APPENDIX

Titanium and scandium.—Titanium-45 has a 3.1-hr half-life. Scandium-44 has a metastable state with a 2.4-day half-life, and a ground state with a 3.9-hour half-life. Scandium-43 has a half-life of 3.9 hours. The target was dissolved in concentrated HCl and heated to drive off all H_2S . Carriers were then added. Scandium was precipitated with saturated oxalic acid. The remaining solution was scavenged five times; then $\text{Ti}(\text{OH})_4$ was precipitated with KBrO_4 . This was dissolved in HCl and reprecipitated with concentrated ammonia. It was ignited, cooled, and weighed after transfer to a tared counting cup. Separation time: 2 hr. Chemical yield: 40–60%. The scandium precipitate was dissolved in concentrated nitric acid and $\text{Sc}(\text{OH})_3$ was precipitated with concentrated ammonia. This was ignited in a platinum crucible, cooled, and weighed after

transfer to a tared counting cup. Separation time: 3 hr. Chemical yield: 30–80%.

Potassium.—The half-life of K^{38m} is 7.7 min. The target was dissolved in a solution of concentrated HCl containing carriers. The solution was buffered to pH 3 and potassium was precipitated with sodium tetraphenylboron. The precipitate was transferred to a tared counting cup after drying. Separation time: 15 min. Chemical yield: 45–80%.

Nitrogen.—The half-life of N^{13} is 10.1 min. The target was dissolved in a solution of concentrated HCl containing carriers. Sodium hydroxide was added and the solution was boiled, NH_3 being distilled into a cooled sodium tetraphenylboron solution. Ammonium tetraphenylboron precipitated and was dried, transferred, and counted. Separation time: 20 min. Chemical yield: 30–60%.

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Effective Charge of Neutrons in Nuclei

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The polarization of closed proton shells by neutrons can be considered as inducing an effective charge on the neutron for some multipoles but not for others. The rough constancy of the effective neutron charge as observed in $E2$ transitions and quadrupole moments in nondeformed odd N -even Z nuclei is discussed, and arguments are presented to explain this phenomenon along the lines of the shell model.

INTRODUCTION

THE coupling scheme of the nuclear shell model is such that an even number of protons or neutrons couples to a zero total angular momentum, both in the ground states of even-even nuclei as well as in the ground states and low excited states of odd- A nuclei. Since a system with zero total angular momentum has vanishing average multipole moments, it follows, as is well known, that in the shell model the average moments of odd-even nuclei are determined, in zeroth order, only by the odd group of nucleons. Furthermore, on that model, transitions between low-lying levels of odd-even nuclei are also determined by the odd group of nucleons, since the even group is assumed to remain unchanged for low excitations as well.

This simple model, despite its many successes, is not adequate to describe some of the electromagnetic properties of nuclei. As is well known, static quadrupole moments, as well as electric quadrupole transitions, in odd- A nuclei do not exhibit any noticeable dependence on whether they occur in odd- Z or odd- N nuclei. The experimental data require that, within the framework of the shell model, the neutron in the nucleus should be

assumed to carry an "effective charge" roughly equal to that of the proton.

It should be pointed out that such conclusions can be derived from the analysis of the electric properties of nuclei; an analysis of their magnetic properties does not necessarily lead to the same conclusion, and may even lead to an opposite one. Thus the "slope" of the magnetic moments of nuclei as a function of their spin is about $1 \text{ nm}/\hbar$ for odd- Z nuclei and vanishes for odd- N nuclei. Since this slope is roughly proportional to the orbital g factor of the nucleons in the odd group, the data on the magnetic moments seem to indicate that the proton and the neutron retain their free electric properties also when bound in nuclei. Thus it seems that the "effective charge" of a neutron in the nucleus cannot be visualized as its sharing charge with the proton due to some exchange forces. Rather it is a concept which is closely connected with the special nuclear feature which is being studied, and a theory which explains this phenomenon should explain, besides the actual value of the effective charge, also its dependence on the multipole in which it manifests itself, and its apparent independence of the nuclear state.

