IN SITU ORIGIN OF LARGE SCALE GALACTIC MAGNETIC FIELDS WITHOUT KINETIC HELICITY?

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ABSTRACT

The origin and sustenance of large scale galactic magnetic fields has been a long standing and controversial astrophysical problem. Here an alternative to the “standard” \( \alpha - \Omega \) mean field dynamo and primordial theories is pursued. The steady supply of supernovae induced turbulence exponentiates the total field energy, providing a significant seed mean field that can be linearly stretched by shear. The observed micro-Gauss fields would be produced primarily within one vertical diffusion time since it is only during this time that linear stretching can compete with diffusion. This approach does not invoke exponential mean field dynamo growth from the helicity \( \alpha \)-effect but does employ turbulent diffusion, which limits the number of large scale reversals. The approach could be of interest if the non-linear Galactic dynamo helicity effect is suppressed independently of the turbulent diffusion. This is an important, but presently unresolved issue.

Subject Headings: Galaxy: general; galaxies: magnetic fields; ISM: magnetic fields
1. Introduction

Magnetic fields are important to dynamics or emission in almost all astrophysical systems. The formation of observed micro-Gauss large scale magnetic fields in the interstellar medium (ISM) of spiral galaxies has been considered a fundamental unsolved astrophysical problem (Beck et al. 1996; Zweibel & Heiles 1997). There is an important distinction between the large and small scale magnetic fields observed in the ISM. Observations indicate that the small scale field is typically ordered on sub-kiloparsec scales and superimposed on that is a large scale, primarily azimuthal field. While it is more or less agreed that the small scale component of this field is injected and sustained by supernovae turbulence (e.g. Beck et al. 1996), the generation of the large scale field is where the controversy lies.

Whether or not the large scale fields are of primordial origin or are produced in situ is difficult to determine observationally, and the “standard” competing explanations both suffer from difficulties. For example even if primordial fields could be produced, in situ processing of the field during the galactic lifetime needs to be addressed, and would likely dominate the present character of the observed field. The standard in situ mean field “$\alpha - \Omega$ dynamo” (Parker 1979; Moffatt 1978; Ruzmaikin et al. 1988) has been threatened by some non-linear simulations (e.g. Tao et al. 1993; Cattaneo & Hughes 1996). While stretching of field lines by differential rotation (which provides the “$\Omega$-effect”) is not controversial, the helical property of the turbulence (which provides the “$\alpha$-effect”) and the turbulent diffusion (the ”$\beta$ effect”) may be suppressed. Here I suppose that the $\alpha$-effect is suppressed but that the $\beta$ effect is not. Whether these two effects can be disentangled is unclear (Field
& Blackman 1997) but the possibility can still be explored.

Turbulent amplification of the small scale field can provide a steadily supplied seed mean field (MF) whose subsequent stretching by differential rotation may produce and sustain large scale Galactic magnetic fields without a dynamo $\alpha$-effect or a primordial seed. I first give the basic magnetic MF equations and summarize the complications of standard MF dynamo theory. The MF equation is then solved without the $\alpha$-effect, but keeping in turbulent diffusion. It is shown that in 1 vertical diffusion time, differential shearing can increase the MF by an order of magnitude. It is also shown that the MF is maximized on a radial scale of a kpc or so: Turbulent diffusion tends to favor large scale field stretching, but this competes with the seed field’s inverse dependence on scale. Simple statistical arguments are then used to predict the most likely number of reversals. That the small scale field is not necessarily dominant on only one scale is also emphasized. The assumptions employed here are no more controversial than in standard MF theory. Generally speaking, MF theory as applied to the Galaxy is most certainly an oversimplification, but it does provide a useful framework from which some understanding can be gained.

2. Aspects of Standard Theory

Writing the magnetic field (and velocity) as a sum of mean and fluctuating components, ie. $\mathbf{B} = \bar{\mathbf{B}} + \mathbf{B}'$, the MF equation can be obtained by spatially averaging the magnetic induction equation over scales large compared to the fluctuations but small compared to the overall scale of the system (e.g. galaxy):

$$\partial_t \bar{\mathbf{B}} = \nabla \times (\mathbf{v} \times \bar{\mathbf{B}}) + \nabla \times \langle \mathbf{v}' \times \mathbf{B}' \rangle + \nu_M \nabla^2 \bar{\mathbf{B}}, \quad (1)$$
where $\mathbf{v}$ is the velocity and $\nu_M$ is the magnetic diffusivity, and the primed and barred (or bracketed) quantities indicate fluctuating and mean values respectively. The turbulent EMF in (1) can be written

