Supersymmetric Models with Product Groups and Field Dependent Gauge Couplings

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Abstract

We study the effective potential which is generated for the dilaton in a wide class of strongly-coupled, asymptotically free, $N = 1$ supersymmetric gauge models. We consider models with product gauge groups, $G_1 \times G_2$, having matter charged under both group factors. These scenarios are rich in new and interesting features, which include mechanisms for stabilizing the dilaton to prevent its runaway solution to weak coupling, even for asymptotically free gauge groups, as well as various possibilities for supersymmetry breaking depending on whether the gauge coupling is field dependent.
1. Introduction

We present findings pertaining to the main phenomenological challenges faced by string theory — the breaking of supersymmetry, the stabilization of the dilaton, and the lifting of the large vacuum degeneracy due to the many moduli which string compactifications typically generate. Great strides have been recently made in the understanding of nonperturbative effects in supersymmetric field theories [1] [2] [3] [4]: Seiberg and other workers have developed methods that allow us to write the exact form of the low-energy superpotential for many supersymmetric gauge theories. Espousing aspects of these methods, we explore features of the dilaton superpotential in a class of $N = 1$ supersymmetric gauge theories with product groups. Our focus is on possible dynamical breaking of supersymmetry and on the stabilization of the dilaton.

The recent resurgence of interest in strongly-coupled supersymmetric models has spawned a number of studies of low-energy effective superpotentials and dynamical supersymmetry breaking (for example, those in refs. [5] [6] [7]), so it is worth stating at the outset what is novel in the analysis we present here. The models which we explore have two defining characteristics. First, they involve a sector consisting of asymptotically-free product gauge groups, $G_1 \times G_2 \times \cdots$, having matter which is charged under more than one group factor. (We call this type of matter ‘doubly charged’ because we typically consider couplings only to two factors of the gauge group.) Second — motivated by string theory — our models incorporate a gauge coupling which is field dependent, with the gauge kinetic term having the form:

$$L_{\text{kin}} = \frac{1}{4} \left( S \, \text{Tr} \, W W \right)_F$$ (1)

where $\left( \cdots \right)_F$ denotes the chiral supersymmetric invariant, $W$ is the gauge-kinetic chiral spinor supermultiplet, and $S$ is the chiral scalar supermultiplet (in string theory, the dilaton) whose v.e.v. determines the value of the gauge coupling.

Earlier workers have typically considered either of these features in isolation (i.e. product gauge groups with doubly-charged matter with the gauge coupling constant [3] [6] [8] [9], or field-dependent gauge couplings with any given matter field carrying charge
for only one of the gauge group factors \([10] [11]\), but not both together. We find that including the two features — field-dependent couplings and doubly-charged matter — leads to models which are rich in interesting phenomena. Our results are as follows:

• 1: Our main result concerns the stabilization of the dilaton. We find asymptotically-free gauge models for which the superpotential defining the vacuum is extremized for finite \(S\). This superpotential has no runaway extrema at weak coupling (i.e. \(\text{Re} \langle S \rangle \to \infty\)) and, on the contrary, gives a vacuum energy which actually increases as the v.e.v. of the dilaton scalar increases. Interestingly, we find that the Kac Moody levels \(k_r\) of the various gauge group factors can play an important role in determining whether the dilaton is unambiguously stabilized at a finite v.e.v.

Our models thus serve as a counter-example to the general wisdom that the potential must always admit an extremum in the weak-coupling limit. As we explain in detail, we are able to evade this general wisdom by choosing examples which do not satisfy an implicit continuity assumption on which it rests.

• 2: We also argue that the inclusion of field-dependent gauge couplings can qualitatively change whether or not a given model spontaneously breaks supersymmetry. The main difference is due to the additional requirement of extremizing the superpotential with respect to the coupling-constant field. For instance, it can happen that a supersymmetry-breaking ground state for fixed gauge coupling becomes supersymmetric once the coupling constant is allowed to relax to minimize the energy.\(^1\) Furthermore, we find that the opposite of this is also possible: supersymmetry can be unbroken for fixed gauge coupling, but become broken once the gauge coupling is considered as a field.

We present our results in the following way. In §2 we outline our general strategy for building models for which the dilaton superpotential is extremized only for finite \(\text{Re} \langle S \rangle\).

\(^1\) This is similar to, but goes beyond, what is known from the simplest case of gaugino condensation for a pure gauge theory having a simple gauge group and no matter multiplets. In this case, for constant gauge couplings, gauginos condense without breaking supersymmetry \([12]\), whereas for field-dependent gauge couplings, the vacuum is at vanishing coupling, for which supersymmetry is again not broken and gauginos do not even condense.
of a runaway extremum for \( \text{Re} \langle S \rangle \rightarrow \infty \). Next, §3 derives expressions for the dilaton superpotential for the models of interest. We explain our procedure by first briefly rederiving standard results for simple supersymmetric gauge theories, and then extending these to the models we wish to explore. Some illustrative cases are examined in §4, including examples for which field-dependent gauge couplings are responsible for (or ruin) the spontaneous breaking of supersymmetry. Finally, §5 summarizes our conclusions.

