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A REMARK ON ONE-DIMENSIONAL QUANTUM ELECTRODYNAMICS *)

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In a recent note, I. Bialynicki-Birula ¹⁾ has shown that the one-dimensional quantum electrodynamics with zero mass electrons can be solved exactly. In what follows we would like to show that this one-dimensional model is essentially equivalent to the Thirring model ²⁾.

The field-equations in this model are as follows :

$$\vec{p} \psi = e \gamma^\mu A_\mu \psi \quad (1a)$$

$$(\square - \mu^2) A^\mu = -j^\mu = -e \bar{\psi} \gamma^\mu \psi \quad (1b)$$

where $\vec{p} = -i \gamma^\mu \partial_\mu$, $\gamma^1 = i \sigma_1$, $\gamma^2 = \sigma_2$ ²⁾, and μ is the mass of the "photons". It will turn out that the case $\mu = 0$ has no physical solution due to the fact that the Coulomb force in the one-dimensional case does not vanish at infinity. (1a,b) should be supplemented with the usual (non-renormalised) canonical commutation relations and the subsidiary condition $\chi^{(+)}(x) |a\rangle = 0$, $\chi = \partial_\mu A^\mu$, $|a\rangle$ being a physically allowed state.

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From (1a) there follow the conservation laws :

$$\partial_\mu s^\mu = 0, \quad \partial_\mu t^\mu = 0,$$

where $s^\mu = \frac{1}{e} j^\mu = \bar{\psi} \gamma^\mu \psi = (\rho_1 - \rho_2, \rho_1 + \rho_2)$, $t^\mu = \bar{\psi} \gamma^\mu \gamma^3 \psi = (\rho_1 + \rho_2, \rho_1 - \rho_2)$, $\rho_\alpha = \psi_\alpha^* \psi_\alpha$, $\gamma = \gamma^2 \gamma^1 = \sigma_3$. With the introduction of new variables $u = t+x$, $v = t-x$, the conservation laws lead to

$$\begin{aligned} \rho_1 &= \rho_1(v) = \psi_1^*(x-t, 0) \psi_1(x-t, 0), \\ \rho_2 &= \rho_2(u) = \psi_2^*(x+t, 0) \psi_2(x+t, 0). \end{aligned} \quad (2)$$

This means that $\square s^\mu = \square t^\mu = 0$, and

$$[j^\mu(x), j^\nu(x')] = 0 \text{ for all } x \text{ and } x'. \quad (3)$$

The last fact follows from (2) and the canonical commutation relations.

From this basic result we see that $\frac{1}{\mu^2} j^\mu(x)$ is a solution of (1b). The most general solution of (1b) will be

$$A^\mu(x) = A_0^\mu(x) + \frac{1}{\mu^2} j^\mu(x), \quad (4)$$

where $A_0^\mu(x)$ satisfies the homogeneous equation $(\square - \mu^2)A_0^\mu(x) = 0$. From (3) and the canonical commutation relations it follows furthermore

$$[A^\mu(x), j^\nu(x')] = [A_0^\mu(x), j^\nu(x')] = 0 \quad (5)$$

and

$$[A^\mu(x), A^\nu(x')] = [A_0^\mu(x), A_0^\nu(x')] = i g^{\mu\nu} \Delta(x-x', \mu^2). \quad (6)$$

This all implies $A_0^\mu = A_{in}^\mu = A_{out}^\mu$, so that the model does not give rise to any creation or scattering of "photons".

Inserting (4) into (1a) we obtain

$$\not{p} \psi = e A_{0\mu} \not{x}^\mu \psi + \frac{e^2}{\mu^2} (\bar{\psi} \not{x}^\mu \psi) \not{x}^\mu \psi. \quad (7)$$

We introduce now the "Ansatz"

$$\psi(x) = \exp \left\{ i \frac{e}{\mu^2} \Lambda(x) \right\} \psi_T(x), \quad (8)$$

where $\Lambda(x) = \partial_\mu A_0^\mu + \not{x}^\mu \epsilon^{\mu\nu} \partial_\nu A_{0\mu} \equiv -i \not{p}^\mu A_{0\mu}$, $\epsilon^{\mu\nu} = -\epsilon^{\nu\mu}$, $\epsilon^{12} = +1$ and $[\Lambda(x), \Lambda(x')] = 0$ for all x and x' by virtue of (6). One sees immediately that $\bar{\psi} \not{x}^\mu \psi = \bar{\psi}_T \not{x}^\mu \psi_T$ and that equation (7) leads to

$$\not{p} \psi_T = \frac{e^2}{\mu^2} (\bar{\psi}_T \not{x}^\mu \psi_T) \not{x}^\mu \psi_T \equiv 2 \frac{e^2}{\mu^2} (\bar{\psi}_T \psi_T) \psi_T. \quad (9)$$

This is exactly the Thirring equation for ψ_T and the coupling constant $g_T = -\frac{e^2}{\mu^2}$ (compare ²).

The "gauge transformation" (8) leading to the Thirring model means that the S-matrix, defined by $\psi_{out} = S^{-1} \psi_{in} S$, $A_{out} = S^{-1} A_{in} S = A_{in}$, is identical with the S-matrix of the Thirring model: $S = \exp \{ i g_T Q_1 Q_2 \}$ (compare ²). It is a unit matrix with respect to the "photons".

In the case $\mu = 0$ equation (1b) can be written in the form $\square A_\pm = -e \varphi_{1,2}$ with $A_\pm = \frac{1}{2} (A^2 \pm A^1)$. The formal solution

$$A_+ = \frac{e}{4} u \int_{v_0}^v \varphi_1(v') dv' + A_+(u, v_0) \text{ contains a factor which}$$

increases linearly with time, so that the asymptotic conditions cannot be satisfied. The physical reason for this behaviour has already been mentioned before.

In conclusion, we would like to thank Dr. W. Thirring and Dr. F.L. Scarf for discussions, which led to the clarification of the physical content of this model.

FOOTNOTES AND REFERENCES

- 1) We are indebted to the author for sending us a preprint of his letter to the Nuovo Cimento.
- 2) W.E. Thirring, Ann.Phys. 9, 91 (1958), Nuovo Cimento 9, 1007 (1958);
V. Glaser, Nuovo Cimento, 9, 990 (1958).