This group of problems drew the attention of many investigators who were able to give them a successful

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and convincing treatment. For nondeformed nuclei there are primarily two approaches. One, initiated by Blin-Stoyle¹ and further elaborated for various cases by Arima, Horie, and others,²⁻⁸ improves the shell-model predictions by including, perturbation-wise, first-order corrections. The other approach, suggested first by Foldy and Milford,⁹ and carried to its full development in the classical work of Bohr and Mottelson¹⁰ and other subsequent investigations,¹¹⁻¹⁵ assumes the existence of a collective motion weakly coupled to the nucleon motion, and ascribes to this collective motion the observed electric properties, or at least a major part of them, in both odd- Z and odd- N nuclei.

Although the great similarity between the electric properties of odd- N and odd- Z nuclei strongly supports the hypothesis of a collective motion being responsible for the major part of these effects, it is still an unsolved problem how the parameters of this motion are related to the forces between nucleons. Furthermore, the surface tension, which appears essentially as an adjustable parameter in these treatments, shows regular variations which can be correlated with a stiffening of the nucleus near closed shells. It is thus conceivable that a future clarification of the collective motion in the weak-coupling limit may explain it in terms of concepts of the shell model, and it therefore seems to us that the attempts to explain the effective charge of the neutron in a nucleus by means of configuration mixing is in a way more "fundamental."

This note is an attempt to reformulate slightly the approach of configuration mixing, so as to leave in the formalism only the essential points required for the study of the specific question of the effective charge of the neutron in a nucleus. In essence what is done is just the evaluation of the polarization of an even number of protons by an odd number of neutrons, and in particular the evaluation of the extent to which this polarization affects various moments, and in what sense can it be replaced by an "effective charge" sitting on the neutrons.

¹ R. J. Blin-Stoyle, Proc. Phys. Soc. (London) **A66**, 1158 (1953).

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⁵ Arima, Horie, and Sano, Progr. Theoret. Phys. Japan **17**, 567 (1957).

⁶ M. Sano, Progr. Theoret. Phys. Japan **18**, 223 (1957).

⁷ R. D. Amado and R. J. Blin-Stoyle, Proc. Phys. Soc. (London) **A70**, 532 (1957).

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⁹ L. L. Foldy and J. F. Milford, Phys. Rev. **80**, 751 (1950).

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¹¹ A. Kerman, Phys. Rev. **92**, 1176 (1953).

¹² K. W. Ford and C. A. Levinson, Phys. Rev. **100**, 1 (1955).

¹³ C. F. Barker, Phil. Mag. **1**, 329 (1956).

¹⁴ W. W. True, Phys. Rev. **101**, 1342 (1956).

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THEORY

We shall use as our starting point the "odd-group" model which asserts that in zeroth order the total angular momentum of the protons (J_p) and that of the neutrons (J_n) are well defined, in addition, of course, to the total angular momentum of the nucleus $\mathbf{J} = \mathbf{J}_p + \mathbf{J}_n$.¹⁶ Recently, a very valid criticism of this model has been put forward by Pandya¹⁷ who made a thorough study of its consequences in odd-odd nuclei. Although his analysis implies pure configurations for both the proton system and the neutron system, an assumption which we shall not have to make here, it is not improbable that his criticism will be valid even under more general conditions. It may, in fact, be due to the relative ease with which configurations of an odd number of protons or neutrons can be excited. If, however, one of the groups has an even number of nucleons it is generally considerably more rigid, and the constancy of J_p and J_n may have a better validity. This is particularly true if the even group is magic, as in the lead or the oxygen isotopes.