2. A Strategy for Eliminating the Runaway Dilaton

Our main purpose in this paper is the construction of supersymmetric gauge models for which the superpotential for the dilaton does not admit any runaway extrema for \( |S| \rightarrow \infty \). Our strategy for doing so is to consider models for which the gauge group is a direct product of factors, and for which the matter multiplets couple to two of these group factors.

2.1) A Class of Product-Group Models

We take the gauge group to be a direct product, \( G = G_0 \times G_1 \times G_2 \), in which case we generalize eq. (1) to

\[
\mathcal{L}_{\text{kin}} = \frac{1}{4} \sum_{r} \left( S_r \Tr W_r W_r \right).
\]  

The index ‘\( r = 0, 1, 2 \)’ labels the gauge-group factor, and the quantity \( S_r \) controls the strength of the dilaton coupling to the corresponding gauge multiplet. For example, in string perturbation theory

\[
S_r = k_r S + \Delta_r,
\]

where \( k_r \) is the Kac-Moody level of the corresponding conformal field theory, and \( \Delta_r \) is a function of the various string moduli which arises at one string loop.

We imagine the matter multiplets of the theory to carry quantum numbers either for the first gauge-group factor, \( G_0 \), or the second two factors, \( G_1 \times G_2 \), but not both. In this
case the total superpotential takes the form

\[ W_{\text{tot}} = W_0 + W_{12}, \]  

where the first term in this expression is the contribution of the \( G_0 \) gauge sector, and \( W_{12} \) is the contribution of the gauge and matter fields which take part in the other two gauge interactions. The construction of \( W_{12} \) is the main topic of the following sections.

If no matter fields were present for any of the gauge-group factors, then this type of model would reduce to ordinary gaugino condensation with a direct-product gauge group, with the result that \( W_{\text{tot}} \) takes the form of a sum of exponentials \[^{[10]}^{[13]}\]:

\[ W_{\text{tot}} = \sum_r C_r e^{-A_r S}, \]

where the \( A_r \) are all positive constants. The scalar potential resulting from eq. (5) generally has a minimum for \( \text{Re } S \to \infty \)[^{[14]}], although local minima at finite values of \( \text{Re } S \) may also be arranged.\[^{2}\] This is an example of the runaway behaviour which has been thought to be generic in low-energy string theory.

In what follows we assume the contribution \( W_0 \) to have this usual runaway-dilaton form:

\[ W_0 = \mu_0^3 e^{-A_0 S_0}, \]

where \( \mu_0 \) is an \( S \)-independent mass scale and \( A_0 \) is a positive constant. One way to satisfy this assumption is to choose this sector to be a pure gauge theory, e.g. \( G_0 = SU(N_{c0}) \), since in this case one finds \( A_0 = 8\pi^2/N_{c0} > 0 \), as required.

Our strategy to remove the runaway problem for \( W_{\text{tot}} \) is to construct models for which \( W_{12} \) increases as \( \text{Re } S \) increases. (Such superpotentials were considered within the context of \( S \) duality in [15], and for non-asymptotically-free models in [16].) One way to do so is

\[^{2}\] It has been argued that the runaway solution might be removed using particular choices for the Kähler potential [11].
to ensure that the superpotential resulting from gaugino and matter condensation has the form

\[ W_{12} = C e^{+AS} \]  

(7)

where \( A \) is a positive constant. As we show in detail in §4, this can be done even when \( G_1 \) and \( G_2 \) are asymptotically-free gauge groups. Once this is inserted into eq. (4), then the dilaton is unambiguously stabilized, and there exists a unique vacuum solution. We also present models for which the matter content is chosen so that \( W_{12} \) by itself admits a finite v.e.v. for the dilaton.

It should also be kept in mind that such models are of practical phenomenological interest even if global supersymmetry should remain unbroken. This is because supersymmetry can break once the same superpotential is incorporated into supergravity, depending on the form taken by the Kähler potential, \( K \). Indeed, with a supersymmetry-preserving superpotential, supersymmetry generally is broken when string-inspired Kähler potentials are involved.

2.2) Understanding the Limit of Weak Coupling

At first sight, a superpotential which blows up as \( \langle S \rangle \to \infty \) seems impossible to obtain, because in this limit the gauge coupling vanishes, and all non-perturbative effects should disappear smoothly leaving the result for a free massless theory (the superpotential of which vanishes). In this section we show why this conclusion need not hold in general.

Here is the main point, which was also sketched briefly in [16]. The general arguments for the existence of a runaway dilaton assume the superpotential to be continuous in the limit of zero coupling. Because the noninteracting theory satisfies \( W_{(g=0)} = 0 \), it is concluded that \( \lim_{g \to 0} W_g \) must also vanish. As the following example demonstrates, this continuity assumption need not hold if the vanishing of the coupling constant qualitatively changes the kinds of vacua to which the system has access.
Consider, then, the following superpotential for a complex superfield, $\phi$:

$$W_g(\phi) = \frac{1}{2} m \phi^2 + \frac{1}{3} g \phi^3,$$

where the constants, $m$ and $g$, are assumed to be real and positive. We are interested in investigating the continuity of the superpotential in the limit $g \to 0$.

