Theory of Protostellar Accretion

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Abstract. This paper reviews current theoretical work on the various stages of accretion in protostars, and the relationship of these ideal stages to the spectral classes of observed protostellar systems. I discuss scaling relationships that have been obtained for models of pre-stellar cores as they evolve by ambipolar diffusion toward a central singularity, and expectations for the dynamical evolution as the cores collapse radially to form a rotating disk. I summarize work that suggests accretion in T Tauri systems may be limited by ionization to disk surface layers, and implications for the variation of $\dot{M}_D$ with radius. Finally, I describe models for the asymptotic structure of free magnetocentrifugal winds, and show that the constant-density surfaces in these winds may be strongly collimated even when the streamlines are not.

INTRODUCTION

The subject of this review is accretion in protostellar systems, which is properly a hot topic in a figurative but not a literal sense. In the other accreting systems discussed at this meeting, the observed high-energy emission often dominates the longer-wavelength flux, and much of the current theoretical effort is devoted to developing dynamical and radiative models of hot accretion flows that are capable of producing the observed spectral distributions. For low-mass protostars and T Tauri stars, on the other hand, the observed X-ray emission of $10^{29} - 10^{31}$ erg s$^{-1}$ [1–4] is much enhanced relative to that of main-sequence stars, but still constitutes just a small fraction ($10^{-4} - 10^{-3}$) of the sources’ bolometric luminosities; it can be understood as thermal emission from $T \sim 10^6 - 10^7$K plasma in the stellar magnetospheres and inner parts of the jets of these systems [5,6]. The bulk of the emission from young, low-mass stellar objects (YSOs) originates instead in a cooler accretion flow, where gas temperatures range from $\sim 10^4$ K (in the boundary layer/hot spot region), to a few $\times 10^3$K (in the inner, centrifugally-supported, accretion disk) to $\sim 10$K (in the outer, radial parts of the flow). As will be described below, many of the
current issues facing theorists interested in protostars have a quite different character from the issues under investigation in higher-energy systems.

In this paper, I will begin by briefly summarizing the standard observational classification of low-mass YSOs, which assigns sources to one of four broad groups based on their spectral energy distributions. As Nuria Calvet describes in detail in her observational review in this volume, other properties of YSOs – such as the relative masses of their disks vs. envelopes, their accretion rates, their outflow momentum flux, etc. – also correlate with their spectral properties. The spectral classification is generally interpreted as an evolutionary sequence corresponding to successive stages of protostellar accretion [7–10]; this paper outlines current understanding of the underlying theory. Since space constraints make a comprehensive review impossible (see instead the volume [11] and its successors), I will focus on some of the ways in which protostellar accretion is different from the other sorts of accretion discussed at this meeting. I will highlight some recent results in the theory of the early (i.e. largest-scale) stages of protostellar accretion, in which the flow is primarily radial. I will also describe recent work suggesting that even Shakura-Sunyaev-type “α”-models of the late, T-Tauri phase may be quite different from the standard (e.g. [12]) \( \dot{M}_D = \text{constant} \) paradigm. In the last section, I will describe models of free, wide-angle YSO winds that are different from the confined, narrow jets found in high-energy systems (cf. the review of Blandford in this volume).

**OBSERVATIONAL CLASSES OF YOUNG STARS**

The spectral classification of YSOs introduced by Lada [13] and expanded by Andre, Ward-Thompson, & Barsony [14] divides them into classes 0-III, as follows:

**Class 0**: These are embedded objects whose spectral energy distributions (SEDs) peak in the sub-millimeter and show little or no IR, and no optical. They are believed to be the earliest protostars, since the masses of their envelopes \( 0.2 - 2 M_\odot \) exceed the masses of their disks + protostars [14–16]. From the relative numbers of these sources compared to other YSO, ages are probably a few \( \times 10^4 \) years.