$$\langle \mathbf{v}' \times \mathbf{B}' \rangle = \alpha_{ij}(\mathbf{B}, \mathbf{v}) \bar{B}_j - \beta_{ijk}(\mathbf{B}, \mathbf{v}) \nabla_j \bar{B}_k + \ldots$$

where “...” indicates higher order gradients. In linear kinematic dynamo theory for isotropic incompressible plasmas, $\alpha \propto \langle \mathbf{v}' \cdot \int \nabla \times \mathbf{v}'(t') dt' \rangle$ and $\beta \propto \langle \mathbf{v}' \cdot \int \mathbf{v}'(t') dt' \rangle$. In the dynamic, non-linear theory, these can be functions of $\mathbf{B}$, and are not necessarily isotropic.

In the standard $\alpha - \Omega$ dynamo ($\alpha \Omega D$) applied to the Galaxy (e.g. Ruzmaikin et al. 1988), supernovae (SN) induced turbulent eddies on $\sim 100$ pc scales stretch field lines into loops or cells. The coriolis force, in principle, conspires to statistically twist all of these loops in the same direction, providing a much larger scale mean loop. This, resulting from the large scale reflection asymmetry, provides the non-vanishing pseudoscalar $\alpha$-effect (Parker 1979; Moffatt 1978). The outer portions of these loops must incur turbulent diffusion to leave a net mean flux in each hemisphere of the Galactic disk (Parker 1979). The large scale field formed in this way is further sheared by differential rotation (the $\Omega$-effect), providing new toroidal field and starting the process again. In principle, this feedback leads to exponential MF growth of a primarily azimuthal field with a growth time $\sim 4 \times 10^8$ yr (Ruzmaikin et al. 1988).

Standard kinematic $\alpha \Omega D$ treatments ignore the backreaction of the growing magnetic field on the turbulence. Because high magnetic Reynolds numbers make the last term in (1) negligible on the energy containing scales, the field exponentially grows to equipartition with the turbulent energy by a fast dynamo (FD) on a time scale much shorter than any
MF evolution time (e.g. Parker 1979) and does not require helicity. In combination with even a weak MF that is \( \gtrsim \rho^{1/2} v'/R_m^{1/2} \) (where \( R_m \) is the magnetic Reynolds number, and \( \rho \) is the density), this may make Lorentz forces (Kulsrud & Anderson 1992; Cattaneo 1994) lock a significant fraction of motions into oscillations. The magnetic fields act like springs that the required turbulent motions must fight against. Although simulations in 2-D show \( \beta \) suppression (Cattaneo 1994), there are not yet simulations that show \( \beta \) suppression in 3-D. There have been some simulations showing \( \alpha \)-effect suppression in 3-D (Tao et al., 1993; Cattaneo & Hughes 1996), and others that do not (Brandenburg & Donner 1997). Intermittency (Blackman 1996; Subramanian 1997) and the nature of the forcing function may play a role in overcoming both \( \alpha \) and \( \beta \) suppression in 3-D. Basically, there is no clear consensus on what happens in fully non-linear mean field dynamo theory with respect to the backreaction, even as to whether suppression of \( \alpha \) and \( \beta \) are intertwined (e.g. Field & Blackman 1997) or independent.

3. Linear Mean Field Growth from a Random Seed Field

SN inject turbulent energy into the ISM, and also inject magnetic field (Ruzmaikin et al. 1988; Rees 1994). Because of the observed dispersal of heavy elements (Rana 1991), the supernova ejecta at least mix with the remnant material. Theoretical estimates for the mean seed field injected from SN range from \( 10^{-13} \) Gauss from simple flux freezing, to \( 10^{-8} \) Gauss for including winding in pulsar winds (Rees 1994).