The scalar potential generated by $W_g$, $V = |W'_g|^2$, has absolute minima at the two extrema of $W_g$: $\phi_0 = 0$ and $\phi_g = -m/g$. The superpotential evaluated at these extrema is: $W_g(\phi_0) = 0$ and $W_g(\phi_g) = m^3/(6g^2)$. Clearly only the minimum at $\phi_0$ agrees in the limit $g \to 0$ with the result, $W_{(g=0)}(\phi_0) = 0$, which obtained by setting $g = 0$ in the superpotential first, followed by extremization with respect to $\phi$. Importantly, the result evaluated at the vacuum $\phi_g$ is discontinuous in the weak-coupling limit, $g \to 0$, because these minima for $V$ cannot arise unless $g$ is nonzero.

A similar situation occurs for the superpotential of the supersymmetric gauge theories considered in the next section, since the qualitative features of the vacua change discontinuously in the zero-coupling ($\text{Re } S \to \infty$) limit. As a result the superpotential has a smooth weak-coupling limit before its arguments are eliminated using their equations of motion, but becomes singular after this elimination is performed to obtain a superpotential for $S$ alone. The resulting discontinuity of the superpotential at zero coupling is what invalidates intuition which is based on the behaviour of the noninteracting theory.

There is an alternative way to see why the models to be presented can generate superpotentials which diverge as $\text{Re } S \to \infty$, based on the idea that the superpotential can only depend on the dilaton through the renormalization-group-invariant scale, $\Lambda(S) \propto e^{-BS}$ (for some positive constant $B$). For a simple gauge group this implies, on dimensional grounds, $W \propto \Lambda^3$, giving a runaway extremum for $\text{Re } S \to \infty$. However, for a product group, with doubly-charged matter, a more complicated dependence is possible since $W$ may now depend on a different scale, $\Lambda_i(S)$, for each of the group factors, $G_i$. We then expect that $W \propto \Lambda_1^3 \Phi(\Lambda_1/\Lambda_2)$. Depending on the form of the function $\Phi$, $W$ may be a positive or negative exponential function of $S$. 

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3. Constructing the Effective Superpotentials

We now collect results for the superpotential for several kinds of gauge theories, following the approach of ref. [17]. We start by rederiving the superpotential for gauge theories having simple gauge groups, and then repeat the construction for the product groups which are of interest here.

3.1) Some Results for Simple Gauge Groups

Let us first consider an $N = 1$ supersymmetric model with gauge group $G$. We represent the matter multiplets with chiral superfields, $Q^i_\alpha \in R$ (and $\tilde{Q}^\alpha_i \in \overline{R}$, if required), where ‘$i$’ is the flavour index, and ‘$\alpha$’ is the gauge index. The microscopic action for the model is given by the dilaton-dependent gauge kinetic terms, eq. (1), plus standard kinetic terms for the matter supermultiplets. The microscopic superpotential relating the matter supermultiplets is taken to vanish identically, $w(Q, \tilde{Q}) = 0$.

We are interested in computing the superpotential for the quantum ‘effective action’ which generates the irreducible correlation functions of the theory (as opposed, say, to the theory’s Wilson action). We may choose as the argument of this superpotential any operator whose correlations we wish to explore. For the present section we choose these to be the following gauge-invariant combinations:

$$ M^i_j \equiv \langle Q^i_\alpha \tilde{Q}^\alpha_j \rangle, \quad \text{and} \quad U \equiv \langle \text{Tr} \, WW \rangle. \quad (9) $$

Of particular interest, however, are those fields which can describe the very light scalar degrees of freedom of the model, since these describe the system’s vacuum moduli and symmetries. In the absence of a microscopic superpotential for the matter fields $Q$ and $\tilde{Q}$, these light degrees of freedom are described classically (and hence also to all orders of perturbation theory) by the $D$-flat directions, which parameterize the zeroes of the classical scalar potential. It is well known that these $D$-flat directions can be parametrized in terms of a suitably chosen set of gauge-invariant holomorphic polynomials [18] [19].
The analyses that we present in §4 have the feature that the gauge invariant polynomials appearing in the effective action suffice to describe the $D$-flat directions. Although the gaugino condensate field, $U$, does not similarly describe a $D$-flat direction, it is nonetheless convenient to keep it as an argument of the effective action.

The dependence of the effective superpotential on its arguments, $W(U, M^i_j)$, is completely determined by the twin conditions of linearity and symmetry under the model’s global flavour symmetries, as follows:

- **Linearity:** As was demonstrated in [17], the fact that $S$ only couples to the microscopic theory via eq. (1) implies, as an exact result, that the effective superpotential necessarily has the form

$$ W = \frac{1}{4}US + f(U, M^i_j). \quad (10) $$

That is, $S$ can only appear linearly, and moreover only in the term $\frac{1}{4}US$.

- **Global Symmetries:** The function $f(U, M^i_j)$ is determined by the various global chiral symmetries of the underlying supersymmetric gauge theory. In the absence of a superpotential for the matter fields, $Q^\alpha_i$ and $\tilde{Q}^\alpha_i$, the underlying gauge theory admits the classical global symmetry $SU(N_f)_L \times SU(N_f)_R \times U(1)_A \times U(1)_B \times U(1)_R$, of which the factors $U(1)_A \times U(1)_R$ are anomalous. Invariance of the effective superpotential under the anomaly-free symmetries implies the fields $M^i_j$ can appear only through the invariant combination $\text{det} M$. The two anomalous symmetries, $U(1)_A$ and $U(1)_R$, then fix the form of the unknown function $f(U, \text{det} M)$.

