Virtual Supersymmetric Corrections in $e^+e^-$ Annihilation

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Abstract

Depending on their masses, Supersymmetric particles can affect various measurements in Z decay. Among these are the total width (or consequent extracted value of $\alpha_s$), enhancement or suppression of various flavors, and left-right and forward-backward asymmetries. The latter depend on squark mass splittings and are, therefore, a possible test of the Supergravity related predictions. We calculate leading order corrections for these quantities considering in particular the case of light photino and gluino where the SUSY effects are enhanced. In this limit the effect on $\alpha_s$ is appreciable, the effect on $R_b$ is small, and the effect on the asymmetries is extremely small.

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1. INTRODUCTION

In recent years several slight anomalies in Z decay have encouraged attention to the possibility of supersymmetric particles in the Z region and below. These are 1) an apparent value of the strong coupling at the Z greater than expected from the extrapolation of low energy measurements, 2) an enhanced value of the Z branching ratio into b quarks relative to the Standard Model (SM) expectations, and 3) deviations from the Standard Model predictions for the left-right asymmetry in polarized electron-positron annihilation and for various forward backward asymmetries. Even if these discrepancies shrink under further analysis as may now be the case with the $R_b$ anomaly, it is useful to consider what constraints they impose on SUSY masses. In this note we treat the strong supersymmetric (SUSY) effects on these quantities from the graph of Fig. 1 involving a virtual squark-squark-gluino triangle correction to the Z coupling to quark-antiquark final states. We assume that, while the gluino might be light relative to the Z mass, the squarks are all above the Z so there are no effects from production of on-shell squarks. The graph of Fig. 1 has been considered earlier [1–4]. These works for the most part ignore sfermion mass splittings crucial to some of the anomalies discussed above and, in any case, do not consider explicit mass splitting schemes currently under discussion. The present work, extending the results of these authors to include these effects, is divided as follows. In section 2 we briefly discuss the experimental situation relative to the above-mentioned anomalies. In section 3 we present the calculation of the graph in Fig. 1, allowing for squark mass splittings. Results and conclusions are given in section 4.

2. ANOMALIES IN Z DECAY

Since 1992, various authors have noted that the apparent value of the strong coupling constant at the Z is greater than the value expected in the Standard Model from extrapolation from lower energies. This was in fact one of the initial proposed positive indications [5–7] of a light gluino since such a particle would slow the running of $\alpha_s$. The effect has been clouded by disagreement over the value of this coupling at both the low and high energy regions but, clearly, the greater is the value of $\alpha_s(M_Z)$ and the lower is its value at lower energies, the more likely one is to be interested in the light gluino possibility. The fitted values quoted from Z decay measurements have ranged from a low of $0.108 \pm 0.005$ [8] to a high of $0.141 \pm 0.017$ [9]. Current values are typified [10] by that from the Z hadronic width.

$$\alpha_s(M_Z)^{\text{apparent}} = 0.123 \pm 0.005$$ (2.1)

Here we have labeled the fitted value as ”apparent” since the value quoted is from a Standard Model analysis and there could be extra (e.g., SUSY) contributions such as that of Fig. 1. We assume that the actual value of $\alpha_s(M_Z)$ is that extrapolated from low energy. Although these numbers are somewhat dependent on the analysis performed, they tend to be systematically below the value in Eq. (2.1). For definiteness we will note the values from [10] in the SM or heavy gluino (HG) case as obtained from an $\Upsilon$ decay analysis

$$\alpha_s(M_Z)^{\text{SM}} = 0.108 \pm 0.010$$ (2.2)

$$\alpha_s(M_Z)^{LG} = 0.114 \pm .005.$$  \hfill (2.3)

Thus the difference between the actual $\alpha_s(M_Z)$ and the apparent value is

$$\delta \alpha_s = -0.009 \pm .007 \text{ (LG case)}$$  \hfill (2.4)

$$\delta \alpha_s = -0.015 \pm .011 \text{ (SM or HG case)}$$  \hfill (2.5)

