Electromagnetic transition form factors of negative parity nucleon resonances *

M. Aiello, M.M. Giannini and E. Santopinto

Dipartimento di Fisica dell’Università di Genova,
I.N.F.N., Sezione di Genova
via Dodecaneso 33, 16164 Genova, Italy
e-mail:giannini@genova.infn.it

Abstract

We have calculated the transition form factors for the electromagnetic excitation of the negative parity resonances of the nucleon using different models previously proposed and we discuss their results and limits by comparison with experimental data.


*Partially supported by EC-contract number ERB FMRX-CT96-0008
1 Introduction

Various Constituent Quark Models have been proposed for the internal structure of baryons [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. A common characteristic is that, although the models use different ingredients, they are able to give a satisfactory description of the baryon spectrum and, in general, of the nucleon static properties.

The problem is that, in any case, the study of hadron spectroscopy is not sufficient to distinguish among the various forms of quark dynamics, that is among the various models, and so other observables, such as the electromagnetic transition form factors and the strong decay amplitudes, are important in testing the models for the internal structure of hadrons.

In order to perform a systematic study of baryon properties it is useful to have some general framework, which allows to formulate or reformulate the various models and compare their results in a consistent way. To this end it has been recently shown [7] that a hypercentral approach to quark dynamics can be used. This method is sufficiently general to investigate new dynamical features, such as three-body mechanisms, and also to reformulate and/or include the currently used two-body potential models.

Within this framework we have performed a study of the helicity amplitudes for the photoexcitation of the nucleon resonances [11], making use of various types of potentials determined in Ref. [7]. The resulting description of the helicity amplitudes is qualitatively good and comparable with the one coming from various models proposed in the literature, including those which take into account some relativistic kinematic corrections [11].

In this paper we present the electromagnetic transition form factors calculated within the general framework of [7] using various potentials. The $Q^2$-behaviour is more sensitive to the quark wave functions, therefore a detailed analysis of the theoretical inputs becomes possible, and moreover the electromagnetic transition form factors are going to be accurately measured at TJNAF(CEBAF) [12].

We study the excitation of the negative parity resonances. Their energies are really well described and therefore any discrepancy with the experimental data cannot be ascribed to
a deficiency in the description of the spectrum, but eventually to some mechanism which is not present in current approaches, such as dynamic and relativistic corrections.

In Sect. 2 we briefly describe the model. In Sect. 3 the electromagnetic transition form factors are evaluated and compared with the data and some discussion on the limits of the present non relativistic description is also given. A brief conclusion is given in Sect. 4.

2 The model

We briefly remind the theoretical framework proposed in [7], which is a three-body force approach to the non-relativistic constituent quark model. The internal quark motion is described by the Jacobi coordinates $\vec{\rho}$ and $\vec{\lambda}$:

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2),$$

$$\vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$

or equivalently, $\rho$, $\Omega_\rho$, $\lambda$, $\Omega_\lambda$. In order to describe three-quark dynamics it is convenient to introduce the hyperspherical coordinates, which are obtained substituting the absolute values $\rho$ and $\lambda$ by

$$x = \sqrt{\rho^2 + \lambda^2}, \quad \xi = arctg\left(\frac{\rho}{\lambda}\right),$$

where $x$ is the hyperradius and $\xi$ the hyperangle. In this way one can use the hyperspherical harmonic formalism [13, 14, 15].

The quark potential, $V$, is assumed to depend on the hyperradius $x$ only, that is to be hypercentral. Therefore, $V = V(x)$ is in general a three-body potential, since the hyperradius $x$ depends on the coordinates of all the three quarks. This class of potentials contains also contributions from two-body potentials in hypercentral approximation [16, 17]. For hypercentral potentials, the Schrödinger equation, in the hyperspherical coordinates, is simply reduced to a single hyperradial equation, while the angular and hyperangular parts of the 3q-states are the known hyperspherical harmonics [14].
There are at least two hypercentral potentials which can be solved analytically. One can observe that the h.o. potential, which has a two-body character, turns out to be exactly hypercentral, since

$$\sum_{i<j} \frac{1}{2} k (\vec{r}_i - \vec{r}_j)^2 = \frac{3}{2} k x^2 = V_{h.o}(x)$$  (3)

The other one is the 'hypercoulomb' potential [18, 19, 16, 20]

$$V_{hyc}(x) = -\frac{\tau}{x}.$$  (4)

This potential is not confining, however it has interesting properties. It leads to a power-law behaviour of the proton form factor [18, 19] and of all the transition form factors [21] and has a perfect degeneracy between the first $0^+$ excited state and the first $1^-$ states [22, 18, 23, 20], which can be respectively identified with the Roper resonance and the negative parity resonances. This degeneracy seems to be in agreement with phenomenology and is typical of an underlying O(7) symmetry [20].