The transformation properties of the microscopic fields under the two anomalous $U(1)$ symmetries may be conveniently written in the combined form:

$$ Q(\theta) \to e^{i\beta R} Q(e^{i\beta} \theta) \quad \tilde{Q}(\theta) \to e^{i\beta R} \tilde{Q}(e^{i\beta} \theta) \quad W(\theta) \to e^{i\beta} W(e^{i\beta} \theta), \quad (11) $$

in which $\beta$ is the transformation parameter and the real quantity $R$ is arbitrary. The anomaly may be expressed as the statement that eq. (11) becomes a *bona-fide* quantum
symmetry if the dilaton simultaneously is shifted by:

\[ S \rightarrow S - \frac{i\beta}{4\pi^2} \left[ \sum_i T(R_i)(R - 1) + T(A) \right]. \tag{12} \]

Here \( \sum_i \) is a sum over matter representations, and \( T(R) \) is defined in terms of the trace of the gauge generators in the matter representation, via \( \text{Tr}[T_R^aT_R^b] \equiv T(R) \delta^{ab} \). \( T(A) \) is the same quantity for the special case of the adjoint representation \((R = A)\). For the gauge group \( SU(N_c) \), the generators are normalized so that \( T(R) = 1/2 \) in the fundamental representation, and so then \( T(A) = N_c \) for the adjoint representation. For supersymmetric QCD with \( N_f \) nonchiral flavours, we therefore have \( \sum_i T(R_i) = N_f \).

The effective superpotential is now nailed down exactly by the requirement that it be of the form given in eq. (10), and that it have an \( R \)-weight of 2 under the transformations which follow from eq. (11) for \( M_{ij} \) and \( U \):

\[ M_j^i(\theta) \rightarrow e^{2i\beta R_j} M_j^i(e^{i\beta} \theta) \quad U(\theta) \rightarrow e^{2i\beta} U(e^{i\beta} \theta), \tag{13} \]

together with eq. (12).

From these considerations, it is clear that \( W \) must have the general structure

\[ W = \frac{1}{4} US + \frac{U}{32\pi^2} \left[ a \log \left( \frac{U}{\mu^5} \right) + b \log \left( \frac{\text{det} M}{\mu^{2d}} \right) + C_0 \right], \tag{14} \]

where \( a \) and \( b \) are constants to be determined from the requirement of \( R \)-symmetry. Symmetry arguments cannot determine the constants \( \mu \) and \( C_0 \). Indeed \( C_0 \) may be chosen to vanish through an appropriate choice for \( \mu \).\(^3\) Finally, \( d \) is defined so that \( 2d \) is the number of factors of matter fields in the invariant; for SQCD, with the invariant being \( \text{det} M \), we have \( 2d = 2N_f \).

Eq. (13) implies the transformation rule \( \text{det} M \rightarrow e^{2id/R} \text{det} M \), and so eq. (14) then has the correct transformation property under eqs. (12) and (13) (for all \( R \)) only if

\[ a = T(A) - \sum_i T(R_i), \quad \text{and} \quad b = \frac{1}{d} \sum_i T(R_i). \tag{15} \]

\(^3\) Alternatively, \( \mu \) can be chosen in a more physical way, such as the scale at which the gauge coupling is given by \( 1/g^2(\mu) = \text{Re} \left\langle S \right\rangle \).
Thus, for $G = SU(N_c)$ with $N_f$ flavours, we find $a = N_c - N_f$ and $b = 1$. Putting these values in Eq. (14), we obtain precisely the expressions obtained in refs. [20], [21].

Since $W$ is the superpotential for the effective action — as opposed to the Wilson action — the correct procedure for ‘integrating out’ fields is to remove them by solving their extremal equations for $W$, rather than by performing their path integral [17]. Furthermore, for supersymmetric theories this should be done using the effective superpotential, $W$, rather than the effective scalar potential $V$ [22]. Performing this operation for the gaugino condensate $U$, using eq. (14), one obtains

$$W = c \left( \frac{\mu^{3a+2bd}}{(\det M)^{b}} e^{-8\pi^2 S} \right)^{1/a} = c' \left( \frac{\Lambda^{3a+2bd}}{(\det M)^{b}} \right)^{1/a}$$

(16)

where $c = -\frac{a}{32\pi^2} \exp \left( \frac{C_0 + a}{a} \right)$, $c' = c \exp \left( -\frac{C_0}{a} \right)$ and the second equality defines the RG-invariant scale, $\Lambda$. For the special case of SQCD, eq. (16) reduces to the familiar expression $W = c' \left( \frac{\Lambda^{3a-3N_c-2N_f}}{(\det M)^{b}} \right)^{1/(N_c-N_f)}$

For completeness, we now briefly summarize the expressions analogous to eqs. (14), (15), and (16) for the other nonexceptional simple Lie groups.