The SM value of the coupling constant adopted here is in agreement with that found from QCD sum rules [12], although these authors quote large errors. Other particular analyses, notably that from $\tau$ decay, could significantly reduce the discrepancy (or even change its sign) but we feel it fair to say that a large body of low energy data prefers negative $\delta \alpha_s$. The $\tau$ analysis in particular has been criticized [13] as being especially vulnerable to non-perturbative effects. In the LG case there is a small positive contribution to $\delta \alpha_s$ from gluino contributions alone to Z decay [14] amounting to $\delta \alpha_s \approx +.002$. Related perhaps to the $\alpha_s$ anomaly, is a possible excess of $b$ quark production in Z decay which has stimulated significant interest in possible manifestations of SUSY below the Z. In particular it has been noted that an appreciable enhancement of b production could be a signal for charginos and stop quarks below the Z [2,15]. Such a non-standard-model contribution to Z decay might also explain the apparent enhancement in $\alpha_s$ at the Z relative to the value expected from extrapolation from lower energies. If the only non-SM contribution to the Z width was in the $b\bar{b}$ channel it would be related to $\delta \alpha_s$ by [16]

$$\delta \alpha_s = \pi \delta R_b \frac{1 + \alpha_s/\pi}{1 - R_b},$$  \hfill (2.6)

where

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})}$$  \hfill (2.7)

and $\delta R_b$ is the deviation of this quantity from its standard model value. However, due to new LEP results above the Z, the likelihood of low mass charginos (at least in the standard
SUSY picture) has diminished. In addition the b anomaly has largely disappeared under reanalysis. The latest experimental analysis yields \[17\]

\[\delta R_b = 0.0009 \pm 0.0011\]  

(2.8)

This value is consistent with zero and it is interesting to see how severely it constrains contributions beyond the Standard Model and, in particular, whether SUSY effects could lead to an enhanced apparent \(\alpha_s\) without enhancing the b decay mode of the Z. With sufficiently light chargino and stop quarks the chargino-stop virtual correction to the Z vertex dominates the SUSY contribution to the Z partial widths and can lead to a \(\delta R_b\) as great as .0018. In the light gaugino scenario, however, with \(m_{1/2} = A = 0\), the stop quarks are expected to be above the top \[18\] and this contribution to \(\delta R_b\) drops below .0005. In this case, the virtual gluino correction could become the dominant SUSY effect. In this article we reconsider the gluino (and photino) corrections with special attention to scheme dependence and dependence on the sfermion mass splitting which have not been part of prior investigations.

We also consider the left-right and forward-backward asymmetries. These are related to the effective coupling of the Z to vector and axial vector currents of particular fermion species. One defines

\[A(f) = \frac{2g_V(f)g_A(f)}{g^2_V(f) + g^2_A(f)}\]  

(2.9)

The left-right asymmetry is defined in terms of the total annihilation cross sections for left-handed and right-handed polarized electrons.

\[A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A(e).\]  

(2.10)

The Forward Backward asymmetry for a particular flavor \(f\) is

\[A_{FB}(f) = \frac{\sigma(\cos \theta > 0) - \sigma(\cos \theta < 0)}{\sigma(\cos \theta > 0) + \sigma(\cos \theta < 0)}.\]  

(2.11)

For an unpolarized beam this is given by

\[A_{FB}(f) = \frac{3}{4} A(e) A(f).\]  

(2.12)

Current asymmetry measurements can be summarized by experimental determinations of the difference between the \(A(f)\) and their standard model predictions \[10\].

\[\delta A(e) = 0.011 \pm 0.005\]

\[\delta A(b) = -0.093 \pm 0.053\]  

(2.13)

\[\delta A(c) = -0.061 \pm 0.090\]

Even if, as we find, the SUSY contributions cannot account for discrepancies at this level, it is useful to record here the SUSY prediction for comparison with future more accurate measurements.
3. SUSY QCD VERTEX CORRECTION

The Born term matrix element for Z decay into a fermion-antifermion pair in the Standard Model takes the form

\[ M^0_{Z \rightarrow f \bar{f}} = \frac{e}{\sin 2\theta} \epsilon_\mu \bar{u}(p) \gamma_\mu (g^0_V - g^0_A \gamma_5) v(q - p), \]  

where

\[ g^0_V = T_3 - 2Q \sin^2 \theta, \quad g^0_A = T_3. \]  

(3.1)