Both theory and simulation (Parker 1979; Piddington 1992; Beck et al. 1996) show that the FD builds up small scale magnetic field energy on a growth time of order the energy
containing eddy turnover time $\sim t/v' \sim 10^7$ yr, where $l \sim 100$pc is usually taken as the energy containing eddy scale and $v' \gtrsim 10$km/sec is a typical observed speed of these eddies. Thus, once the Galactic volume is full of a weak seed magnetic field from the first set of SN, the next generation of eddies stirs the field to equipartition. The SN remnants fill the Galactic disk (height $\sim 500$pc by radius 12kpc) every $10^7$ years given their observed rate of $\sim 0.02$yr$^{-1}$ (Ruzmaikin et al. 1988), and this maintains a steady random field energy. How the field actually mixes from the SN to the ambient ISM is complicated. The amount of magnetic annihilation, the amount of enhancement, the geometry/topology of the injected field (e.g. Ruzmaikin et al. 1988), and the role of boundary instabilities are all subtle issues. Despite these complications, the basic picture of SN seed field injection and subsequent stirring as described above, leading to equipartition fields at a time $\lesssim 2.5 \times 10^8$yr (where this upper limit comes from using the $10^{-13}$G seed value given above) and all subsequent times, is consistent with observations: The magnitude of the random field is observed to be $B \sim 5 - 10 \times 10^{-6}$ G (e.g. Heiles 1994; Rand & Kulkarni 1989). Rand & Kulkarni (1989) impose a single cell model whose best fit small scale over which the field is ordered is 50-100pc. Though this has become the standard quoted range for the small scale, it will be emphasized later why multiple and larger cell sizes (Ferrière 1994) are important.

The above small scale field would give a corresponding MF of magnitude $\bar{B}_0 \sim B/N^{1/2}$ where $N$ is the number of small scale coherence volumes in the region of averaging. Such a residual large scale field has been argued to be a viable source of seed field for the Galactic $\alpha\Omega D$ (Ruzmaikin et al. 1988; Rees 1994), but below I suggest that even if $\alpha$ is suppressed, an appropriate large scale field can still be produced. Assume $\alpha$ is suppressed
well below the critical value required (Parker 1979) for the standard $\alpha - \Omega$ dynamo growth, 

ie., $\alpha << \alpha_{\text{crit}} \sim \beta^2/\Omega h^3$ (where $h$ is the disk height), so we can then ignore it in what follows. The MF induction equations in cylindrical coordinates become

$$\frac{\partial}{\partial t} \vec{B}_r = \beta \nabla^2 \vec{B}_r$$  \hspace{1cm} (2)$$

$$\frac{\partial}{\partial t} \vec{B}_{\phi} = r \frac{\partial}{\partial r} \Omega \vec{B}_r + \beta \nabla^2 \vec{B}_{\phi},$$  \hspace{1cm} (3)$$

$$\frac{\partial}{\partial t} \vec{B}_z = \beta \nabla^2 \vec{B}_z,$$  \hspace{1cm} (4)$$

Eqs. (2) and (4) are decoupled from (3), implying pure diffusion of $\vec{B}_r$ and $\vec{B}_z$. As in standard treatments (e.g. Ruzmaikin et al. 1988), the $\Omega$-effect (differential rotation) increases the azimuthal field in (3) linearly on a time of order the rotation time. For the Galaxy this is $\Omega^{-1} \sim 3.3 \times 10^7$ yr. Because the rotational energy far exceeds that which can be transferred into the magnetic field during the age of the universe, the $\Omega$-effect is not controversial; there is no back-reaction on the large scale rotational motion. As shown below, this $\Omega$-effect can generate a factor of $\sim 10$ increase in the large scale field. This is sufficient without an $\alpha$-effect since the seed field is continually supplied.

To solve the equations and determine the dominant scale of the MF, I assume that $\vec{B} = \vec{B}_t e^{(i\mathbf{k}\cdot \mathbf{x})}$, where $k$ is the wave vector of the MF and the subscript $t$ labels the time dependence. The MF equations (e.g. Ruzmaikin et al. 1988) without an $\alpha$-effect and with
homogeneous $\beta$ for the azimuthal and radial fields are then

$$\partial_t \bar{B}_{\phi t} = \bar{B}_{rt} f \Omega - \beta k^2 \bar{B}_{\phi t},$$  \hspace{1cm} (5)$$

and

$$\partial_t \bar{B}_{rt} = -\beta k^2 \bar{B}_{rt},$$  \hspace{1cm} (6)$$

where $f \Omega = r \partial \Omega / \partial r$. Solving (6) gives $\bar{B}_{rt} = \bar{B}_{r0} \text{Exp}[-k^2 \beta t]$, so (5) gives

$$\partial_t \bar{B}_{\phi t} = f \Omega \bar{B}_{r0} \text{Exp}[-k^2 \beta t] - k^2 \beta \bar{B}_{rt}.$$  \hspace{1cm} (7)$$

Multiplying both sides of (7) by $\text{Exp}[k^2 \beta t]$, using the chain rule and solving gives

$$\bar{B}_{\phi t} = B(f \Omega t + 1)(3N)^{-1/2} \text{Exp}[-(k_0^2 + k_z^2 + k_r^2)\beta t]$$

$$\simeq B(f \Omega t + 1)[(3hDR)(4\pi l^3/3)]^{-1/2} \text{Exp}[-(D^{-2} + h^{-2} + R^{-2})\beta t],$$  \hspace{1cm} (8)$$

where I have taken $\bar{B}_{r0} \sim \bar{B}_{\phi 0} \sim \bar{B}_0/3^{1/2} = B/(3N)^{1/2}$, $N \sim hDR/(4\pi l^3/3)$, and $h$ is the Galactic scale height, while $D$ and $R$ are the azimuthal and radial mean field gradient lengths corresponding to their wave vectors, and defined only for scales $\gtrsim h$.