**Class I**: These are embedded objects which are strong far-IR emitters, with a peak in the SED at \( 10 - 100 \mu \). They are believed to correspond to late protostars or “pre-T-Tauri” stars since their envelope masses are typically \( 0.1 - 0.3 M_\odot \) or less [15]. From the relative number of these sources, ages are estimated at a few \( \times 10^5 \) years (e.g. [17]).

**Class II**: These are optically-revealed objects whose spectral energy distributions consist of a stellar photosphere plus significant “excess” IR and UV. These are interpreted as systems containing a star, an accreting disk, a boundary layer/hot spot, and a remnant envelope. Disk masses derived from mm
continuum during this stage are 0.01-0.1 $M_\odot$ [18] Ages of classical T Tauri stars (CTTS) (nearly coincident with this class) are $1 - 4 \times 10^6$ yrs [19].

Class III: These sources present stellar blackbodies, with no sign of active accretion; this class is essentially the same as the weak T Tauri stars (WTTS).

**STAGES OF PROTOSTELLAR ACCRETION**

**Ambipolar Diffusion and Density Restructuring**

Molecular clouds in our Galaxy contain hundreds of thermal Jeans masses ($M_J = c_s^3(\pi/G)^{3/2}\rho^{-1/2}$); the fact that the observed star formation rate is orders of magnitude below the ratio of the gas mass to the gravitational collapse time is used to argue that molecular clouds are primarily supported by magnetic pressure and tension rather than thermal pressure (e.g. [7,20,21]). Under these circumstances, dynamical collapse to form a star occurs only when the ratio of mass to magnetic flux exceeds a certain critical value [22,23]; for the central flux tube the corresponding ratio of surface density to magnetic field is $\Sigma/B = 1/(2\pi\sqrt{G})$. A given column density for a cloud implies a certain critical magnetic field strength $B_{\text{crit}}$, and vice versa. Using a field strength of $20\mu G$, and a size scale of $\sim 0.1$ pc, the critical mass (for a spherical cloud) corresponds to $\sim M_\odot$, comparable to the minimum masses of dense cores where low mass stars are observed to be forming.

As Mestel and Spitzer pointed out in the 1950s, the first stage in the protostellar accretion process under present Galactic conditions is expected to be the inward diffusion of neutrals relative to ions and the magnetic field, in order to create a supercritical core within a subcritical cloud. Thus the first accretion timescale is the ambipolar diffusion timescale $t_{\text{AD}} = L/v_{\text{drift}}$, which can be estimated by equating the drag force on ions from inflowing neutrals with the magnetic force on ions; the result is [24]

$$t_{\text{AD}} \approx 2 \times 10^6 \text{yrs} \left(\frac{n_i/n_{H_2}}{10^{-7}}\right) \left(\frac{B_{\text{crit}}}{B}\right)^2.$$  \hfill (1)

The ambipolar-diffusion/density restructuring stage has been studied by Nakano [25] and Lizano and Shu [26] in the quasistatic limit (applicable to early phases), and by Mouschovias and his collaborators [27–29] with a full dynamical treatment. For the latter numerical calculations, evolution is typically followed until the central density reaches $10^9 \text{cm}^{-3}$, 1-2 orders of magnitude below the limit at which an opaque “protostar” would form. The same phase of evolution of initially subcritical cores has also recently been studied semi-analytically by modeling magnetic stresses in an angle-averaged sense [30]. In complementary studies, Tomisaka [31] has performed ideal-MHD (i.e. no ambipolar diffusion) simulations of the evolution of magnetized clouds which...
are initially supercritical, beginning from a cylindrically-stratified equilibrium. While the results of all these authors differ in detail depending on their assumptions and parameter regime, the similarities in the results are more striking; I will concentrate on the latter. In both the initially-subcritical and the initially-supercritical simulations, the model clouds evolve towards flattened (disk-like), infalling structures in which the surface density $\Sigma \propto 1/R$, the magnetic field strength $B \propto 1/R$, and the midplane density $\rho_c \propto 1/R^2$, where $R$ is the cylindrical radial coordinate. The peak inflow speed is 2-3 times the sound speed (which for $T = 10K$ is $c_s = 0.2 \text{ km s}^{-1}$).