Let us assume, then, that a nuclear state ψ is given by

$$\psi = |\alpha_p J_p, \alpha_n J_n; \alpha J M\rangle + \delta\psi, \quad (1)$$

where J_p , J_n , and J stand for the total angular momenta of the protons, the neutrons, and the whole nucleus, respectively, and the α are further quantum numbers which may be required for the complete specification of the proton, the neutron, and the nuclear states. We do not assume anything about the structure of either the proton states or the neutron states and we may take them, if we wish, as the exact eigenstates of the Hamiltonians built, respectively, from the protons alone (including their mutual interaction) and the neutrons alone. The modifications in $|\alpha_p J_p, \alpha_n J_n; \alpha J M\rangle$ due to the proton-neutron interactions are assumed to be small and are included in $\delta\psi$. Any effective charge which the neutrons may possess due to their interaction with the protons will, of course, be described by $\delta\psi$, and in order to have an estimate of this effective charge we have to evaluate the contribution of $\delta\psi$ to an operator through which the effective charge will show itself. The "cleanest" possible situation is that in which *all* the effect is due to $\delta\psi$; an example will be an electric quadrupole transition between two states i and f in an odd N -even Z nucleus for which $J_p^{(i)} = J_p^{(f)} = 0$, $\alpha_p^{(i)} = \alpha_p^{(f)}$. More generally, we shall consider a tensor operator, $e_p \Omega_p^{(l)}$, of rank $l > 0$ which operates on the proton coordinates only, and we shall calculate the transition amplitude for this operator between the

¹⁶ This model is slightly more general than the shell model in that it does not assume a pure shell structure for either the protons or the neutrons; it requires only the weakness of the interaction between the protons and the neutrons but not of the interactions of the protons or neutrons among themselves.

¹⁷ S. P. Pandya, Phys. Rev. **108**, 1312 (1957).

states ψ_i and ψ_f . Clearly,

$$\begin{aligned} & \int \psi_f^* e_p \Omega_p^{(l)} \psi_i \\ &= \langle J_p^{(f)}, J_n^{(f)}; J_f M_f | e_p \Omega_p^{(l)} | J_p^{(i)}, J_n^{(i)}; J_i M_i \rangle \\ &+ \langle J_p^{(f)}, J_n^{(f)}; J_f M_f | e_p \Omega_p^{(l)} | \delta \psi_i \rangle \\ &+ \langle \delta \psi_f | e_p \Omega_p^{(l)} | J_p^{(i)}, J_n^{(i)}; J_i M_i \rangle + O(\delta \psi)^2. \quad (2) \end{aligned}$$

In (2) we have omitted the α 's, and we shall write them explicitly only when it will be actually required. We further assume that our states ψ_i and ψ_f are such that $J_p^{(i)} = J_p^{(f)} = 0 \equiv J_p^{(0)}$. The first matrix element on the right-hand side of (2) vanishes, since we assumed $l > 0$, and neglecting terms of order $(\delta \psi)^2$ we are left with

$$\begin{aligned} \langle f | e_p \Omega_p^{(l)} | i \rangle &= \langle J_p^{(0)}, J_n^{(f)}; J_f | e_p \Omega_p^{(l)} | \delta \psi_i \rangle \\ &+ \langle \delta \psi_f | e_p \Omega_p^{(l)} | J_p^{(0)}, J_n^{(i)}; J_i \rangle, \quad (3) \end{aligned}$$

where we have rewritten (2) for the reduced matrix

$$\begin{aligned} c_k^{(i)} &= \frac{\langle \alpha_p^{(k)} J_p^{(k)}, \alpha_n^{(f)} J_n^{(f)}; J_i M_i | V_{pn} | \alpha_p^{(0)} J_p^{(0)}, \alpha_n^{(i)} J_n^{(i)}; J_i M_i \rangle}{\Delta E_p^{(k)} - \Delta E_n}, \\ c_k^{(f)} &= \frac{\langle \alpha_p^{(k)} J_p^{(k)}, \alpha_n^{(i)} J_n^{(i)}; J_f M_f | V_{pn} | \alpha_p^{(0)} J_p^{(0)}, \alpha_n^{(f)} J_n^{(f)}; J_f M_f \rangle}{\Delta E_p^{(k)} + \Delta E_n}, \end{aligned} \quad (5)$$

$$\begin{aligned} \Delta E_p^{(k)} &= E(\alpha_p^{(0)} J_p^{(0)}) - E(\alpha_p^{(k)} J_p^{(k)}), \\ \Delta E_n &= E(\alpha_n^{(f)} J_n^{(f)}) - E(\alpha_n^{(i)} J_n^{(i)}). \end{aligned} \quad (6)$$

