Reconstructing the Source in Heavy Ion Collisions from Particle Interferometry

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The preliminary CERN SPS NA49 Pb+Pb 158 GeV/A one- and two-particle \(h^-\)-spectra at mid-rapidity are consistent with a source of temperature \(T \approx 130\) MeV, lifetime \(\tau_0 \approx 9\) fm/c, transverse flow \(\eta_t \approx 0.35\), and a transverse geometric size which is twice as large as the cold Pb nucleus.

1. Introduction

Hadronic one- and two-particle spectra provide for each particle species information about the phase-space distribution \(S(x, K)\) of the hadronic emission region [1]. Once reconstructed from the measured spectra, \(S(x, K)\) allows to distinguish between different dynamical scenarios of heavy ion collisions. It provides an experiment-based starting point for a dynamical back extrapolation into the hot and dense early stage of the collision, where quarks and gluons are expected to be the relevant physical degrees of freedom.

Our reconstruction of \(S(x, K)\) is based on the hadronic one- and two-particle spectra

\[
E \frac{dN}{d^3p} = \int d^4x \ S(x, p) = \frac{1}{2\pi} \int \frac{d^2N}{dp_\perp dp_\parallel} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n \cos n(\phi - \psi_R) \right],
\]

\[
C(K, q) = 1 + \frac{\int d^4x \ S(x, K) e^{iq \cdot x}}{\int d^4x \ S(x, p_1) \int d^4y \ S(y, p_2)} = 1 + \exp \left[ -\sum_{ij} R_{ij}^2(K) q_i q_j \right].
\]

The triple-differential hadronic one-particle spectrum (1) tests the momentum-dependence of \(S(x, K)\) only. Its azimuthal \(\phi\)-dependence with respect to the reaction plane \(\psi_R\) is parametrized by the harmonic coefficients \(v_n\) [2]. Information about the space-time structure of \(S(x, K)\) can be obtained from the relative momentum dependence of the two-particle correlator \(C(K, q)\), \(q = p_1 - p_2\). This \(q\)-dependence is usually parametrized via the Hanbury-Brown Twiss (HBT) radius parameters \(R_{ij}^2(K)\) which depend on the average pair momentum \(K = \frac{1}{2}(p_1 + p_2)\). Depending on the Gaussian parametrization adopted, the indices \(i, j\) in (2) run either over the Cartesian directions long (parallel to the beam), out (parallel to the transverse component \(K_\perp\)) and side, or over the corresponding Yano-Koonin coordinates \(q_\perp = \sqrt{q_2^2 + q_3^2}, q_3\) and \(q^0 = q \cdot K/K_0\) [1].
This model assumes local thermalization at freeze-out with temperature $T$ and transverse flow $\eta_f$ account for the same one-particle slope.

2. Reconstructing an azimuthally symmetric source

A typical data analysis starts from a simple ansatz for the phase space distribution $S(x,K)$ in terms of very few, physically intuitive fit parameters, [1,4]:

$$S_n(x,p) = S^\text{dir}_n(x,p) + \sum_R S_{R\to n}(x,p),$$

(3)

$$S^\text{dir}_n(x,p) = \frac{2J_i+1}{(2\pi)^3} P_n(x) \exp\left(-\frac{p_n(x)}{T}\right) \exp\left(-\frac{r^2}{2R^2} - \frac{\eta_f^2}{2(\Delta \eta)^2} - \frac{(\tau - \tau_0)^2}{2(\Delta \tau)^2}\right).$$

(4)

This model e.g. assumes local thermalization at freeze-out with temperature $T$ within a space-time region of transverse Gaussian width $R$, emission duration $\Delta \tau$, longitudinal extension $\tau_0$, and average emission time $\tau_0$. The model allows for dynamical source correlations via the hydrodynamic flow 4-velocity $u_\mu(x)$. We assume a linear transverse flow profile $\eta_f(r) = \eta_f \left(\frac{r}{R}\right)$ with variable strength $\eta_f$, and Bjorken scaling of the flow component in the longitudinal direction, $v_l = z/t$, $\eta_l \equiv \frac{1}{2} \ln\left[\left(1 + v_l\right)/\left(1 - v_l\right)\right] = \eta$. Resonances are produced in thermal abundances with proper spin degeneracy $2J_i + 1$ for each particle species $i$. Their contribution to the pion yield is obtained by propagating them along their classical path $x^\mu = X^\mu + \frac{p_\mu}{M} \tau$ according to an exponential decay law [4],

$$S_{R\to n}(x,p) = \int_\mathbb{R} \int d^4X \int d\tau \Gamma e^{-\Gamma\tau} \delta^{(4)}\left[x - \left(X + \frac{p}{M} \tau\right)\right] S^\text{dir}_R(X,P),$$

(5)

where $S_R$ is the integral over the available resonance phase space for isotropic decays. We include all pion decay channels of $\rho$, $\Delta$, $K^*$, $\Sigma^*$, $\omega$, $\eta$, $\eta'$, $K^0$, $\Sigma$ and $\Lambda$ with branching ratios larger than 5 percent. The model parameters $T$, $\eta_f$, $R$, $\Delta \eta$, $\Delta \tau$, $\tau_0$ can then be determined via the following strategy:

1. **transverse one-particle spectrum** $dN/dM^2_\perp$ determines blue-shifted temperature $T_{\text{eff}}$:

   The slope of $dN/dM^2_\perp$ is essentially given by $T_{\text{eff}} = T\sqrt{(1 + \eta_f)/(1 - \eta_f)}$. Hence, different combinations of $T$ and $\eta_f$ can account for the same data, see Fig. 1.