The dominant contribution to the MF at any one time is that produced within a vertical diffusion time, $\tau_{dv}$, from the observation time. Thus $\beta t \sim 1/h^2$, and from (8)

$$\bar{B}_{\phi \tau_{dv}} = B(f \Omega h^2/\beta + 1)[3hDR/(4\pi l^3/3)]^{-1/2} \text{Exp}[-(1 + h^2/D^2 + h^2/R^2)].$$  \hspace{1cm} (9)$$

The scale height of the disk is fixed at $h \sim 500$pc, but $\bar{B}_{\phi \tau_{dv}}$ can be extremized as a function of $R$ and $D$, giving a maximum at $R = D = 2(\beta t)^{-1/2}$. Using $\beta \sim 10^{26}$ cm$^2$/sec
(e.g. Ruzmaikin et al. 1988) a Galactic disk scale-height of $h \sim 500\text{pc}$, $\Omega \sim 10^{-15}\text{sec}^{-1}$, and $f \sim 1$ this gives $D = R = 1\text{kpc}$. Thus $\bar{B}_{\phi\tau_{av}} = 1.42 \times 10^{-6} (B/5 \times 10^{-6}\text{G})\text{G}$. If instead we take $D = 3\text{kpc}$, to match the radial scale measured by Faraday rotation (e.g. Rand & Lyne 1994), this becomes $1.02 \times 10^{-6} (B/5 \times 10^{-6}\text{G})\text{G}$. The approximate magnitude of the local MF (e.g. Heiles 1994) may therefore be reproduced without the $\alpha$-effect.

Note that $\bar{B}_{\phi\tau_{av}}$ depends on the value of the characteristic averaging azimuthal distance as $D^{-1/2}\text{Exp}[-h^2/D^2]$, on the characteristic small scale structure size $l$ to the $-3/2$ power, and linearly on $B$. The actual small scale field of the Galaxy has been shown from observations to be inconsistent with a single scale size (Rand & Kulkarni 1989): The statistical dispersion between the observed field and their model large scale field shows little evidence of fall off with the distance to pulsars as it should if the the single cell size model were appropriate. This highlights the importance of super-bubbles and other larger scale fluctuations known to be important to the field structure (Ferrière, 1996). If the region over which the measured MF were composed of primarily $\gtrsim 200\text{pc}$ instead of $100\text{pc}$ structures, then the estimate given above is magnified by an additional factor $\gtrsim 2^{3/2} = 2.8$.

Also, Heiles (1994) finds that the total magnetic energy does not scale simply with the mean azimuthal field as measured by Faraday rotation in different parts of the Galaxy. This can be explained in the present model, since the mean field is proportional to the RMS field divided by $N^{1/2}$, where $N$ is the number of small scale cells of uniform field in the region determining the Faraday rotation measure. Regions of different cell sizes would therefore produce different observed mean fields even if the total magnetic energy density were the same. All of this highlights the possible importance of multiple small scale sizes.
4. Discussion of Mean Field Reversals

The calculation of section 3 shows that a scale height of \( h = 500 \text{pc} \), maximizes the azimuthal MF for a radial averaging scale of \( D = R = 1 \text{kpc} \). This results from two competing effects: [1] The time for shear to increase the field strength by an order of magnitude is relatively independent of scale. Thus large scales are preferentially sheared in a fixed time because the competing turbulent diffusion depends on scale squared. [2] However, the initial seed field depends inversely on the averaging scale. The scale of \( 1 \text{kpc} \) optimizes [1] and [2]. This defines the minimum radial scale over which the maximum average azimuthal field could reverse sign. This does not mean that there would be necessarily be reversals every \( 1 \text{kpc} \). It means that between \( 1 \text{kpc} \) annuli, the mean field may or may not reverse. Within a \( 10 \text{kpc} \) Galactic radius, there are \( \sim 9 \) interfaces between \( 1 \text{kpc} \) annuli. The probability \( P(n) \) of observing \( n \) reversals by Faraday rotation would be \( 9 \) ’choose’ \( n \), i.e. \( 9!/(9-n)!n! \), which is maximized for \( n=4 \) or 5.