We shall assume that the proton-neutron interaction is a sum of two-body interactions $V_{pn} = \sum_{i,j} V_{p_i n_j}$, where p_i runs over all protons and n_j over all neutrons in the nucleus. Furthermore we decompose the two-body interaction V_{ij} into a sum of scalar products of irreducible double tensor operators of rank¹⁹ m (their separate space and spin tensorial characters are not

elements, the M dependence of the matrix elements being trivially given by a proper Clebsch-Gordan coefficient.¹⁸

The functions $\delta \psi_i$ and $\delta \psi_f$ can be expanded in a complete set of states of the proton and neutron systems. It is convenient to choose this complete set so that the states $\alpha_p^{(0)} J_p^{(0)}$, $\alpha_n^{(i)} J_n^{(i)}$, and $\alpha_n^{(f)} J_n^{(f)}$ are among its basic vectors. Thus we can write

$$\begin{aligned} \delta \psi_i &= \sum_k c_k^{(i)} | \alpha_p^{(k)} J_p^{(k)}, \alpha_n^{(f)} J_n^{(f)}; J_i M_i \rangle + \dots, \\ \delta \psi_f &= \sum_k c_k^{(f)} | \alpha_p^{(k)} J_p^{(k)}, \alpha_n^{(i)} J_n^{(i)}; J_f M_f \rangle + \dots, \end{aligned} \quad (4)$$

where we have written down explicitly only terms which can contribute to (3); since $\Omega_p^{(l)}$ operates on the proton coordinates only, there will be no contributions to (3) from those components of $\delta \psi_i$ which include other neutron states than $\alpha_n^{(f)} J_n^{(f)}$, and similarly for $\delta \psi_f$. The coefficients $c_k^{(i)}$ and $c_k^{(f)}$ are determined by the proton-neutron interaction V_{pn} , and to first order in perturbation theory they are given by

important at this stage).

$$V_{ij} = \sum_m f_m(|r_i|, |r_j|) T^{(m)}(i) T^{(m)}(j). \quad (7)$$

We now note that, since we assumed that $J_p^{(0)} = 0$, the only contributions to (3) can come from excited proton configurations in which $J_p^{(k)} = l$ since $J_p^{(0)}, l$ and $J_p^{(k)}$ have to satisfy the triangular condition. Using this remark we conclude immediately that in (7) only the term with $m = l$ can contribute anything to (5), since again $J_p^{(0)}, T^{(m)}(p)$, and $J_p^{(k)}$ should satisfy the triangular condition. Thus we find

$$\begin{aligned} c_k^{(i)} &= \frac{\langle \alpha_p^{(k)} J_p^{(k)}, \alpha_n^{(f)} J_n^{(f)}; J_i M_i | \sum_{i,j} f_l(r_{p_i}, r_{n_j}) T^{(l)}(p_i) T^{(l)}(n_j) | \alpha_p^{(0)} J_p^{(0)}, \alpha_n^{(i)} J_n^{(i)}; J_i M_i \rangle}{\Delta E_p^{(k)} - \Delta E_n}, \\ c_k^{(f)} &= \frac{\langle \alpha_p^{(k)} J_p^{(k)}, \alpha_n^{(i)} J_n^{(i)}; J_f M_f | \sum_{i,j} f_l(r_{p_i}, r_{n_j}) T^{(l)}(p_i) T^{(l)}(n_j) | \alpha_p^{(0)} J_p^{(0)}, \alpha_n^{(f)} J_n^{(f)}; J_f M_f \rangle}{\Delta E_p^{(k)} + \Delta E_n}. \end{aligned} \quad (8)$$

We can now define an average value f_l for $f_l(r_p, r_n)$, and noting that

$$\sum_{i,j} T^{(l)}(p_i) T^{(l)}(n_j) = [\sum_i T^{(l)}(p_i)] [\sum_j T^{(l)}(n_j)] = T^{(l)}(p) T^{(l)}(n),$$

¹⁸ See, for instance, M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley and Sons, Inc., New York, 1957), p. 85.

