Anthropic considerations in multiple-domain theories and the scale of electroweak symmetry breaking

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Abstract

One of the puzzles of the Standard Model is why the mass parameter which determines the scale of the Weak interactions is closer to the scale of Quantum Chromodynamics (QCD) than to the Grand Unification or Planck scales. We discuss a novel approach to this problem which is possible in theories in which different regions of the universe can have different values of the physical parameters. In such a situation, we would naturally find ourselves in a region which has parameters favorable for life. We explore the whole range of possible values of the mass parameter in the Higgs potential, $\mu^2$, from $+M_{Pl}^2$ to $-M_{Pl}^2$ and find that there is only a narrow window, overlapping the observed value, in which life is likely to be possible. The observed value of $\mu^2$ is fairly typical of the values in this range. Thus multiple domain theories in which $\mu^2$ varies among domains may give a promising approach to solving the fine tuning problem and explaining the closeness of the QCD scale and the Weak scale.
In our present theory of physics, there are only three parameters in the fundamental Lagrangian which are dimensionful. Two of these are associated with General Relativity, i.e. the Planck Mass \(M_P^2 = G_N^{-1} = (10^{19} \text{ GeV})^2\), and the cosmological constant, which is presently bounded to be \(\Lambda \leq 10^{-120} M_P^4\). The third is the mass parameter in the Higgs potential of the Standard Model, \(\mu^2\), which leads to a vacuum expectation value for the Higgs field \(v = \sqrt{-\mu^2/\lambda} = 246 \text{ GeV}\). (\(\lambda \sim 1\).) The expectation value \(v\) is the origin of the masses of all of the quarks, leptons and gauge bosons. A fourth mass scale does not appear in the Lagrangian, but enters indirectly as the energy at which the “running” strong coupling constant becomes of order unity. This QCD scale is roughly 200 MeV. Because the QCD coupling varies only logarithmically with the energy, it is natural that the QCD scale is much smaller than the Planck Mass. However, the smallness of the cosmological constant and the Higgs mass parameter are severe problems for our present understanding.

The Higgs vacuum expectation value is not only small compared to the Planck scale, \(v \sim 10^{-17} M_P\), but it is also problematic because it receives large quantum corrections. If the Standard Model is the appropriate description up to some scale \(\Lambda_{SM}\), then \(\mu^2\) receives radiative corrections of order \(\Lambda_{SM}^2\). For the Standard Model to be valid to high energies \(\Lambda_{SM} \gg v\), one requires a highly fortuitous cancellation of the bare parameter and its radiative corrections in order to produce a low physical value of \(\mu^2\). The puzzling smallness of \(\mu^2\) is often referred to as the “hierarchy problem”, and the sensitivity to quantum corrections as the “fine-tuning problem”[1]. The smallness and fine-tuning of the cosmological constant is even more dramatic [2].

The problem of the Higgs mass parameter is one of the key issues in modern particle physics, and has led to the widespread expectation that new physics beyond the Standard Model must be present at energies \(\Lambda_{SM} \sim 1\) TeV. Prime candidates are supersymmetric theories [3] or theories without fundamental Higgs fields [4]. The search for this new physics is a prime goal of theoretical and experimental efforts.

However, there is the possibility of an entirely different hypothesis, in which one posits certain new cosmological features which would naturally imply “anthropic” [5] constraints on some parameters. In exploring theories of inflation, the possibility has emerged that different domains of the universe could involve different values of the fundamental parameters. In such theo-
ries, typical of chaotic inflation [6], dynamical Higgs-like fields can get fixed at various vacuum expectation values, defining low-energy theories with different parameters. Our observed universe would be entirely within one such domain. The idea of multipledomains may be more general than chaotic inflation and may potentially be realizable in other contexts also [7]. With our present limited information, it is not any more scientific to assume that only one unique domain exists than it is to explore the possibility of multiple domains. The idea that multiple domains may exist takes the Copernican revolution to its ultimate limit — even our universe may not be the center of the Universe.