In terms of these parameters the zeroth order Z width into a fermion anti-fermion pair, neglecting the fermion mass, is

\[ \Gamma^0_{f \bar{f}} = \frac{\alpha}{\sin^2 2\theta} M_Z [(g^0_V)^2 + (g^0_A)^2]. \]  

(3.3)

A second quantity of some interest is

\[ A^0(f) = \frac{2g^0_V g^0_A}{(g^0_V)^2 + (g^0_A)^2} \]  

(3.4)

since the forward-backward asymmetry of fermion species \( f \) is given by eq. (2.12), and the left-right asymmetry of the total hadronic cross section with a longitudinally polarized electron beam is \( A_{LR} = A^0(e) \). The existing discrepancies between these measured quantities and the Standard Model predictions make a study of the supersymmetric contributions of interest. The lowest order SUSY QCD corrections to the Z-fermion vertex arise from the squark-squark-gluino triangle graph of Fig. 1. The corresponding matrix element is

\[ \delta M^0_{Z \rightarrow f \bar{f}} = \frac{e}{\sin 2\theta} \epsilon_\mu \bar{u}(p) \gamma_\mu (\delta g^0_V - \delta g^0_A \gamma_5) v(q - p), \]  

where

\[ \delta g^0_V = [g^0_V (I^r_L + I^r_R) + g^0_A (I^r_L - I^r_R)] C_F \alpha_s(\mu) \frac{4\pi}{\mu^2}, \]  

\[ \delta g^0_A = [g^0_A (I^r_L + I^r_R) + g^0_V (I^r_L - I^r_R)] C_F \alpha_s(\mu) \frac{4\pi}{\mu^2}, \]  

(3.5)

(3.6)

(3.7)

and \( C_F \) is the eigenvalue of the quadratic Casimir operator of the gauge group in the fundamental representation (for SU(3), \( C_F = 4/3 \)). The \( I^r_{L,R} \) are the contributions from left and right squarks corresponding to the graph of Fig. 1. The \( r \) indicates that the ultraviolet renormalization has been made. Adding the SUSY correction to the Born term matrix element is equivalent to replacing \( g^0_{V,A} \) by

\[ g^0_{V,A} = g^0_{V,A} + \delta g^0_{V,A}. \]  

(3.8)

These lead to the \( O(\alpha_s) \) equations
\[ g_V^2(f) + g_A^2(f) = [(g_V^0(f))^2 + (g_A^0(f))^2]\left\{1 + C_F \frac{\alpha_s(\mu)}{2\pi} \left[(I_L^f + I_R^f) + A^0(f)(I_L^f - I_R^f)\right]\right\} \] (3.9)

\[ A(f) = A^0(f) + C_F \frac{\alpha_s(\mu)}{2\pi} \left[I_L^f - I_R^f\right] \] (3.10)

Using dimensional regularization [19] with the dimension of space-time \( d = 4 - 2\epsilon \) and a standard Feynman parametrization, the unrenormalized \( I_{L,R} \) can be written as follows.

\[
I_{L,R} = \frac{16i(2\pi)^{d-4}\mu^2}{d-4} \int d^4k \int_0^1 dx_1dx_2\theta(1 - x_1 - x_2) \frac{C_{L,R}}{(C_{L,R} - k^2)^3}
\]

or

\[
I_{L,R} = \Gamma(\epsilon)(4\pi)^\epsilon \int_0^1 \frac{C_{L,R}}{\mu^2}\frac{1}{(C_{L,R} - k^2)^3} dx_1dx_2\theta(1 - x_1 - x_2),
\] (3.11)

where

\[
C_{L,R} = \tilde{\mu}^2x_2 + \tilde{m}_{L,R}^2(1 - x_2) - Q^2x_1(1 - x_1 - x_2).
\] (3.12)

Here \( \tilde{\mu} \) is the gluino mass and \( \tilde{m}_{L,R} \) are the left and right squark masses of the same flavor as the final state fermions. \( Q^2 \) is the center-of-mass energy squared \((M_Z^2 at the Z pole)\) and \( \mu \) is t'Hooft’s unit of mass [19].