Besides the two analytical solutions, we have studied three-body potentials of the form [7]

$$V(x) = -\frac{\tau}{x} + \frac{\kappa}{x^2} + \beta x.$$  (5)

The hypercentral equation is solved numerically. Starting from any potential $V(x)$ and solving the corresponding hyperradial equation, one can construct a complete basis of antisymmetric three-quark states, analogously to what is done in standard h.o. models [3], combining the $SU(6)$-spin-flavour configurations with the space wave functions [7]. In order to account for the splitting within each $SU(6)$-multiplet, a hyperfine interaction has been added and treated as a perturbation and therefore each resonance is a superposition of $SU(6)$-configurations. In this way, one obtains [7] a description of the observed spectrum for different choices of the potential parameters of Eq. 5.

### 3 Electromagnetic transition form factors

The electromagnetic transition form factors, $A_{1/2}(Q^2)$ and $A_{3/2}(Q^2)$, are defined as the transition matrix elements of the transverse electromagnetic interaction, $H_{e.m.}^t$, between
the nucleon, $N$, and the resonance, $B$, states:

$$A_{1/2}(Q^2) = \langle B, J', J'_z = \frac{1}{2} | H_{\text{em}} | N, J = \frac{1}{2}, J_z = -\frac{1}{2} \rangle$$

$$A_{3/2}(Q^2) = \langle B, J', J'_z = \frac{3}{2} | H_{\text{em}} | N, J = \frac{1}{2}, J_z = \frac{1}{2} \rangle$$

The transition operator is assumed to be

$$H_{\text{em}} = -\frac{3}{2} \sum_{i=1}^{3} \left[ \frac{e_j}{2m_j} (p_j^i \cdot \vec{A}_j + \vec{A}_j \cdot p_j^i) + 2\mu_j s_j^i \cdot (\vec{\nabla} \times \vec{A}_j) \right],$$

where spin-orbit and higher order corrections are neglected [24, 25, 11]. In Eq. 7, $m_j$, $e_j$, $s_j^i$, $p_j^i$ and $\mu_j = \frac{g_{em}}{2m_j}$ denote the mass, the electric charge, the spin, the momentum and the magnetic moment of the $j$-th quark, respectively, and $\vec{A}_j = \vec{A}_j(r_j)$ is the photon field. For the transverse interaction it is sufficient, without loss of generality, to consider photons with right-handed polarization ($\epsilon_+ = -\frac{1}{\sqrt{2}} (1, i, 0)$) and momentum along the $z$-axis. Taking into account the antisymmetry of states, which permits to write $H_{\text{em}} = 3H_{\text{em}}(3)$, the transverse coupling is given by

$$H_{\text{em}} = 6 \sqrt{\frac{\pi}{k_0}} \mu_p \epsilon(3) e^{ikz} \left[ k(s_{3,x} + is_{3,y}) + \frac{1}{g}(p_{3,x} + ip_{3,y}) \right],$$

where $\mu_p$ is the proton magnetic moment and $(k_0, \vec{k})$ is the virtual photon tetramomentum.

We calculate the transition form factors in the Breit frame, using the relation

$$\vec{k}^2 = Q^2 + \frac{(W^2 - M^2)^2}{2(M^2 + W^2) + Q^2},$$

where $M$ is the nucleon mass, $W$ is the mass of the resonance and $Q^2 = \vec{k}^2 - k_0^2$. The matrix elements of the e.m. transition operator between any two 3q-states are expressed in terms of integrals involving the hyperradial wavefunctions and are calculated numerically.

The computer code has been tested by comparison with the analytical results obtained with the h.o. model of Refs. [24, 25] and with the analytical model of Ref. [21].