Within such a theory it is an obvious requirement that out of the ensemble of all domains we could only find ourselves in domains in which physical parameters are such as to allow the development of life — we will call these "viable" domains. This may drastically narrow the range of allowed values for the mass parameters. Weinberg has already used this form of reasoning to argue [7] that the "anthropic" need for the clustering of galaxies can only be possible for cosmological constants which are smaller than a value which is close to the present bound. In this paper, we argue that under the assumption that life requires the complex elements to be formed in the universe one has a constraint that only allows values of $\mu^2$ close to the QCD scale and in a range near that found in our domain. If the multiple-domain cosmological theories are correct, this limited allowed range would plausibly provide an explanation for the observed small value of the mass scale of the Standard Model [8].

In the process, these considerations will also illuminate another puzzle posed by the Standard Model. Even if a different kind of mechanism to solve the fine-tuning problem is found, and a hierarchy of scales is allowed, it is puzzling that, out of all the available parameter space, the weak scale is intertwined with the QCD scale. Quark and lepton masses (manifestations of the weak scale) appear at values both below and above the QCD scale, and to describe the physical world we need important inputs from both weak scale physics and QCD physics. Within the Standard Model, there is no need for these scales to be close. As far as we know, even in extended theories there is no known explanation for this curious fact. Logically, the fine-tuning problem, the hierarchy problem and this "intertwined scales" problem are all distinct, although they are all aspects of our need to understand the scale of weak symmetry breaking.
We consider all values of $\mu^2$ from $-M^2_P$ to $+M^2_P$, under the condition that all dimensionless parameters of the Standard Model are held fixed at the unification or Planck scale. Many of our arguments could be adapted to situations where more parameters vary, although without knowing more about the underlying theory one cannot be sure which parameters should be treated as variable. Our results are displayed compactly in Fig. 1, and the rest of this paper is devoted to explaining this figure. The key ideas are relatively simple to present, and we provide more details in a longer paper [9]. We label the values of parameters found in our domain by a subscript zero, i.e. $\mu^2_0$ and $v_0$.

The impact of the variable values of $\mu^2$ and $v$ are transmitted to the structure of the chemical elements largely through the quark and lepton masses, since these are linearly proportional to $v$, i.e. $m = m_0(v/v_0)$. The most important of these are the up and down quarks (with $m_u/m_d = 0.6$, $m_d \sim 7$ MeV) and the electron ($m_e \sim 0.5$ MeV). Despite the electromagnetic mass shift which enhances the proton mass ($m_p - m_n)_{EM} \sim 1.7$ MeV), the neutron is heavier than the proton because of the larger down quark mass. The quark masses also play a role in the nuclear force, most importantly through the attractive long-range pion-exchange potential which has a range $r \sim 1/m_\pi$, with the pion mass-squared roughly linearly proportional to the light quark masses, $m^2_\pi \propto (m_u + m_d)$.

If we start close to the observed values, we note that smaller values of $v$ appear to be allowed. As $v$ becomes smaller, the nuclear binding becomes more effective (see the discussion below) and for $v$ less than a critical value, which we estimate to be about $0.7v_0$ to $0.85v_0$, the di-neutron and di-proton become bound. This has a large impact on the relative abundances of elements [10], but does not prevent the existence of complex nuclei. Stellar evolution is greatly affected. It is amusing to note that below $v/v_0 = 0.5$ the proton is heavier than the neutron and decays $p \rightarrow n + e^+ + \nu$. In such a domain there would be no hydrogen, and much of matter would consist of neutrons. However, deuterium and the complex elements would still exist and could have enough potential to produce life of some form. We see no clear reason why domains with $v < v_0$, and even close to zero, would not be biologically "viable".

For values of $v$ larger than $v_0$, the elements will become increasingly unstable. The first key element to become unbound will be the deuteron, which is just barely bound in nature. As the nuclear force becomes shorter range
with increasing $v$, we estimate that deuterium becomes unstable against the strong decay $d \rightarrow p + n$ at some value of $v/v_0$ in the range 1.4 to 2.7 depending on the model used for the nucleon-nucleon potential. This presents an obstacle to the formation of the elements, as both nucleosynthesis in the early universe and in the burning of stars requires a stable deuteron for the initial processes. Beyond this critical value of $v/v_0$, a domain would likely lack most of the elements required for life. However, even if there were a way to form the elements, a more severe problem develops at a value of $v/v_0$ around 5. At values larger than this the neutron is heavier than the proton by more than the nucleon’s binding energy in nuclei, so that even bound neutrons would decay to protons. (Of course, as $N$ becomes less than $Z$ in this way, the change in the nuclear fermi energies make $n \rightarrow p e^−\bar{\nu}$ less exothermic, but our understanding of nuclear structure indicates that nuclei with $Z \gg N$ are not bound anyway.) Such a domain would contain only protons, would not form complex nuclei, and would be chemically sterile, and therefore probably not viable. This yields our first bound on $\mu^2$ on the left side of Fig. 1. It is interesting that the existence of neutrons close enough in mass to the proton to be stable in nuclei appears to be a requirement for life to exist.