- $Sp(2N_c)$: For symplectic groups with $2N_c$ colours, if the generators for the fundamental representation are normalized so that $T(R) = 1/2$, then $T(A) = N_c + 1$. In a theory with $2N_f$ fundamentals $Q^i_\alpha$, $i = 1, \ldots, 2N_f$, one has $a = N_c + 1 - N_f$. There is a classical global symmetry $U(2N_f) \times U(1)_R$. As with the SQCD case, the $U(1)_R$ as well as the $U(1)$ factor of the chiral symmetry $U(2N_f)$ are both anomalous. These symmetries can be rendered non-anomalous by suitable shifts of the field $S$, as in eq. (12). A set of gauge invariants is provided by $M^j_i = Q^i_\alpha Q^j_\beta J^{\alpha\beta}$, where $J^{\alpha\beta}$ is the $2N_c \times 2N_c$ antisymmetric invariant tensor of the symplectic groups. The only possible chiral invariant is Pf $M$. Thus one substitutes the Pfaffian for $\det M$ in eq. (16). As to $d$, defined as being half the number of factors of matter fields in the chiral invariant, it is given by $d = N_f$. Thus, from eq. (15), we have $b = 1$. With the above values for $a$, $b$ and $d$, we obtain results that agree with those found in Refs. [8] and [9], except that those authors did not include the dilaton field in their effective superpotentials.
• $SO(N_c)$: For special orthogonal groups with $N_c$ colours, if the generators for the fundamental representation are normalized so that $T(R) = 1$, then $T(A) = N_c - 2$. In a theory with $N_f$ fundamentals $Q_i$, $i = 1, \ldots, N_f$, one has $a = N_c - 2 - N_f$. There is a classical global symmetry $U(N_f) \times U(1)_R$. Of course, the values of $d$ and $b$ depend on the choice of invariants used to parametrize the low-energy theory.

3.2) Some Results for Product Gauge Groups

We now turn to the construction of $W_{12}$ for the models having product gauge groups, which are the main focus of this paper. We do so for matter which carries the quantum numbers of both factors of the gauge group $G_1 \times G_2$. Such models were studied in refs. [3] [6] [8] [9], although not with field-dependent gauge couplings. Our construction again follows the method of ref. [17], which involves temporarily maintaining the fiction that the quantities $S_r$ are independent fields, with the connection to the single dilaton field, $S$, through eq. (3), deferred to the final expressions.

For concreteness, imagine that the matter multiplets transform in complex representations of the gauge group factors, and that there is one copy of the representation $Q_{\alpha \dot{\alpha}} \in (R_1, R_2)$ and $\tilde{Q}^{\dot{\alpha} \alpha} \in (\overline{R}_1, \overline{R}_2)$. We use here $\alpha$ as the gauge index of the first group, and $\dot{\alpha}$ as that of the second group. We again assume there to be no superpotential for $Q$ or $\tilde{Q}$ in the microscopic lagrangian.

The first step is to choose the fields whose correlations are of interest, and so which appear as arguments of the effective superpotential. In this section we focus on the following gauge-invariant polynomials:

$$M \equiv \langle Q_{\alpha \dot{\alpha}} \tilde{Q}^{\dot{\alpha} \alpha} \rangle, \quad U_r \equiv \langle \text{Tr} W_r W_r \rangle, \quad r = 1, 2. \quad (17)$$

In the examples of later interest, the polynomial $M$ suffices to parameterize the $D$-flat directions.

In order to determine the superpotential generated as a function of $M$ and $U_r$, we again ask that a shift of the $S_r$ cancel the anomalies induced by the abelian $R$-transformations of
the model. The major difference between the present case and the analysis of the previous section is that the $R$ symmetries have separate anomalies with each of the gauge group factors, $G_r$. (All mixed anomalies – involving both gauge groups simultaneously – vanish so long as the gauge group generators are traceless.) The anomalies of the R symmetries then may be separately cancelled by corresponding shifts of the $S_r$, so long as these fields are regarded as independent of one another. Given the transformations of eq. (11), the required shifts are

$$S_r \rightarrow S_r - \frac{i\beta}{4\pi^2} \left( n_r T(R_r)(R - 1) + T(A_r) \right)$$

(18)

where now $n_r$ represents the number of “flavours” seen by the group $G_r$, as might be determined by setting the other gauge coupling to zero, for example.

An argument identical to that used previously for simple gauge groups therefore leads to a superpotential of the following form

$$W_{12} = \frac{1}{4} U_1 S_1 + \frac{U_1}{32\pi^2} \left[ a_1 \log \left( \frac{U_1}{\mu_1^3} \right) + b_1 \log \left( \frac{M}{\mu_1^2} \right) \right]$$

$$+ \frac{1}{4} U_2 S_2 + \frac{U_2}{32\pi^2} \left[ a_2 \log \left( \frac{U_2}{\mu_2^3} \right) + b_2 \log \left( \frac{M}{\mu_2^2} \right) \right].$$

(19)

The constants $a_r$ and $b_r$ are again determined by requiring this expression to have R-weight 2 under the combined transformations of eqs. (13) and (18). In writing eq. (19) the freedom to redefine the $\mu_r$’s has been used to absorb constants which could have appeared additively in each of the square brackets.