¹⁹ G. Racah, Phys. Rev. **62**, 438 (1942).

where $\sum_i T^{(l)}(p_i) \equiv T^{(l)}(p)$, etc., we get, using the well-known formulas developed by Racah,¹⁹

$$\begin{aligned} c_k^{(i)} &= (-1)^{2J_f+l} f_l \frac{\langle \alpha_n^{(f)} J_n^{(f)} \| T^{(l)}(n) \| \alpha_n^{(i)} J_n^{(i)} \rangle}{[(2l+1)(2J_i+1)]^{\frac{1}{2}}} \times \frac{\langle \alpha_p^{(k)} J_p^{(k)} \| T^{(l)}(p) \| \alpha_p^{(0)} J_p^{(0)} \rangle}{\Delta E_p^{(k)} - \Delta E_n}, \\ c_k^{(f)} &= (-1)^{2J_i+l} f_l \frac{\langle \alpha_n^{(i)} J_n^{(i)} \| T^{(l)}(n) \| \alpha_n^{(f)} J_n^{(f)} \rangle}{[(2l+1)(2J_f+1)]^{\frac{1}{2}}} \times \frac{\langle \alpha_p^{(k)} J_p^{(k)} \| T^{(l)}(p) \| \alpha_p^{(0)} J_p^{(0)} \rangle}{\Delta E_p^{(k)} + \Delta E_n}. \end{aligned} \quad (9)$$

From (9), (4), and (3) we finally get for the first-order contribution to the matrix elements of $\Omega_p^{(l)}$

$$\langle f \| e_p \Omega_p^{(l)} \| i \rangle = \langle J_n^{(f)} \| T^{(l)}(n) \| J_n^{(i)} \rangle \omega_l = \langle \alpha_p^{(0)} J_p^{(0)}, \alpha_n^{(f)} J_n^{(f)}; J_f \| T^{(l)}(n) \| \alpha_p^{(0)} J_p^{(0)}, \alpha_n^{(i)} J_n^{(i)}; J_i \rangle \omega_l, \quad (10)$$

where

$$\begin{aligned} \omega_l &= \frac{f_l}{2l+1} \sum_k \left\{ \frac{\langle \alpha_p^{(0)} J_p^{(0)} \| T^{(l)}(p) \| \alpha_p^{(k)} J_p^{(k)} \rangle \langle \alpha_p^{(k)} J_p^{(k)} \| e_p \Omega_p^{(l)} \| \alpha_p^{(0)} J_p^{(0)} \rangle}{\Delta E_p^{(k)} + \Delta E_n} \right. \\ &\quad \left. + \frac{\langle \alpha_p^{(0)} J_p^{(0)} \| e_p \Omega_p^{(l)} \| \alpha_p^{(k)} J_p^{(k)} \rangle \langle \alpha_p^{(k)} J_p^{(k)} \| T^{(l)}(p) \| \alpha_p^{(0)} J_p^{(0)} \rangle}{\Delta E_p^{(k)} - \Delta E_n} \right\}. \end{aligned} \quad (11)$$

Strictly speaking, the ω_l 's in Eqs. (10) and (11) depend on the particular initial and final neutron states. This dependence enters both through the energy denominators $\Delta E_p^{(k)} + \Delta E_n$, as well as through the average radial-integral factor f_l . However, since we have assumed that in zeroth order the low excitations are due to neutrons only, we have to require, for consistency, that

$$|\Delta E_n| \ll |\Delta E_p^{(k)}|, \quad (12)$$

and we can thus neglect ΔE_n in (11). The dependence of f_l on the particular neutron states involved is harder to analyze. However, so long as we are dealing with transitions between low-lying states it can be argued that such states are mostly concentrated on the same spherical shell. (They differ, of course, considerably in their angular distribution on this shell.) Therefore we can expect that f_l will depend only slightly on the particular neutron states involved. Consequently ω_l can be considered, with the above limitations, to be independent of the neutron states, and to represent the first-order contribution to $\Omega_p^{(l)}$ from all possible proton excitations.