Schematically, these common properties can be understood from dimensional analysis as follows: First, marginally supercritical disks obey $B = 2\pi G^{1/2} \Sigma$ locally, so that once a region of the cloud becomes supercritical it will maintain $B \propto \Sigma$ because the ambipolar diffusion time is long compared to the dynamical time. Second, in local vertical equilibrium, an isothermal disk has central density $\rho_c = \pi G \Sigma^2/(2c_s^2)$ [32]; hence we can expect $B \propto \Sigma \propto \rho_c^{1/2}$ for the dynamically-evolving part of the flow. Finally, near-balance of the inward force of gravity ($\sim GM/R^2 \sim G\Sigma$) against the large-scale gradients of magnetic pressure and tension ($\sim B^2/\rho_c R \rightarrow G\Sigma^2/(\rho_c R) \rightarrow c_s^2/R$) and thermal pressure ($\sim \rho_c c_s^2/\rho_c R \rightarrow c_s^2/GR$) would imply $\Sigma \sim c_s^2/(GR)$, and hence $B \propto 1/R$ and $\rho_c \propto 1/R^2$. Ideas along these lines have been developed with more rigor by Basu [33] (see also this volume), who constructs an approximate self-similar dynamical solution that matches well to his numerical solutions for this stage of accretion.

Dynamical Collapse and Centrifugal Disk Formation

What comes next? How does dynamical collapse proceed in the centrally-condensed structures, so as to appear as the observed Class 0 and Class I objects? At this point, there is not the same convergence of theoretical models for the dynamical collapse stage (in which speeds approach free-fall in the inner parts) as there is for previous restructuring stage (which involves speeds no greater than a few km s$^{-1}$). Because the result of the previous stage yields a density profile $\rho \propto r^{-2}$, though, we may anticipate close resemblance to the family of (unmagnetized) isothermal-sphere self-similar collapse models whose best-known representatives are the Larson [34]/Penston [35] and Shu [36] solutions.

To gain insight into the dynamical collapse process, Galli & Shu [37] calculated solutions for collapsing clouds as perturbations with nonzero magnetic field strength about the inside-out singular isothermal sphere collapse. They found that, even with an initially uniform magnetic field and spherical density distribution, magnetic pinching forces lead to the formation of a growing equatorial “pseudodisk.” A small-amplitude magnetic field, however, does not alter the accretion rate; this remains equal to the value $0.975c_s^3/G$ of [36]. In
an opposite limit, Li & Shu [38] demonstrate that an initially static disk with initial magnetic and surface density profiles $B \propto \Sigma \propto 1/R$ (i.e. the asymptotic scalings obtained from the evolution in the “restructuring” stage) undergoes an inside-out collapse. The accretion rate is given by $\dot{M} \approx (1 + H_0)c_s^3/G$, where $1 + H_0$ is the density overfactor allowed by magnetic support in the initial equilibrium compared to an unmagnetized state ($H_0 = 0$). Although the calculation was performed in the $H_0 \gg 1$ thin-disk limit, more realistic cases where $H_0 = 1 - 10$ may be expected to show similar behavior for the dynamical collapse that follows the formation of a central density singularity.