Adding SUSY self energy corrections on the external quark legs has the effect of subtracting from the result of Fig.1 its value at \( Q^2 = 0 \) [4]. Then

\[
I_{L,R} = -\int_0^1 dx_1dx_2\theta(1 - x_1 - x_2) \ln\left[1 - \frac{Q^2x_1(1 - x_1 - x_2)}{\tilde{\mu}^2x_2 + \tilde{m}_{L,R}^2(1 - x_2)}\right].
\] (3.13)

This function has the series expression

\[
I_{L,R}^{CWZ} = \sum_{n=1}^{\infty} \frac{Q^{2n}}{n} B(n + 1, n + 1) \int_0^1 dx_2 \frac{(1 - x_2)^{2n+1}}{[\tilde{\mu}^2x_2 + \tilde{m}_{L,R}^2(1 - x_2)]^n}.
\] (3.14)

Two simple cases can be studied:

\[
I_{L,R}(\tilde{\mu} = 0) = \sum_{n=1}^{\infty} \left(\frac{Q^2}{\tilde{m}_{L,R}^2}\right)^n \frac{B(n + 1, n + 1)}{n(n + 2)} = \frac{1}{18} \left(\frac{Q^2}{\tilde{m}_{L,R}^2}\right) + \frac{1}{240} \left(\frac{Q^2}{\tilde{m}_{L,R}^2}\right)^2 + \cdots
\] (3.15)

and

\[
I_{L,R}(\tilde{\mu} = \tilde{m}_{L,R}) = \sum_{n=1}^{\infty} \frac{Q^2}{\tilde{m}_{L,R}^2}^n \frac{B(n + 1, n + 1)}{n(2n + 2)} = \frac{1}{24} \frac{Q^2}{\tilde{m}_{L,R}^2} + \cdots
\] (3.16)

In each case the series are rapidly converging and the first term gives about 85% of the full result even down to \( \tilde{m}_{L,R}^2 = Q^2 \). One can see from these results that although \( \tilde{I} \) falls rapidly with increasing squark mass it is much less sensitive to the gluino mass. Since an ultra-light gluino is still a possibility [20], in the following section we concentrate on the numerical
results for the $\hat{\mu} = 0$ case. The one-loop SUSY QCD corrected decay rates and asymmetries into $f \bar{f}$ are to $O(\alpha_s)$

$$\Gamma_{f \bar{f}} = \frac{\alpha}{\sin^2 2\theta} m_Z (g_V^2 + g_A^2) = \Gamma_{f \bar{f}}^0 \left[ 1 + \frac{3}{4} C_F \alpha_s \pi + C_F \frac{\alpha_s \pi}{\pi} \left( \frac{I_L^f + I_R^f}{2} + A^0(f) \frac{I_L^f - I_R^f}{2} \right) \right]$$

(3.17)

$$A(f) = \frac{2 g_V g_A}{g_V^2 + g_A^2} = A^0(f) + C_F \frac{\alpha_s \pi}{\pi} \left[ I_L^f(f) + I_R^f(f) + A^0(f)(I_L^f(f) - I_R^f(f)) \right]$$

(3.18)

In $\Gamma_{f \bar{f}}$ we have included the QCD order $\alpha_s$ correction. The total hadronic decay rate to this order, obtained by summing the partial rates over $f$, can be written as follows.

$$\Gamma_{\text{tot}} = \Gamma_{\text{tot}}^0 \left( 1 + \frac{\alpha_s^{\text{apparent}}}{\pi} \right)$$

(3.19)

where the "apparent" $\alpha_s$, the one that would result from a Standard Model analysis, is

$$\delta \alpha_s = \alpha_s - \alpha_s^{\text{apparent}} = \pi \sum_f \delta_f$$

(3.20)

where

$$\delta_f = C_F \frac{\alpha_s \Gamma_{f \bar{f}}}{2 \pi \Gamma_{\text{tot}}} \left[ I_L^f(f) + I_R^f(f) + A^0(f)(I_L^f(f) - I_R^f(f)) \right].$$

(3.21)

$\Gamma_{\text{tot}}^0$ being the zeroth order total hadronic decay width.

