J(\zeta)
\]

(10)

is the frequency of electron-electron collisions for degenerate electrons with \(\varepsilon_{\nu} \gg mc^2\), \(\alpha = e^2/\hbar c = 1/137\) is the fine structure constant,

\[
J(\zeta) \simeq \begin{cases} 
51(1 - 19.5\zeta^{-1} + 296\zeta^{-2}) & \text{at } \zeta \geq 20, \\
0.23\zeta^{1.8} & \text{at } 1 < \zeta < 20, \\
(1/3)\zeta^{3}\ln(2/\zeta) & \text{at } \zeta \leq 1,
\end{cases}
\]

(11)

\[
\zeta = 2\sqrt{\frac{\alpha}{\pi}} \frac{\varepsilon_{\nu}}{kT_s} \simeq 0.1 \frac{\varepsilon_{\nu}}{kT_s}.
\]

(12)

For \(\varepsilon_{\nu} \approx 20\) MeV, \(R \approx 10^9\) cm, \(\Delta r_{\nu} \approx 10^{-10}\) cm and \(n_e \approx 10^{35}\) cm\(^{-3}\), from Eqs. (8) – (12) the flux of \(e^+e^-\) pairs from the bare quark surface of a strange star is

\[
\dot{N}_{\pm} \simeq 3 \times 10^{59} \left(\frac{kT_s}{\varepsilon_{\nu}}\right)^3 \exp\left(-\frac{2mc^2}{kT_s}\right) J(\zeta) \text{ s}^{-1}
\]

\[
\simeq \exp(-1.2 \times 10^{10} K/T_s) \times \left\{ \begin{array}{l} 
10^{56}\ln(T_s/10^{10} K) \text{ s}^{-1} \text{ at } T_s \geq 10^{10.3} K, \\
10^{55.5}(T_s/10^{10} K)^{1.2} \text{ s}^{-1} \text{ at } 10^9 K \lesssim T_s < 10^{10.3} K, \\
10^{57.3}(T_s/10^{10} K)^{3} \text{ s}^{-1} \text{ at } T_s < 10^9 K
\end{array} \right.
\]

(13)

within a factor 2-3 or so. The thermal energy of the star is a source of energy for the process of pair creation.

At high temperatures, \(T_s \gtrsim 10^{10}\) K, the thickness, \(\Delta r_{\nu}\), of the region with a strong electric field decreases with increase of \(T_s\) [8]. On the other hand, the electron density, \(n_e\), increases with increase of \(T_s\). As a result, the value of \(\Delta r_{\nu}/n_e\) is more or less independent of \(T_s\), and therefore, in Eq. (13) we ignored the temperature dependences of both \(\Delta r_{\nu}\) and \(n_e\).

The density of created \(e^+e^-\) pairs at the stellar surface is \(n_{\pm} \approx \dot{N}_{\pm}/4\pi R^2 c \lesssim 10^{38}\) cm\(^{-3}\) that is much smaller than the typical density of electrons in the electron layer, \(n_e \ll n_{\pm}\). Therefore, the production of \(e^+e^-\) pairs by the Coulomb barrier does not affect essentially the structure of the barrier itself. Our consideration of the process of \(e^+e^-\) pair creation is valid only for strange stars with bare (or nearly bare) quark surfaces.

At extremely high temperatures, \(T_s \approx 10^{11}\) K, the flux of pairs \(\dot{N}_{\pm}\) is as high as the upper limit

\[
\dot{N}_{\pm}^{\text{max}} \approx 4\pi R^2 \Delta r_{\nu} \dot{W}_{\pm} \approx 4 \times 10^{50} \text{ s}^{-1}
\]

(14)

in the vacuum approximation when any suppression of pair creation rate in comparison with the vacuum value \(\dot{W}_{\pm}\) given by Eq. (2) is neglected. In Eq. (14) to get the last equality the field strength of \(5 \times 10^{17}\) V cm\(^{-1}\) is used.

The luminosity in \(e^+e^-\) pairs which are created by the Coulomb barrier is \(L_{\pm}^b \approx \varepsilon_{\pm} \dot{N}_{\pm}\), where \(\varepsilon_{\pm} \approx mc^2 + kT_s\) is the mean energy of created particles and \(\dot{N}_{\pm}\) is given by Eq. (13). At \(T_s \sim 10^{11}\) K, we have \(\varepsilon_{\pm} \approx 10^{-5}\) erg and \(L_{\pm}^b \approx 3 \times 10^{51}\) ergs s\(^{-1}\). The total (time integrated) energy converted to \(e^+e^-\) pairs during the first ten seconds after the strange star formation may be as high as a few \(10^{51}\) ergs (for details, see Ref. [12]). This energy coincides with the typical energy output of \(\gamma\)-ray bursters [14] if these enigmatic astronomical objects are cosmological in origin [15] as it was suggested in Ref. [16].