In the very innermost regions, the solution must approach free-fall, and the asymptotic collapse solution for a perfectly cold, unmagnetized disk with initial surface density $\Sigma_i \propto R^{-1}$ is informative. For disk with initial profile $\Sigma_i(R) = M_i(R)/(2\pi R^2) \propto R^{-1}$ where $M_i \propto R$, the resulting asymptotic scalings (for large time $t$ and small radius $R$) are $\Sigma \propto t^{-1/2} R^{-1/2}$, $v \propto t^{1/2} R^{-1/2}$, and $M = \text{const.} = M_i(R)/t_{\text{collapse}}(R) = 2^3(\pi G)^{1/2}(\Sigma_i(R)R)^{3/2}$. Using the surface density profile $\Sigma \approx c_s^2/(GR)$ of [29], one obtains $\dot{M} = 14c_s^3/G$, or about $2 \times 10^{-5} M_\odot/\text{yr}$ for typical conditions. The associated collapse time $\sim 10^5 \text{yr}$ is comparable to the observationally-estimated ages of embedded sources, giving some confidence that such dynamical collapse models may indeed be associated with the Class 0 and Class I objects.

As collapse proceeds, infalling material will finally create a centrifugal disk whose radius increases in time due to the larger specific angular momentum of collapsing material at later times. Previous analysis of the growth of the centrifugal disk (see [39–41]) may be modified in the future to account for the differences between magnetized and unmagnetized (exterior) radial collapse models. Although the theory of the dynamics in this accretion stage remains rather uncertain, observations seem to have identified at least one case – the source HL Tau – where a large-scale, radially-infalling “pseudodisk” imaged in gas [42] surrounds a much more compact (presumably centrifugal) disk imaged in dust [43,44].

**Accretion in Protostellar Centrifugal Disks**

In the earlier stages of their evolution, protostellar disks are massive and self-gravity must be important to their evolution. A number of workers have investigated analytically and numerically the growth, development, and saturation of large-scale, low-$m$ gravitational instabilities (i.e. spiral density waves) in disks with protostellar properties (see e.g. [45] for recent calculations and references). Others have taken steps toward characterizing the action of gravitational torques by an effective “$\alpha$” parameter (e.g. [46] and references therein); this approach is more fruitful for local instabilities than global disk modes. In addition, still others have discussed how the presence of binary companions (present in the majority of systems) or planets can induce accretion through
externally-excited gravitational torques (e.g. [47,48]). At present, however, there is no generalized theory for how the processes of density-wave driven accretion and binary companion growth may develop in tandem or in competition out of self-gravitating disk perturbations. Significant future efforts will be needed before this stage of disk evolution is well-understood theoretically.

In the latter stage of centrifugal disk accretion, on the other hand, disk self-gravity is less important, and we believe we understand the relevant processes better. At present, the leading contender for a physical mechanism to produce the “viscous” torques in accretion disks around compact objects is the saturated Balbus-Hawley instability (see the contributions of Balbus and Gammie in this volume). Does the same process work in the disks of YSOs as well? The issue here is whether, with the low temperature and high surface density conditions of protostellar disks, the gas is sufficiently ionized so that resistive dissipation does not suppress the instability. In the inner parts ($R \lesssim 0.1$ AU) of protostellar disks, the central disk temperature exceeds 1000K and collisional ionization of alkali metals provides sufficient conduction electrons. But in the low-temperature outer parts of these disks, some source of external ionization is needed.

Recently, Gammie [49] argued that if cosmic rays alone provide the outer disk ionization, just the upper and lower surfaces to a depth of 100 g cm$^{-2}$ will be sufficiently ionized to couple magnetic fields to the disk matter, and that the result would be a “layered model” where disk surfaces accrete and the equatorial portion is inert. In corresponding “alpha” models of disk accretion, the “active layer” surface density $\Sigma_a$ is effectively constant with $R$, while the mass accretion rate $\dot{M}$ varies with $R$ depending on the local opacity. The result is a relatively low value of accretion

$$\dot{M} = 1.8 \times 10^{-8} M_\odot yr^{-1} \left( \frac{\alpha}{0.01} \right)^2 \left( \frac{\Sigma_a}{100 \text{g cm}^{-2}} \right)^2$$

in the innermost region, and the possibility of mass accumulation in the outer disk if the infall rate exceeds the carrying capacity of the surface layers. The low predicted value of $\dot{M}$ in the “layered” model appears consistent with recent spectral modeling of observed classical T Tauri disks (see the chapter by Calvet). If the difference between the infall rate and accretion rate is large enough, the mass buildup in the outer disk could periodically be purged via gravitational instabilities; such a mechanism could potentially be responsible for the observed FU Ori outbursts (see the chapter of Calvet for discussion of these phenomena).