Extremizing eq. (19) with respect to $U_r$ gives the following expressions for the gaugino condensates:

$$U_r = \frac{\mu_r^3}{e} e^{-8\pi^2 S_r/a_r} \left( \frac{M}{\mu_r^2} \right)^{-b_r/a_r},$$

(20)

where $e$ is the base of the natural logarithms. This, when plugged back into eq. (19), gives the expression for $W_{12}(S_r, M)$:

$$W_{12} = -\frac{a_1 \mu_1^3}{32\pi^2 e} \exp \left( -\frac{8\pi^2 S_1}{a_1} \right) \left( \frac{M}{\mu_1^2} \right)^{-b_1/a_1}.$$
\[-\frac{a_2\mu_2^3}{32\pi^2\epsilon} \exp\left(-\frac{8\pi^2 S_2}{a_2}\left(\frac{M}{\mu_2^2}\right)^{-b_2/a_2}\right).\] (21)

The connection with string phenomenology is then made by using eq. (3) to express the $S_r$ in terms of $S$ and any other moduli.

We remark in passing that although it might seem problematic at first glance to treat the $S_r$ as independent when cancelling anomalies, and then to subsequently use eq. (3), the result is nonetheless justified. One way to see that this is so is to imagine the constants $k_r$ in eq. (3) to be chiral superfields which, like the other fields, can also transform under the R-symmetry. Once the algebraic form of the effective superpotential, as a function of all the fields (including the $k_r$), has been found, we then set the $k_r$ equal to their original (integer) values. This last step corresponds to setting the fields equal to their v.e.v.s. This procedure can be considered another application of the method exploited in Ref. [23].

4. Illustrative Examples

In this section, we present several examples to illustrate our findings concretely.

4.1) $a_1$ and $a_2$ both different than zero

Consider the gauge group $G_1 \times G_2 = SU(N_1) \times SU(N_2)$, with $N_1 > N_2 + 1$ and a single flavour, $N_f = 1$, of matter supermultiplets tranforming as $Q_{\alpha \dot{\alpha}} \in (N_1, N_2)$ and $\tilde{Q}^{\alpha \dot{\alpha}} \in (\overline{N}_1, \overline{N}_2)$. One finds

\[a_1 = N_1 - N_2, \quad a_2 = N_2 - N_1, \quad b_1 = N_2, \quad b_2 = N_1\] (22)

For this group the $D$-flat directions break the gauge group from $G = SU(N_1) \times SU(N_2)$ down to $H = SU(N_1 - N_2)$, where we assume that $N_1 \geq N_2 + 1$. In this case the number of complex $D$-flat directions, $D$, is the number of complex scalar fields, $S = 2N_1N_2$, minus the number of generators of $G/H$, $B = [N_1^2 + N_2^2 - 2] - [(N_1 - N_2)^2 - 1] = 2N_1N_2 - 1$. 14
That is, there is $D = 1$ $D$-flat direction which may be parameterized by the ‘meson’ field, $M$. The effective superpotential, from eq. (19), is

$$W_{12}(U_r, S, M) = \frac{1}{4} U_1 S_1 + \frac{U_1}{32\pi^2} \left[ (N_1 - N_2) \log \left( \frac{U_1}{\mu_1^2} \right) + N_2 \log \left( \frac{M}{\mu_1^2} \right) \right] + \frac{1}{4} U_2 S_2 + \frac{U_2}{32\pi^2} \left[ (N_2 - N_1) \log \left( \frac{U_2}{\mu_2^2} \right) + N_1 \log \left( \frac{M}{\mu_2^2} \right) \right],$$

(23)

implying the following expressions for the gaugino condensates:

$$U_1 = \frac{\mu_1^3}{e} e^{-8\pi^2 S_1/(N_1 - N_2)} \left( \frac{M}{\mu_1^2} \right)^{-N_2/(N_1 - N_2)}$$

$$U_2 = \frac{\mu_2^3}{e} e^{+8\pi^2 S_2/(N_1 - N_2)} \left( \frac{M}{\mu_2^2} \right)^{+N_1/(N_1 - N_2)}.$$  

(24)

The result for $W_{12}(S, M)$ becomes:

$$W_{12}(S, M) = -D \mu_1^3 e^{-8\pi^2 S_1/(N_1 - N_2)} \left( \frac{M}{\mu_1^2} \right)^{-N_2/(N_1 - N_2)} + D \mu_2^3 e^{+8\pi^2 S_2/(N_1 - N_2)} \left( \frac{M}{\mu_2^2} \right)^{+N_1/(N_1 - N_2)}$$

(25)

with $D = (N_1 - N_2)/(32\pi^2 e)$.