The contribution to $R_b$ is

$$\delta R_b = \delta_b - R_b^0 \sum_f \delta_f.$$  

(3.22)

with the analogous expression for $\delta R_c$. The $I_{L,R}$ depend on the masses of the SUSY partners of the left and right handed quarks of flavor $f$ as well as on the gluino mass. The squark masses, in the SUGRA scheme are given by

$$\tilde{m}_{f,L}^2 = m_0^2 + m_f^2 + M_Z^2 \cos 2\beta (T_{3,f} - Q_f \sin^2 \theta)$$

(3.23)

$$\tilde{m}_{f,R}^2 = m_0^2 + m_f^2 + M_Z^2 Q_f \cos 2\beta \sin^2 \theta$$

(3.24)

where $m_0$ is the universal scalar mass. Neglecting the quark masses, and $M_Z^2$, the $I_{L,R}$ become independent of flavor. In this case the lowest order contributions to $\delta R_b$ and to the asymmetries vanish although there is still a significant effect on the $Z$ total width.

4. RESULTS AND CONCLUSIONS

Since the experimental evidence for discrepancies from the Standard Model are so slight (two standard deviations or less), at this point our results should be interpreted as predictions of low energy SUSY rather than as suggestions from the data. Our main point is that the graph of Fig. 1 can be effective in enhancing the hadronic decay rate of the $Z$ without
enhancing the relative contribution from b quarks. The variations of $\delta R_b$ and $\delta \alpha_s$ are shown in Figs. 2 and 3 respectively as functions of the universal scalar mass $m_0$, using $\alpha_s(M_Z) = 0.113$, $\tan \beta = 1.6$ and $\sin^2 \theta = 0.2317$. The contribution to $\delta R_b$ and to $\delta \alpha_s$ are negative and slowly varying. The SUSY QCD contribution to $\delta R_b$ therefore tends to cancel the chargino-stop contribution to $\delta R_b$ while still leaving an appreciable negative contribution to $\delta \alpha_s$ consistent with the experimental indications discussed in section 2.

The contributions to the forward-backward asymmetries behave similarly although they are orders of magnitude smaller than the current experimental errors. The SUSY predictions are very close to those of the Standard Model and are two standard deviations from the current experimental results. In table 1 we summarize the results in the case $m_0 = 106\,\text{GeV}$ and $\tilde{\mu} \approx 0$ as suggested in recent phenomenological studies of the Fermilab jet transverse energy distributions, scaling ratio, and top quark events [21,18]. $\delta A_{\exp}$ is defined as $A_{\exp} - A^0$ and $\delta \Gamma$ is the SUSY correction to the $Z$ width from eq. (3.17). The left-right asymmetry is obtained by replacing $C F \alpha_s$ by $\alpha$ in eq. (3.18). Since the Standard Model result is suppressed by a factor of $4 \sin^2 \theta - 1$ one could expect the experiment to be sensitive to the SUSY contribution. However in the SUGRA related model for sfermion mass splittings, the selectron mass splitting is also suppressed by the same factor. This fact plus the proportionality to the fine structure constant makes this experiment very insensitive to SUSY.

The analysis presented here can be readily extended to the higher $Q^2$ values of LEP II. The early results from CERN are consistent with the Standard Model but, in view of the large fluctuations seen in the early results from LEP I discussed in the introduction to this article and in view of the sharply reduced statistics at higher energy and complications from the "radiative return" and W pair production we prefer to defer this analysis to a later date. Parity violating effects in hadronic collision from SUSY virtual contributions such as those treated here in $Z$ decay have also received recent attention [22].

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**TABLE I. Results for different schemes.**

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<th>$f$</th>
<th>$\tilde{m}_L$</th>
<th>$\tilde{m}_R$</th>
<th>$\Gamma^0$</th>
<th>$\delta \Gamma \times 10^4$</th>
<th>$\delta R_f \times 10^5$</th>
<th>$A^0$</th>
<th>$\delta A \times 10^4$</th>
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<td>.667</td>
<td>0.826</td>
<td>.06±.09</td>
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<tr>
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<td>107</td>
<td>.370</td>
<td>6.78</td>
<td>-3.85</td>
<td>.935</td>
<td>-1.06</td>
<td>.10±.05</td>
</tr>
</tbody>
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FIG. 2. Results for different schemes vs universal scalar mass
FIG. 3. Effect in $\alpha_s$ vs universal scalar mass
REFERENCES