DISCUSSION

Equation (10) gives the first-order contributions to the matrix elements of an operator which operates on protons only. In zeroth order these elements vanish, since we limited ourselves to "neutron transitions" in which the protons' state remains inert. To the extent that ω_l can be considered as independent of the initial and final neutron states, Eq. (10) tells us that the first-order effects of the "proton operator" $e_p \Omega_p^{(l)}$ could be simulated by zeroth order effects of a "neutron operator" $\omega_l T^{(l)}(n)$. If $\Omega^{(l)}$ is a single-particle operator $\Omega_p^{(l)} = \sum_i \Omega^{(l)}(p_i)$ and if further $T^{(l)}(x) = g \Omega^{(l)}(x)$, then we can introduce the concept of an "effective neutron charge" $e_n = g \omega_l$ and calculate matrix elements, in zeroth order, of the new operator $\Omega^{(l)} = e_p \Omega_p^{(l)} + e_n \Omega_n^{(l)}$. These

matrix elements will include the first-order polarization effects and no further correction due to such effects should be applied to them. We see therefore that the possibility of introducing the concept of effective charge depends on the operator we are considering and on the character of the interaction between protons and neutrons. Thus, for the electric multipoles $\Omega_p^{(l)}$ is, apart from factors, a sum of spherical harmonics of order l : $\Omega_p^{(l)} = c \sum Y_l(p_i)$. $T^{(l)}(n)$ will also be a sum of spherical harmonics of order l if, for instance, the interaction $v_{ij} = \sum_m f_m T^{(m)}(p_i) T^{(m)}(n_j)$ [see Eq. (7)] is of the Wigner type. Thus we conclude that Wigner forces between protons and neutrons lead to a meaningful effective charge for the neutron so long as electric multipoles are considered. For magnetic multipoles, however, the first-order corrections due to Wigner forces do not take the form of an effective charge; to induce an effective charge in these cases one needs forces which are either spin dependent and/or velocity dependent. These forces are probably present in nuclear forces to a lesser degree than Wigner forces and one may thus have a qualitative explanation of the absence of "effective charge" in the strict sense in magnetic multipoles. This does not mean, of course, that configuration interaction does not contribute to magnetic multipoles; indeed one knows now that these contributions are quite significant. We have shown only that for magnetic multipoles these corrections cannot be represented by an effective charge, in agreement with experimental observation.

It is interesting to estimate the value of the effective charge of the neutron, assuming Wigner forces, and considering electric transitions. In this case we can take

$$T^{(l)} = [4\pi/(2l+1)]^{\frac{1}{2}} Y^{(l)}, \quad (13)$$

in which case $f_l(r_1, r_2)$ will be given by

$$f_l(r_1, r_2) = \frac{2l+1}{2} \int_{-1}^1 V(|\mathbf{r}_1 - \mathbf{r}_2|) P_l(\cos \omega_{12}) d(\cos \omega_{12}). \quad (14)$$

Since numerical factors multiplying $\Omega^{(l)}$ do not change the value of the effective charge [see Eqs. (10) and (11)] and since we are considering electric transitions, we can also take

$$\Omega^{(l)} = T^{(l)},$$

thus getting for the ratio of the effective charge e_n to the proton charge e_p the expression

$$\frac{e_n}{e_p} \approx \frac{2}{2l+1} f_l \sum_{k \neq 0} \frac{|\langle \alpha_p^{(0)} J_p^{(0)} \| T^{(l)}(p) \| \alpha_p^{(k)} J_p^{(k)} \rangle|^2}{\Delta E_p^{(k)}}. \quad (15)$$