2. **$dN/dM^2_\perp$ and $R_{\perp}(M_\perp)$** disentangle temperature and transverse flow. $R_{\perp}$ fixes transverse extension $R$:

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Figure 1. LHS: $\chi^2$ contour plot of a fit to the NA49 [3] $h^{-}$-spectrum. Dashed lines are for constant values of $\eta_f^2/T$. RHS: different combinations of temperature $T$ and transverse flow $\eta_f$ account for the same one-particle slope.
The $M_\perp$-slope of $R_\perp$ is proportional to $\eta_f^2/T$, $R_\perp^2 \approx R^2/(1 + M_\perp \eta_f^2/T)$ [6,7]. This slope allows in combination with $dN/dM_\perp^2$ to specify $T$ and $\eta_f$, see Fig. 2. The overall size of $R_\perp$ determines the Gaussian width $R$. We find $\eta_f \approx 0.35$, $T \approx 130$ MeV, $R \approx 7$ fm.

3. $R_\parallel$ determines $\tau_0$, width of rapidity distribution $dN/dY$ determines $\Delta \eta$:
In principle, $R_\parallel$ depends on $\tau_0$, $\Delta \eta$ and $\Delta \tau$ [7]. We have fixed $\Delta \eta = 1.3$ by matching the width of the pion rapidity distribution. The data then favour clearly a lifetime of $\tau_0 \approx 9$ fm/c (see Fig. 2) but are not very sensitive to the emission duration $\Delta \tau$ [8]. The present plots are obtained with $\Delta \tau = 1.5$ fm/c.

4. $R_0$ discards opaque sources:
For the model (3-5), the YKP-parameter $R_0$ is mainly sensitive to the temporal aspects of the source. The large statistical uncertainties for $R_0$ do not allow to constrain the model parameter space further, see Fig. 2. Models of opaque sources including an opacity factor in (4) are excluded already by the present data [8].

The radius of a cold Pb nucleus corresponds to $R_{0\text{B}}(\text{cold}) \approx 3.5$ fm in the Gaussian parametrization (4), i.e., the experimental data indicate a very large source $R \approx 2 R_{0\text{B}}(\text{cold})$. This is dynamically consistent with a collision scenario in which the initially produced pressure gradients result in a significant transverse flow $\eta_f = 0.35$, driving the expansion of the system over a time of 9 fm/c to twice its initial size. The temperature may have decreased substantially during this expansion; the data indicate 130 MeV at freeze-out. These conclusions are further supported by the analysis presented by G. Roland [9], which is based on approximate analytical formulas. We have extracted the model parameters by comparison to numerical model calculations, following the above strategy. They are not obtained from a simultaneous fit to all observables, and hence we do not quote errors.

3. Particle interferometry for collisions with finite impact parameter

The reaction plane analysis of triple-differential one-particle spectra (1) has been discussed extensively in this conference e.g. by A. Poskanzer, J.-Y. Ollitrault, and S.
Voloshin. The general strategy for linking this analysis to an azimuthally sensitive particle interferometry is based on the $\Phi$-dependence of the HBT-radii \[10\], where $\Phi$ measures the azimuthal angle of $\vec{K}_\perp$ relative to the reaction plane,

\[ R_{ij}^2(K_\perp, \Phi, Y) = R_{ij,0}^2(K_\perp, Y) + 2 \sum_{n=1}^{\infty} R_{ij,n}^2(K_\perp, Y) \cos n\Phi + 2 \sum_{n=1}^{\infty} R_{ij,n}^2(K_\perp, Y) \sin n\Phi. \]

Here, the many harmonic coefficients $R_{ij,n}$ make a direct comparison to experimental data impossible. However, various relations hold amongst these coefficients, since the leading anisotropy in realistic source models $S(x, K)$ can be quantified by very few parameters only. Using the symmetries of the system and assuming that elliptic deformations dominate we find \[10\]

\[ 0 \approx R_{i,m}^2 \approx R_{s,m}^2 \approx R_{o,m}^2 \approx R_{ol,m}^2 \approx R_{os,m}^2 \approx R_{sl,m}^2 , \quad m \geq 1 , \quad (7) \]

\[ \alpha_1 \approx \frac{1}{3} R_{o,1}^2 \approx R_{s,1}^2 \approx -R_{os,1}^2 , \quad (8) \]

\[ \alpha_2 \approx -R_{o,2}^2 \approx R_{s,2}^2 \approx R_{os,2}^2 . \quad (9) \]

A violation of Eqs. (8)-(9) by experiment would indicate strong higher order deformations and rule out many model scenarios. On the basis of Eqs. (7)-(9), an azimuthally sensitive parametrization of the two-particle correlator involves only two additional fit parameters,

\[ C_{\psi_R}(\mathbf{K}, q) \approx 1 + \lambda \exp \left[-R_{o,0}^2 q_o^2 - R_{s,0}^2 q_s^2 - R_{l,0}^2 q_l^2 - 2 R_{o,0} q_o q_l \right] \]

\[ \times \exp \left[-\alpha_1 (3 q_o^2 + q_s^2) \cos(\Phi - \psi_R) + 2 \alpha_1 q_o q_s \sin(\Phi - \psi_R) \right] \]

\[ \times \exp \left[-\alpha_2 (q_o^2 + q_s^2) \cos(\Phi - \psi_R) + 2 \alpha_2 q_o q_s \sin(\Phi - \psi_R) \right]. \quad (10) \]

The anisotropy parameter $\alpha_1$ vanishes at mid-rapidity or if the source contains no dynamical correlations. It characterizes anisotropic dynamics. The parameter $\alpha_2$ characterizes the elliptic geometry. The parameters $\alpha_1$ and $\alpha_2$ can be determined from event samples in spite of the uncertainty in the eventwise reconstruction of the reaction plane. For details, see Ref. \[10\].

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REFERENCES

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9. G. Roland for the NA49 Coll., these proceedings.