Domains with $v/v_0$ above 5 and below another critical value near $10^3$ would appear as sterile “proton domains”. In domains with $v/v_0$ above around $10^3$ the only stable baryons would be $\Delta^{++}$ particles, which, being atomically equivalent to helium, would be even more chemically inert. This transition to “$\Delta$ domains” happens when the $d − u$ mass difference is large enough that the $\Delta^{++}$ (i.e. $uuu$) is lighter than the proton ($uud$) despite the QCD hyperfine energy which shifts the $\Delta$’s up in mass by about 300 MeV compared to the proton. We have estimated the non-relativistic binding energy of 6 ultra-heavy $u$ quarks in a single object and find that almost certainly it would fission to two $\Delta^{++}$’s. (At the transition point between “proton domains” and “$\Delta$ domains”, there is a narrow range of $v/v_0$ where the electron mass would stabilize both $p$ and $\Delta^{++}$, but even this somewhat richer chemical environment seems unlikely to support life processes.)

Where $\mu^2$ has the opposite sign from that in our domain, the Higgs potential does not lead to electroweak symmetry breaking; rather, the $SU(2)_L$ symmetry is broken by the chiral dynamics of QCD. This leads to light $W^\pm$ and $Z^0$ gauge bosons ($M_W \sim 50$ MeV). It also leads to a tiny value for $v \sim f_π/\mu^2$, so all the quarks and leptons are nearly massless. This leads to domains which are very different from our own, hard to analyze definitively,
but with several features that appear to disfavor the possibility of life.

All energy scales in chemistry are set by the electron mass, which for \( \mu^2 > |\mu_0^2| \) would be smaller by more than a factor of a billion. Chemical binding energies would therefore be very small. It is clear that chemical life cannot emerge until the time, \( t_{chem} \), when the temperature of the universe cools below typical biochemical reaction energies; otherwise (to put it picturesquely) life would be fried by the primordial cosmic background radiation. For electron-dominated chemistry we estimate

\[
t_{chem} \sim 10^{23} \text{yrs} \left( \frac{\mu^2}{|\mu_0^2|} \right)^{\frac{3}{2}}.
\]

This timescale could be reduced by a factor of up to 50 if the valence electrons were replaced by muons and/or tau leptons, which are effectively stable due to their small mass. In any event \( t_{chem} \) is a long time and several factors relevant to the development of chemical life would be altered. For example, if life is to evolve it must do so before all the baryons decay, or before all stars reach the end of their evolutionary paths.

It is likely that baryons can decay. The unification of gauge couplings [11] suggests the existence of gauge bosons of mass \( 10^{16} \) GeV whose exchange leads to violation of baryon number. Even without this, it is plausible that Planck scale physics leads to baryon decay. We therefore parameterize the baryon decay rate as \( \Gamma_B = \frac{m_B}{M^4} \), where \( M \) is assumed to lie between \( 10^{16} \) GeV and \( 10^{19} \) GeV. In comparing \( t_{chem} \) to \( \Gamma_B \) we must include the thermalized energy from the decaying baryons, which was left out of Eq. 1. The temperature at the epoch of baryon decay will be \( T_{rad} \sim \left( \Gamma_B M_P \right)^{1/2} \). If \( T_{rad} \) is greater than some fraction (which in our universe is of order \( 10^{-3} \)) of the energy binding leptons to atoms, then life based on chemistry will be impossible. This constraint rules out the larger positive values of \( \mu^2 \) as not being biologically viable, as shown for \( M = 10^{16} \) GeV and electron chemistry in our Figure. This constraint could be much stronger if baryon decay involves the exchange of fermions (as in many supersymmetric models) or weak interaction processes known as sphalerons [12], both of which are suppressed in our world but may be allowed in a world with ultralight quarks and and QCD-mass-scale weak bosons.