In the absence of the pure gauge factor, $G_0$, the conditions for a supersymmetric vacuum using this superpotential would be

$$\frac{\partial W_{12}}{\partial S} \propto k_1 \mu_1^3 e^{-8\pi^2 S_1/(N_1 - N_2)} \left( \frac{M}{\mu_1^2} \right)^{-N_2/(N_1 - N_2)}$$

$$+ k_2 \mu_2^3 e^{+8\pi^2 S_2/(N_1 - N_2)} \left( \frac{M}{\mu_2^2} \right)^{+N_1/(N_1 - N_2)} = 0,$$

(26)

and

$$\frac{\partial W_{12}}{\partial M} \propto N_2 \mu_1 e^{-8\pi^2 S_1/(N_1 - N_2)} \left( \frac{M}{\mu_1^2} \right)^{-N_1/(N_1 - N_2)}$$

$$+ N_1 \mu_2 e^{+8\pi^2 S_2/(N_1 - N_2)} \left( \frac{M}{\mu_2^2} \right)^{+N_2/(N_1 - N_2)} = 0.$$  

(27)
There are several noteworthy features of these two equations.

• 1: First, eqs. (26) and (27) are incompatible unless \( k_1/k_2 = N_2/N_1 \), signalling the spontaneous breaking of supersymmetry in this case. Notice that supersymmetry would not have been broken in this way if the gauge couplings were not fields, since then one of the mutually inconsistent conditions, \( \partial W_{12}/\partial S = 0 \), would not be imposed. In this way we come upon an example of our observation, mentioned in the introduction, that the inclusion of field-dependent gauge couplings can introduce dynamical supersymmetry breaking into a model which would otherwise not break supersymmetry.

• 2: Second, notice that it is clear at the outset that \( S \rightarrow \infty \) is not a solution to eqs. (26) and (27) for any value of \( M \), so the weak-coupling limit is not a supersymmetric solution. To see why this is so, it is instructive to solve eq. (27) to obtain \( M \) as a function of \( S \). One finds

\[
\left( M^{N_1+N_2} \begin{pmatrix} N_2 \mu_1 \\ \mu_1^2 \mu_2 \end{pmatrix} \right) = \left( N_2 \mu_1 \frac{N_1}{\mu_1 \mu_2} \right)^{N_1-N_2} e^{-8\pi^2(S_1+S_2)},
\]

(28)

so, as \( S \) tends to infinity, \( M \) tends to 0, and can do so quickly enough (depending on \( k_r \) and \( N_r \) – see below) to overwhelm the explicit \( S \)-dependence which appears in \( W_{12}(S,M) \) in this limit. The superpotential is not extremized by large \( S \) because of the interplay of the two limits, \( S \rightarrow \infty \) and \( M \rightarrow 0 \). This is as expected when the superpotential is not behaving continuously in the large-\( S \) limit.

• 3: When the relation \( k_1/k_2 = N_2/N_1 \) is satisfied, the solutions to eqs. (26) and (27) are a family of degenerate field configurations given by eq. (28).

The picture changes somewhat once the superpotential due to the pure-gauge sector, \( G_0 \), is included. In this case eq. (27) remains unchanged, but eq. (26) acquires a new term

\[
\frac{\partial W_{tot}}{\partial S} = \frac{\partial W_{12}}{\partial S} - A_0 \mu_0^3 \ e^{-A_0 S_0} = 0.
\]

(29)

Solving \( \partial W_{tot}/\partial M = \partial W_{12}/\partial M = 0 \) for \( M \) gives eq. (28), which, when substituted
back into eq. (25) gives

\[
W_{12} = N \exp \left[ \frac{8 \pi^2 (N_2 S_2 - N_1 S_1)}{N_1^2 - N_2^2} \right],
\]

for constant \(N\). Using \(S_r = k_r S + \Delta_r\) gives an exponent in eq. (30) the sign of whose coefficient of \(S\) is controlled by the combination

\[
\frac{k_1 N_1 - k_2 N_2}{N_2^2 - N_1^2},
\]

which is always negative if \(k_1 = k_2\), but can be positive if \(k_2/k_1 > N_1/N_2 > 1\) (or the same condition with 1 \(\leftrightarrow\) 2). This coefficient vanishes for \(k_1/k_2 = N_2/N_1\).

Thus we see that, after integrating out the \(M\) field and after adding the contribution from \(G_0\), we are left with a superpotential which stabilizes the dilaton at a finite value

\[
< S > = \frac{1}{p_0 + p_{12}} \log \left( \frac{p_0 P_{12}}{P_0 P_{12}} \right) \text{ with } P_0 = \mu^3, P_{12} = D_1 \left( 1 + \frac{b_1}{b_2} \right) \left( \frac{b_2 D_2}{b_1 D_1} \right)^{-b_1/(b_1+b_2)}, \quad p_0 = 24 \pi^2 k_0/B_0, \quad p_{12} = \left[ \frac{8 \pi^2}{a} \left( \frac{b_1 k_2 - b_2 k_1}{b_1+b_2} \right) \right].
\]

We have gaugino and chiral condensates with supersymmetry unbroken and chiral symmetry broken.

In this example, we have shown that for product gauge groups \(SU(N_1) \times SU(N_2)\) with a single flavour of doubly charged matter and with field-dependent gauge couplings, the Kac-Moody levels play a role in whether supersymmetry is dynamically broken or left unbroken. Moreover, we have seen that the Kac-Moody levels also determine the sign of the coefficient of the dilaton in the exponential in \(W_{12}\), possibly leading to a potential which increases as dilaton field strength increases even if we consider only asymptotically free gauge groups.