Following up on these ideas, Glassgold, Najita, & Igea [50] have shown that since YSOs are strong X-ray emitters, the surface layers of their accretion disks will be ionized to a thickness comparable to Gammie’s estimate, even if the cosmic ray flux is inhibited by strong, magnetized T Tauri winds. Further work on these sorts of “layered” models will be needed to determine whether
both the observed low mean accretion rate of T Tauri stars and their high-accretion-rate outbursts events can be explained within a unified framework.

**PROTOSTELLAR WINDS**

My final topic is the nature winds from protostellar systems, and their connection to the narrow Herbig-Haro jets and broader massive molecular outflows which, observationally, seem to be an inescapable side-effect of protostellar accretion. Optically-visible Herbig-Haro objects are believed to represent shocked regions in dense, high-velocity gas that emerges from the very innermost part of protostellar accretion disks (see contributions by Frank and Bally in this volume). Although there is general agreement that the surrounding molecular outflows consist of ISM material that has been mobilized by an interaction with a wind, there is equally general disagreement about the nature of that primary wind, and its relationship to observed H-H jets.

In most current models, the primary wind is believed to be magnetocentrifugally driven, from the inner disk edge where the accretion flow interacts with the stellar magnetosphere (see [51] and the review of Stone in this volume), and/or from the surface of the disk at larger radii (see the review [52]). The controversial issue is whether the primary wind consists solely of an isolated, highly collimated flow – seen as the ionized, optical jet – or whether there is also a lower-density, wider-angle, neutral wind surrounding the observed jet.

In the first “isolated jet” picture, molecular outflows are essentially the wings of bow shocks in the jet head or beam (see e.g. [53]); hence the gas motions transverse to the jet are driven by pressure gradients. One difficulty with such models is that the cooling under interstellar conditions may be too strong to allow a very broad bow shock to form (e.g. [54,55]). Another potential difficulty is that, depending on the variation of magnetic field strength with radius at the base of the flow, it may not even be possible to drive a wind which is both strongly collimated and fast [56]. A related difficulty is that since magnetocentrifugal wind models typically have internal Alfvén speeds which approach (or even exceed) their flow speeds, they may not be able to remain collimated as they propagate into the low-pressure ambient medium – where the sound speed is 1-3 orders of magnitude smaller than the wind speed. Adam Frank’s contribution to this volume describes ongoing numerical studies of jet/cloud interactions which address these and other issues.

In the second, “wide-angle wind” picture, molecular outflows are interpreted as the pileup from a momentum-driven “snowplow” [57]. A wide-angle wind would occur if a magnetocentrifugal outflow is unable to self-collimate, or becomes decollimated as it emerges from a dense core into a medium of low ambient pressure. Such a wind would expand laterally to fill the whole solid angle between the equator and whatever natural boundary exists near the pole. Because cold magnetocentrifugal outflows only occur on field lines having
angles less than \(60^\circ\) with respect to the equator, it is natural to expect that there would remain some unloaded axial fields filling the space interior to the wind towards the pole, which originate in the disk at angles greater than \(60^\circ\) with respect to the equator. In particular, when the wind is formed by tearing open a magnetosphere \([51,59]\), the axial magnetic flux interior to the wind will be comparable to the poloidal magnetic flux in the wind itself. If the wind field lines are instead open field lines carried in with the accretion flow, then the ratio of interior magnetic flux to wind magnetic flux would depend on the distribution of the field in the disk.