Even if baryons exist, it is not clear if or how they would form nuclei appropriate for chemical life to evolve. Since all the quarks are light: a) the
ground-state baryons will contain 27 members, including the neutron and proton, and b) there will be a host of neutral mesons with masses less than a KeV (for $\mu^2 > |\mu_0^2|$). Nuclear forces will be long-range, although short-range repulsive forces would still lead to a saturation of nuclear density. The large number of nucleon species will produce lower fermi levels in nuclei. Since weak forces have a range of several fermis, their contribution to the electrostatic energy must be included. For intermediate size nuclei (a few $< A < $ a few hundred) we find that $Z \sim A/4$. The finite range of the weak force means that in very large nuclei $Z$ and $N$ will adjust to minimize electrostatic rather than electroweak energy, and thus $Z$ will be much smaller than $A$. Given the uncertainties, it is unclear whether or not there is a maximum nuclear size beyond which spontaneous fission occurs.

The long range of mesonic nuclear forces suggests that nucleosynthesis will proceed rapidly. However, in a thermal bath the effective mass of the mesons will be significant, and the range of nuclear forces will be reduced. It is unclear whether electrostatic coulomb and weak potentials will provide an effective barrier to nuclear reactions in a plasma. If they do, then primordial nucleosynthesis will halt at modest charges and nuclear sizes. There will be ample fuel for stars and a plausible elemental mix for life. If not, then primordial nucleosynthesis will run away either to the equivalent of trans-iron elements or to super-heavy nuclei with very low ratios of charge to mass. It is questionable if either of the last two scenarios would lead to biologically viable domains.

Even if nucleosynthesis produces an appropriate mix of elements, there is a question of stellar evolution and finding an environment and energy source for life to develop. With extremely light leptons, objects with mass less than a solar mass ($M < M_\odot$) will condense to planets supported by non-relativistic degenerate leptons. As larger objects cool, the leptons become relativistic before they become degenerate, and so such objects will condense to stars and burn nuclear fuel. The cooling time during the pre-ignition phases of stellar evolution will be dominated by photon diffusion at a time when the internal temperature is comparable to the electron mass (which maximizes the compton cross-section). We estimate $t_{cool} \approx 10^{17} \mu^2/\|\mu_0^2\|$ yr. This is less than $t_{chem}$, but not by so much that stars may not be important as energy sources for life.

If electrostatic coulomb barriers are effective in a plasma of charged leptons and neutral mesons, thermonuclear reactions will support the star at
temperatures of $1 - 10$ KeV. Because of the ultra-light charged leptons, radiative opacities will be large. Therefore, given the small $W^\pm$ and $Z^0$ masses, such an object will cool by neutrino pair emission. We estimate nuclear burning lifetimes for $M \sim M_\odot$ of roughly a year, and much less for larger stars. This is very much less than $t_{\text{chem}}$.

Thus, within this crude treatment of stellar evolution, stars are expected to form slowly, and then burn nuclear fuel very quickly. But both timescales appear to be too small for there to be stars left when the tempretaure of the universe will allow biochemistry. However, it is possible that other sources of energy may be available, eg, gravitational energy of stars collapsing to the main sequence, “geothermal” energy, energy from radioactive decay, etc. It is therefore plausible, but by no means certain, that elemental and stellar evolutionary considerations exclude life in $\mu^2 > 0$ domains in the remaining area of Figure 1.

In conclusion, in a universe which has a domain structure, and in which some parameters have different values in different domains, life may only be able to develop in some domains and not others. If this is the case, we would expect that the parameters of our domain should be typical of the “viable” range. We have found that within the overall structure of the Standard Model there is a relatively small acceptable viable range for the Higgs parameter $\mu^2$. It seems that $\mu^2$ must be negative and of absolute magnitude close to what is observed. A multiple domain scenario in which $\mu^2$ varied could alleviate the fine-tuning problem. In an ensemble of different domains, the Higgs mass parameter will occasionally fall into the viable range without having to be fine-tuned in general.

If $\mu^2$ is positive, then lepton masses are extremely small, as $v$ is then set by QCD chiral symmetry breaking. Therefore, biochemical energies are also small, and the universe may be so old before it has cooled sufficiently to allow biochemical life that baryons have all decayed away, or stars have ceased to form and burn.