Galactic Faraday rotation observations can determine the sign of the large scale field in the line of sight (cf. Beck et al. 1996; Zwiebel & Heiles 1997). (Unlike Galactic measurements, where pulsar dispersion measures can be used, extragalactic measurements require independent determinations of the density to obtain any information from Faraday rotation. The data for external galaxies are therefore less reliable (Heiles 1994; Zwiebel & Heiles 1997).) Generally, a large scale theoretical Galactic field model is statistically compared to observations (e.g. Rand & Kulkarni 1989). Field reversals seem to occur in each of the two interarm regions immediately inside of the solar circle (Beck et al. 1996;
Heiles 1994) with perhaps two more outside. The reversals are not necessarily periodic between spiral arms (Vallée 1996). Also, because of fluctuations in rotation measure data for some quadrants (Rand & Lyne 1994; Beck et al. 1996) averaging over smaller scales then shows smaller intermediate scale reversals. This again highlights that intermediate scales (Rand & Kulkarni 1989) from 50-500 pc complicate theoretical and observational interpretations. The precise structure of the large scale field in spiral galaxies is difficult to conclusively determine (Beck et al. 1996).

Note that turbulent diffusion is distinct from dissipation. The former describes a transfer of magnetic energy between scales, whereas dissipation is a removal of magnetic energy. Turbulent motions on sub-kiloparsec scales both randomize the mean field and amplify the small scale field, thereby re-seeding the mean field. Though a turbulent cascade drains energy to the dissipation scale, the magnetic energy is steadily replenished by the FD and the total magnetic energy density remains steady.

Previous work has recognized the importance of diffusion for reversal reduction (Poezd et al. 1993). In fact, the weaker the $\alpha$-effect in dynamo models, the less vigorously the $\alpha\Omega D$ can compete with turbulent diffusion and the fewer reversals that survive. Primordial models are sometimes employed with the assumption that turbulent diffusion is not operating (Zweibel & Heiles 1997). Though this seems unlikely, other proposed mechanisms would then be needed to eliminate reversals (c.f. Zweibel & Heiles 1997). Another possibility is that the winding of a proto-galactic field in the subsequently formed galaxy (Howard & Kulsrud 1997) generates the correct number of reversals.
For some external galaxies (e.g. NGC6946), observations indicate that the large scale field is actually stronger in the interarm regions (Beck & Hoernes 1996). In the present approach, the deficit of large scale field in the spiral arms would be the result of a reduced shear there (Elmegreen, 1994) and thus an $f < 1$ in Eq. (5). This is generally consistent with rotation curves of NGC6946 and other galaxies which show reduced differential rotation in spiral arms (Sofue 1986; Rubin et al. 1980). In contrast, an enhanced MF strength might result in the arms if their increased electron density dominates the effect of reduced shear. The total magnetic energy can be larger in spiral arms if the turbulent energy is higher there. Varying density complicates the interpretation of rotation measures of external galaxies if the density variation cannot be independently measured.

5. Discussion

It is important to understand whether $\alpha$ and $\beta$ can actually be disentangled. If so, the main point herein is to suggest that it may not be absolutely certain that the observed large scale Galactic magnetic field requires a dynamo $\alpha$-effect even if the mean field is produced in situ. The well-known exponential growth of small scale field by the FD and its steady replenishing of seed MF, combined with the subsequent linear growth of large scale azimuthal field by the $\Omega$-effect might supply a large scale Galactic field without requiring the dynamo $\alpha$-effect. The linear growth may be sufficient because it proceeds faster than the time for the field to diffuse below micro-Gauss values. The observed field of any spiral galaxy would be that produced within $\tau_{dw}$ of the time of observation, and since the field is steadily replenished, this statement is true at any time in a galaxy’s lifetime $\geq 10^8$yr from the time of the galaxy’s birth. The most likely number of reversals in the large scale field
within 10kpc radius would be of order 4-5 in the presence of turbulent diffusion for the simplest approach. (Unlike the cellular model of Michel & Yahil (1973), here turbulence is important.) If flux tubes were present with a small volume filling fraction, and/or if the energy containing small scale of the field were much reduced from the semi-empirically determined 100pc scale, too many mean-field reversals might be produced by the present approach. A small filling fraction may also aid the standard dynamo (Blackman 1996; Subramanian 1997), making the approach herein less useful. However, unlike the Sun, the tube filling fraction in galaxies may be large if the average particle pressure does not overwhelm the magnetic pressure (Blackman 1996).

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