4.2) Fixing the Dilaton with Quantum-Deformed Moduli Spaces: \(a_1 = 0\)

We now discuss another mechanism for fixing the dilaton. Consider some model with a product gauge group, for which \(a_1 = 0\) while \(a_2 \neq 0\), and in which the effective superpotential can be parametrized using only a single holomorphic gauge-invariant polynomial
of the matter fields, $N$. Then we have, from eq. (19),

$$W_{12}(U_r, S, M) = \frac{1}{4} U_1 S_1 + \frac{U_1}{32\pi^2} b_1 \log \left( \frac{N}{\mu_{1}^{2d}} \right)$$

$$+ \frac{1}{4} U_2 S_2 + \frac{U_2}{32\pi^2} \left[ a_2 \log \left( \frac{U_2}{\mu_{2}^{3}} \right) + b_2 \log \left( \frac{N}{\mu_{2}^{2d}} \right) \right], \quad (32)$$

where $2d$ equals the number of factors of matter fields in $N$. The equation of motion for $U_1$ yields the relation

$$N = \mu_1^{2d} \exp \left( -\frac{8\pi^2 S_1}{b_1} \right) \quad (33)$$

Substituting this back into $W_{12}$ gives

$$W_{12} \propto \exp \left( -\frac{8\pi^2 S_1}{a_2} \left[ S_2 - (b_2/b_1)S_1 \right] \right). \quad (34)$$

Clearly, if models can be found which satisfy the relations $b_2 k_1 > b_1 k_2$, $a_1 = 0$ and $a_2 \neq 0$, then the coefficient of $S$ in $W_{12}$ is positive, as in eq. (7). Combining $W_{12}$ with the contribution to the superpotential due to the group factor $G_0$ (as in eq. (4)), one obtains a total superpotential which stabilizes the dilaton at a finite value. Note that the ability of this mechanism to stabilize the dilaton depends on the values of the $k_r$.

The situation in which the constant $a_1$ vanishes can be arranged by choosing the matter appropriately. Since in such models there is no term $U_1 \log U_1$ in the effective superpotential, the equation of motion for the $U_1$ field results in a quantum deformed moduli space, expressed through a relation such as eq. (33) above, namely, $N = \mu_1^{2d} \exp \left( -\frac{8\pi^2 S_1}{b_1} \right)$. Such a case is said to be quantum deformed because (with $S \neq 0$) the point $N = 0$, which is available in the classical model, is disallowed in the quantum version. This quantum constraint is usually expressed in the effective superpotential using the artifice of a Lagrange multiplier field; however, our eq. (32) also realizes this constraint, using instead the physically meaningful gaugino condensate field.

4.3) 3-2 Model

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In §4.1, we examined models that can break a supersymmetry which was unbroken for constant gauge couplings. Let us note that the opposite is also true, that is, in models of dynamical global supersymmetry breaking with constant gauge couplings, supersymmetry can be restored by the incorporation of the dilaton (i.e. by the field dependence of the gauge couplings). To illustrate this sort of mechanism, consider the canonical example of dynamical global supersymmetry breaking, the so-called 3-2 model of Affleck et al [3] [5], in which the gauge group is $SU(3) \times SU(2)$. The fundamental matter spectrum is such that the $SU(2)$ factor is quantum constrained (in the sense discussed in §4.2). The quantum constraint is of the form $YZ = \Lambda_2^4$. It is shown in [5] that, if we suppose the condensation scale for the $SU(2)$ factor to be much greater than that for the $SU(3)$ factor, and if we suppose a certain superpotential in the microscopic theory, then the effective superpotential can be written as

$$ W = XY + \lambda(YZ - \Lambda_2^4) .$$

(35)

One can easily see that the equation of motion for $X$ implies $Y = 0$ and that the equation of motion for the Lagrange multiplier $\lambda$ implies $YZ = \Lambda_2^4$. For the case of constant gauge couplings ($\Lambda_i =$constant), the relations cannot be simultaneously satisfied and supersymmetry is said to be dynamically broken. However, for the case of field dependent gauge couplings ($\Lambda_i = \mu_i e^{-c_i S_i}$), the relations are satisfied by the runaway vacuum $S \to \infty$, for which $\Lambda_i = 0$. Therefore we learn that in this model, supersymmetry is restored by a runaway dilaton if the gauge couplings are conceived to be field-dependent.

5. Summary

We have considered $N = 1$ supersymmetric models with product gauge groups $G = G_0 \times G_1 \times G_2$, having matter charged under both of the final two groups, and with field dependent gauge couplings. In these models, we have found a number of scenarios which remove the dilaton runaway to weak coupling, without recourse to the non-asymptotically free models considered in ref. [16].
We have also shown how the status of supersymmetry breaking changes from the more familiar constant gauge couplings scenarios once the gauge coupling is made field dependent. We find examples for which field-dependent couplings introduce an otherwise unavailable supersymmetry-preserving minimum, and we also find models for which the introduction of field dependence in the gauge couplings breaks an otherwise unbroken supersymmetry.

Another of our observations is that in the class of models that we have considered, the important issues of the stabilization of the dilaton and the breaking of supersymmetry are affected critically by the values of the Kac-Moody levels $k_i$, a point which may have significance in guiding the search for phenomenologically viable string vacua.

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6. References


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