The calculations are performed using various models:

1) the potential of Eq. 5 retaining only the hypercoulomb and the linear confinement terms [7] with the parameters, $\tau = 4.59$ and $\beta = 1.61 \text{ fm}^{-2}$, fitted to the spectrum, plus a standard hyperfine interaction [3];
2) the analytical model of [21], which corresponds to the potential of Eq. 5 with \( \tau = 6.39 \) and \( \beta = 0.15 \, fm^{-2} \) plus a hyperfine interaction with a smooth \( x \)-dependence; the spin-spin interaction is assumed to be

\[
V^S(x) = A e^{-ax} \sum_{i<j} \vec{\sigma}_i \cdot \vec{\sigma}_j =
A e^{-ax} \left( 2 S^2 - \frac{9}{4} \right),
\]

(10)

where \( S \) is the total spin of the 3-quark system, and the tensor interaction

\[
V^T(x) = B \frac{1}{x^3} \sum_{i<j} \left[ \frac{\left( \vec{\sigma}_i \cdot (\vec{r}_i - \vec{r}_j) \right) \left( \vec{\sigma}_j \cdot (\vec{r}_i - \vec{r}_j) \right)}{|\vec{r}_i - \vec{r}_j|^2} - \frac{1}{3}\left( \vec{\sigma}_i \cdot \vec{\sigma}_j \right) \right],
\]

(11)

with \( A = 140.7 \, MeV \), \( a = 1.53 \, fm^{-1} \) and \( B = 14 \, MeV \, fm^3 \). All these parameters are fixed by the reproduction of the spectrum.

3) the potential of Eq. 5 with \( \tau = 1.6 \) and \( \kappa = -0.875 \, fm \), plus the same hyperfine interaction as in model 1); although not confining this potential has the property of reproducing exactly the dipole fit of the proton form factor [26];

4) the harmonic oscillator with the parameter \( \alpha = 0.229 \, GeV \) which reproduces the proton charge radius [3];

5) the harmonic oscillator with the parameter \( \alpha = 0.410 \, GeV \) corresponding to a confinement radius of the order of 0.5 \( fm \), required in order to reproduce the \( A^{2}_{3/2} \) at \( Q^2 = 0 \) for the \( D_{13}(1520) \)-resonance [24, 25].

We give the results for the transition form factors of the negative parity resonances, considering the excitations for which there is some experimental information, that is the \( D_{13}(1520) \), \( S_{11}(1535) \), \( S_{11}(1650) \), \( S_{31}(1620) \) and \( D_{33}(1700) \). The experimental data are in some case available up to \( Q^2 = 3 \, (GeV/c)^2 \) [27]. Although the CQMs are in principle not applicable to such high \( Q^2 \)-values, nevertheless we consider that it could be interesting to show the theoretical calculations even for \( Q^2 \) higher than 1 \( (GeV/c)^2 \) and discuss their limits.

In Fig. 1 we report the proton helicity amplitudes of the \( D_{13}(1520) \)-resonance. The two potentials 1) and 2) give rise to similar values. They both fit the energy levels and
lead to a confinement radius of the order of 0.5 fm. The medium $Q^2$-behaviour is good but they fail to reproduce well the data at low $Q^2$ especially in the $A_{1/2}^p$ case.

In the h.o. model 5) the 0.5 fm value for the radius is imposed by hand in order to reproduce the experimental value of the $D_{13}$-resonance $A_{1/2}^p$ at $Q^2 = 0$ [24, 25]. The results are however very different from potential 1) and 2) and in the $A_{1/2}^p$ case also far from the data. The potential which reproduces exactly the dipole form factor, 3), gives too damped results; the same happens for the h.o. with the correct proton radius, which causes a too strong damping in the wave functions.

Similar conclusions can be drawn from Fig. 2, where we show the results for the $S_{11}(1535)$-resonance. The two potentials 1) and 2) give a reasonable account of data, while the dipole-fit potential 3) is again too low with respect to the experimental values. Here we report only one h.o. calculation, the one with the smaller confining radius, taking into account the mixing of $SU(6)$-configurations for the $S_{11}$ states [28].