If $\mu^2$ negative, as in our domain, it seems that the whole range of values for $v$ from $M_P$ down to about 5 (or perhaps even down to 1.4) times the value in our domain can be excluded. Any domain which had a value of $v$ in most of that range (down to about $10^3 v_0$) would contain only sterile, helium-like atoms whose nuclei were $\Delta^{++}$. There would be essentially no reactions either chemical or nuclear. For the lower part of the excluded range, there would be virtually no nuclei other than protons, and the $pp$ and $pn$ processes that
are needed for nucleosynthesis would be endothermic as the deuteron would not be stable.

Thus we see that the natural viability requirement present within multiple domain theories provides a plausible approach to the fine-tuning problem, the hierarchy problem and the intertwined scales problem, as well as possibly the cosmological constant problem[7].

Finally, let us comment on the ability of these ideas to be tested. Negatively, we can say that if the weak scale is governed by “anthropic” considerations, there would be no need to invoke supersymmetry or technicolor or other structure at the weak scale to make the fine-tuning “natural” [1,13]. If no such structure is found, it would be a point in favor of anthropic explanations; indeed, in that case there would be few if any alternatives. Positive evidence is harder to come by. Of course, we are not able to explore other domains in the universe. However, theories which generate multiple domains may be testable by other, more conventional means. Because the class of theories which lead to multiple cosmological domains is not yet well understood theoretically, this will certainly be challenging and will not be completed in the near future. However, the community is hoping to be able to test the details of inflationary theories through cosmological measurements, and this may possibly inform us on the correctness of chaotic inflation. Likewise, as with any theory of physics beyond the Standard Model, we will require direct physical experimentation to eventually sort out the correct underlying theory. Through standard means we may be able to learn if the fundamental theory in fact produces multiple domains, and whether $\mu^2$ can vary among those domains. If so, then the hypothesis we propose in this paper automatically becomes relevant. Until such time, our conclusion must be modest: the observed value of the weak scale is reasonably typical of the biologically viable range.

References


2. For a comprehensive review see S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).


8. R. Cahn, *Rev. Mod. Phys.* 68, 951 (1996), makes arguments similar to some of ours, but not in a multiple domain framework, and not as an approach the gauge hierarchy problem. There fermion masses are held fixed, whereas here the Yukawa constants are held fixed and $\mu^2$ varied.


13. Note however that the anthropic arguments are also compatible with supersymmetry and may illuminate the intertwined scales problem and the ranges of the many parameters within this theory.
Figure 1: The figure shows a summary of arguments that $|\mu^2| \ll M_P$ is necessary for life to develop. $\mu^2$ is the mass parameter of the Higgs field of the Standard Model, and $v$ is its vacuum expectation value. $\mu^2$ can range from $+M_P^2$ to $-M_P^2$. The abscissa is defined to allow both signs of $\mu^2$ to be shown on the same log plot. For $\mu^2 < 0$, $v \propto (-\mu^2)^{1/2}$, and thus large values of $|\mu^2|$ imply large masses for leptons, quarks, and baryons. The increasing difference between the light quark masses, $m_d - m_u \propto v/v_0$, implies universes with but a single species of stable nucleus ($p$ or $\Delta^{++}$), which we argue would not allow for chemistry rich enough to support life. There is a narrow band where both $p$ and $\Delta^{++}$ are stable, but the chemical equivalent of a mix of hydrogen and helium is probably also sterile. For $\mu^2 > 0$, quark chiral condensates lead to $v \propto f_\pi^3/\mu^2$ (where $f_\pi \sim 100$ MeV) and quark and lepton masses become very small. Light lepton masses imply that biochemical processes cannot occur until cosmologically late times, when baryons may have already decayed. We show a constraint for a baryon lifetime estimated from exchange of intermediate GUT scale ($\approx 10^{16}$ GeV) particles. Even if baryons are stable, formation of a biologically acceptable mix of elements or the nature of stellar evolution may make development of life improbable. What is left is a rather narrow range of $\mu^2 < 0$ which includes the physical values in our domain.
Our Domain

$p, \Delta^{++}$ stable

\[ \text{no stable nuclei?} \]

\[ \text{stellar evolution?} \]

\[ \tau_B < t_{\text{chem}} \]

\[ \text{sgn}(\mu^2) \log(1+|\mu^2/f_\pi^2)^{1/2} \]

\[ \log \left( \frac{v}{v_0} \right) \]