The asymptotic structure of a wide-angle MHD wind, for the case of a flow along opened magnetospheric field lines emerging nearly isotropically from the corotation region (the “x-wind”), has been analyzed by Shu, Najita, Ostriker, & Shang \([60]\). They show that, while the streamlines collimate only slowly with distance, the density structure in the wind becomes nearly cylindrically stratified. Briefly, the reasons for these results can be explained as follows: In the asymptotic regime, the largest term in the internal stress is that associated with the toroidal magnetic field \(B_\phi\). In order to have a locally force-free configuration, \(\partial (B_\phi R)/\partial \theta \approx 0\) must hold, where \(R = r \sin \theta\) is the cylindrical distance from the axis, and \(\theta\) is the polar angle. This implies

\[
B_\phi = -C(r)/R
\]  

(3)

for some (positive) function \(C(r)\). From angular momentum conservation and field freezing, we have

\[
\frac{B_x}{B_p} = -\frac{\Omega R}{v_p} \quad \text{and} \quad B_p = \beta \rho v_p,
\]

(4)

where \(v_p\) and \(B_p\) are respectively the poloidal flow speed and poloidal magnetic field, \(\Omega\) is the angular rotation rate of the footpoint of the local field line, \(\rho\) is the density, and \(\beta\) represents the ratio of magnetic flux to mass flux (which remains constant on any streamline). From these equations, we find

\[
\rho = \frac{C(r)}{\Omega \beta R^2}.
\]

(5)

Using \(\nabla \cdot \mathbf{B}_p = 0\), it can be shown that \(C\) is a slowly-varying function of \(r\). Thus, if \(\beta\) and \(\Omega\) vary little between streamlines (as is true in the x-wind solutions of Najita and Shu \([61]\)), then the density in the wind will vary approximately as \(\rho \propto R^{-2}\) at large distances. Note that while the scaling \(\rho \propto r^{-2}\) arises kinematically from the continuity equation for a constant-speed radial flow, the scaling \(\rho \propto (\sin \theta)^{-2}\) comes about because hoop stresses redistribute the streamlines in \(\theta\) until the magnetic field is force-free (cf. eq. 3). The actual value of the function \(C(r)\) may be evaluated by balancing the pressure at the inner edge of the wind \(B_\phi^2/(8\pi)\) against the pressure provided by interior axial
fields, $B^2_{\text{pol}}/(8\pi)$, and applying the constraint of mass outflow conservation. The value of the poloidal speed on any streamline can be obtained from energy conservation; in terms of the streamline’s footpoint radius $R_0$ and Alfvén radius $R_A$, $v_p = \Omega (2R_A^2 - 3R_0^2)^{1/2}$.

For any variation of $\Omega$, $\beta$, $R_0$, and $R_A$ with magnetic flux $\Phi$ (this variation must be derived self-consistently for the wind inside the fast-MHD surface), the foregoing considerations can be applied to find the asymptotic structure of a “free” MHD wind – whether the wind originates in a narrow or broad region of the disk. It can be shown, for example, that disk winds which leave the disk with $\rho \propto R_0^{-q}$ and $B \propto R_0^{-(1+q)/2}$ also have slowly-collimating (i.e. pointing close to radially) streamlines but nearly cylindrically-stratified density structure near the pole, at large distances from the disk. Figure 1 shows an example of the asymptotic structure of a wind in which all streamlines originate at the same radius $R_0$ and have $R_A = 2R_0$, and for which the interior axial magnetic flux along the pole equals the wind poloidal magnetic flux.

Wide-angle “snowplow” models with winds of this sort have been successful at explaining many general characteristics of outflows [62], as well as particular outflow systems [63].

Further work on both observational and theoretical fronts is still needed to identify which set of conditions leads to a wide-angle wind, and which to an isolated jet, or perhaps to conclude instead that alternative models are
warranted to represent the primary winds from protostars.

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