We can estimate f_l by assuming that those nucleons which contribute to it are all concentrated in a spherical shell $R_1 \leq r \leq R_2$ and that their distribution there is uniform. We further assume $V(r_{ij})$ to be a square well of depth V_0 and range ρ_0 , $R_2 - R_1 < \rho_0 < R$, where R is the nuclear radius. The expression for f_l then becomes

$$f_l = \frac{\int_{R_1}^{R_2} \int_{R_1}^{R_2} f_l(r_1, r_2) r_1^2 dr_1 r_2^2 dr_2}{\int_{R_1}^{R_2} \int_{R_1}^{R_2} r_1^2 dr_1 r_2^2 dr_2} = \frac{\frac{2l+1}{2} V_0 \int_{R_1}^{R_2} \int_{R_1}^{R_2} \int_{\beta}^1 P_l(x) dx r_1^2 dr_1 r_2^2 dr_2}{\int_{R_1}^{R_2} \int_{R_1}^{R_2} r_1^2 dr_1 r_2^2 dr_2}, \quad (16)$$

where

$$\beta = (r_1^2 + r_2^2 - \rho_0^2) / 2r_1 r_2. \quad (17)$$

This integral can be evaluated, and for $(R_2 - R_1)/R \ll 1$ one gets, to lowest order in $(\rho_0/R)^2$, for $l=2$,

$$f_2 = 2V_0(\rho_0/R)^2, \quad (18)$$

and similar expressions for other values of l . Introducing further an average excitation energy $\langle \Delta E \rangle_{Av}$ by

$$\frac{1}{\langle \Delta E \rangle_{Av}} = \sum_k \frac{|\langle \alpha_p^{(0)} J_p^{(0)} \| T^{(l)}(p) \| \alpha_p^{(k)} J_p^{(k)} \rangle|^2}{\Delta E_p^{(k)}}, \quad (19)$$

we get, for $l=2$,

$$\frac{e_n}{e_p} \approx \frac{4}{5} \frac{V_0 \rho_0^2}{\langle \Delta E \rangle_{Av} R^2}, \quad (20)$$

or taking $V_0 \rho_0^2 = 1 \times 10^{-24}$ Mev cm² and $R = 1.2 \times 10^{-13} A^{\frac{1}{3}}$ cm,

$$\frac{e_n}{e_p} \approx \frac{60}{\langle \Delta E \rangle_{Av} A^{\frac{2}{3}}} \quad (E \text{ in Mev}). \quad (20a)$$

The estimates of $\langle \Delta E \rangle_{Av}$ are naturally not very reliable; however, it can be expected that as a function of A it will be proportional to $A^{-\frac{2}{3}}$. Equation (20) or (20a) then shows that the effective charge of the neutron is roughly the same for light and heavy nuclei (provided our approximations are valid; this is certainly not so for highly deformed nuclei). For the lead isotope a reasonable value for $\langle \Delta E \rangle_{Av}$ is anything between 4 and 8 Mev. Taking $\langle \Delta E \rangle_{Av} = 6$ Mev, we find $e_n/e_p \approx 0.3$. This value agrees reasonably well with the experimental values found in this region. Thus for the 516-kev $E3$ transition in Pb^{206} , Alburger and Pryce²⁰ find exactly this value for the effective charge of the neutron.

This extremely good agreement with experiment should not be taken too seriously in view of the very rough approximations which we have employed. A better estimate of the effective charge should rely on its detailed evaluation from Eq. (15). Such programs were carried out in great detail by various authors¹⁻⁸ (see in particular the work of Amado,⁸ where further references are also mentioned). Our aim was just to show the possible usefulness of the concept of neutron effective charge in some cases and its poor validity in others, as well as to try to understand why, when this concept is valid, the effective charge of the neutron is of the same order of magnitude for diagonal elements as for nondiagonal elements in different states in the same nucleus, as well as for different nuclei.

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²⁰ D. E. Alburger and M. H. L. Pryce, Phys. Rev. **95**, 1482 (1954).