Another process of \(e^+e^-\) pair creation outside the compact objects (neutron and strange stars) is the neutrino-antineutrino annihilation, \(\nu + \bar{\nu} \rightarrow e^+ + e^-\) [17]. The luminosity of a strange star in \(e^+e^-\) pairs which are created in this process is [6]

\[
L_{\pm}^\nu \approx 2 \times 10^{50} \left(\frac{T_s}{10^{11} K}\right)^9 \left(\frac{R}{10^9 \text{ cm}}\right)^3 \text{ ergs s}^{-1}.
\]

(15)

This luminosity may be more than the luminosity of the Coulomb barrier in \(e^+e^-\) pairs only during the first second after the strange star formation [18] when the temperature \(T_s\) at the stellar center is higher than \(\sim 10^{11}\) K. Here and below, it is taken into account that strange stars with \(T_s \gtrsim 10^{11}\) K are nearly isothermal (e.g., Ref. [6]).

The value of \(L_{\pm}^\nu\) rapidly decreases with the temperature decrease (see Eq. (15)) while the value of \(L_{\pm}^b\) varies quite slowly (roughly, \(L_{\pm}^b \propto T_s \ln T_s\) at \(10^{10} \lesssim T_s \lesssim 10^{11}\) K). For a strange star with \(T_s \ll 10^{11}\) K, we have \(L_{\pm}^b > L_{\pm}^\nu\), i.e., in this case, \(e^+e^-\) pairs are mainly produced by the Coulomb barrier. At \(T_s \approx T_c = 10^{10}\) K, for example, from Eqs. (13) and (15) the ratio \(L_{\pm}^\nu/L_{\pm}^b\) is \(\sim 10^{-8}\). As to neutron stars, their luminosity in \(e^+e^-\) pairs created due to neutrino-antineutrino annihilation [7,17] is not more than the value given by Eq. (15). Therefore, for a fixed temperature \(T_s\) at the stellar center, \(T_s \ll 10^{11}\) K, we also have \(L_{\pm}^b > L_{\pm}^\nu\), where \(L_{\pm}^b\) and \(L_{\pm}^\nu\) are the luminosities in \(e^+e^-\) pairs for a strange star and a neutron star, respectively.

Recently, some criteria were suggested [9] for a compact X-ray source to be considered as a strange star candidate. The considered effect of \(e^+e^-\) pair creation by the Coulomb barrier may be an additional observational
signature of strange stars with nearly bare quark surfaces. In connection with this, it is interesting to note that the emission feature at \( \sim 0.5 \) MeV was observed \cite{19} in the spectrum of 1E 1740.7–2942 which answers the criteria formulated in Ref. \cite{9} and may be, in fact, a strange star, not a black hole as it is usually assumed. This feature is commonly believed to be related to \( e^+e^- \) annihilation into two photons. To match the data on this feature, the \( e^+e^- \) pair flux of \( \sim 10^{43} \) s\(^{-1} \) from the source of pair plasma into outer space where pairs annihilate is required. This flux is equal to the pair flux from a strange star (see Eq. 13)) if the surface temperature of the star is \( T_s \approx 5 \times 10^8 \) K. This temperature is consistent with the data on the X-ray spectrum of 1E 1740.7–2942 \cite{19}.

\[\text{References}\]

Fig. 1. Creation of $e^+e^-$ pairs by the Coulomb barrier at the quark surface. The electric field of the barrier is very strong, $E > E_{cr}$, in the region $r_{\text{min}} < r < r_{\text{max}}$. Two thick solid lines given by equations $\text{Energy} = \pm mc^2 - eV(r)$ restrict the classically forbidden zone. The part of this zone where the tunneling of electrons from the "Dirac sea" is allowed at $T_s > 0$ is marked by dots.
\[ \text{Distance from the stellar center} \]

\[ \begin{align*}
\text{Energy} & = mc^2 \\
& = 0 \\
& = -mc^2 \\
& = -e\Delta V_c + mc^2 \\
& = -e\Delta V_c - mc^2 \\
\end{align*} \]

\[ r_{\text{min}} \quad R \quad r_- \quad r_+ \quad r_{\text{max}} \]

Dirac sea