The transition form factor for the $S_{11}(1650)$-resonance (see Fig. 3) is non zero because of the configuration mixing. The two potentials 1) and 2) have different hyperfine interactions, nevertheless their results are still quite similar. The h.o. is the same as in Fig. 2, while the dipole-fit potential is omitted for the reasons stated above. In Figs 4, 5 and 6 we give the transition form factors for the $S_{31}(1620)$- and $D_{33}(1700)$-resonances, respectively. Here again the first two potentials give similar transition form factors.

All these results seem to favour potentials leading to wave functions which are localized in a small confinement region, of the order of 0.5 fm. On the other hand, in this approach one can see that the reproduction of the elastic form factor is not a guaranty of describing also the transition form factors.

It should be stressed that our main concern is the study of the possibilities and limits of quark models in the description of dynamical properties at low $Q^2$. From the analysis of our results it is evident that none of the present CQMs can explain adequately the transition form factors at low momentum transfer. In our opinion, this discrepancy indicates that some important effect at low momentum transfer is missing, such as the polarization effect of the Dirac sea, which is not included in CQMs.
The calculations, at variance with what expected, are in agreement with the few existing data at $Q^2 = 1 - 2 \ (GeV/c)^2$, that is outside the range of applicability of a non relativistic description.

The problem of a relativistic description is still open. The approaches used up to now are mainly of two types. In the first one, relativity is taken into account consistently [29], but the corresponding results are, for the moment, available only for the meson spectrum. In the second approach, relativistic corrections are included within a light-cone method [30, 31] or considered as higher order contributions to the e.m. current [32, 33, 34]. From these works one can see that the relativistic corrections modify slightly the high-$Q^2$ behaviour, that is at $2 - 3 \ (GeV/c)^2$, of the form factors [30, 34]. It has been shown that significant modifications are induced by relativistic corrections at low $Q^2$ [30, 34], although the results are still not able to reproduce the data.

4 Conclusions

We have calculated the transition form factors for the excitation of the negative parity nucleon resonances using the general framework provided by the three-body force approach of ref. [7] which allows to investigate the predictions of new three-body models.

The h.o. results depend strongly on the confinement radius and the $Q^2$-behaviour is not realistic (see also Ref. [35]).

The dipole-fit potential (model 3)) reproduces exactly the elastic proton form factor, however it fails in the case of the transition amplitudes.

From the analysis of our results, one sees that a potential containing a hypercoulomb and a linear confinement term plus a hyperfine term (models 1 and 2) ), is able to give a reasonable description of the transition form factor data, specially at medium values of the momentum transfer $Q^2$, that is $1 - 2 \ (GeV/c)^2$. The agreement at medium $Q^2$ is not expected to be modified by the inclusion of relativistic corrections, since, according to the discussion in the previous Section, in this range the relativistic corrections are expected to be not so important [30, 34].
Figure 1: Comparison between the experimental data for the transition form factors $A_{3/2}^p$, $A_{1/2}^p$ for the $D_{13}(1520)$-resonance and the calculations with the potentials 1) (full curve), 2) (dot-dashed curve), 3) (dashed curve), 4) (the dotted curve with the stronger damping) and 5) (the dotted curve with the softer damping). The data are from the compilation of Ref. [26].

We have observed that the potentials 1) and 2) still have problems for low $Q^2$-values. This can be an indication that further degrees of freedom, as $q\bar{q}$-pairs [36], should be included in the CQM in a more explicit way.

References


Figure 2: The same as in Fig. 1, for the $A_{1/2}^p$ of the $S_{11}(1535)$-resonance, calculated with the potentials 1), 2), 3) and 5). The data are from the compilation of Ref. [26].

Figure 3: The same as in Fig. 1, for the $A_{1/2}^p$ of the $S_{11}(1650)$-resonance, calculated with the potentials 1), 2) and 5). The data are from the compilation of Ref. [26].
Figure 4: The same as in Fig. 1, for the $A_{1/2}^p$ of the $S_{31}(1620)$-resonance, calculated with the potentials 1), 2) and 3). The data are from the compilation of Ref. [26].

Figure 5: The same as in Fig. 1, for the $A_{1/2}^p$ of the $D_{33}(1700)$-resonance, calculated with the potentials 1), 2), and 3). The data are from the compilation of Ref. [26].
Figure 6: The same as in Fig. 1, for the $A_{3/2}^p$ of the $D_{33}(1700)$-resonance, calculated with the potentials 1), 2), and 3). The data are from the compilation of Ref. [26].


