ELECTROMAGNETIC FORM FACTORS OF THE NUCLEON AND PION-PION INTERACTION

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We wish to propose a simple model for the electromagnetic structure of the nucleon, based on dispersion theory and on a strong pion-pion interaction. The model is a synthesis of several theoretical ideas proposed by Frazer and Fulco, Nambu, and Chew.

Let us first of all summarize some general properties of the nucleon form factors. We write the interaction of the nucleon with the electromagnetic field in the form:

\[ \langle p' | j_\mu | p \rangle A_\mu = -i\bar{u}(p') [G_1(t)\gamma_\mu + G_2(t)\sigma_{\mu\nu}k^\nu]u(p)A_\mu, \tag{1} \]

where \( p', p, \) and \( k \) are the four-momenta of the final nucleon, initial nucleon, and photon, respectively, and \( t = k^2 = (p' - p)^2. \) The \( G_i \) still are operators in the isospin space:

\[ G_i = G_i^S + G_i^V T_3, \]

and so

\[ G_i^p = G_i^S + G_i^V; \quad G_i^n = G_i^S - G_i^V. \]

As is well known, the separation into the isoscalar and the isovector current is very useful because only an even number of pions contribute to \( G^V \) and an odd number to \( G^S. \) At \( t = 0 \) the \( G_i \) functions tend to the static charge and magnetic moment of the nucleon:

\[ G_1^p(0) = e, \quad G_1^n(0) = 0, \]

\[ G_2^p(0) = \mu = \frac{e\hbar}{2M}, \quad G_2^n(0) = \mu = \frac{e\hbar}{2M}, \]

\[ G_1^S(0) = G_1^V(0) = e/2, \]

\[ G_2^S(0) = (\mu + \mu)/2 = \frac{e\hbar}{2M}, \quad G_2^V(0) = (\mu - \mu)/2 = \frac{e\hbar}{2M}, \]

\[ g_p = 1.79, \quad g_n = -1.91, \quad g_S = -0.06, \quad g_V = 1.85, \] \tag{2}

The functions \( G(t) \) are related to the usual Hofstadter form factors \( F(t) \) by the following definitions:

\[ G_i^{p,n}(t) = G_i^{p,n}(0)F_i^{p,n}(t). \tag{3} \]

Dispersion theory allows one to write the different functions \( G(t) \) in the following form:

\[ G(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(t')}{t'-t} dt'. \tag{4} \]
The spectral functions $g(t)$ are of fundamental theoretical importance because they are related to the weight with which the different many-particle states contribute to the nucleon form factors. Therefore, if there is no bound state formed by pions, $g(t)$ will be different from zero only for $t$ larger than $4m_{\pi}^2$ for the vector part and $9m_{\pi}^2$ for the scalar part.

If there is no strong correlation between the pions, $g(t)$ is related just to the statistical weight of the many-particle state with c.m. energy $E = \sqrt{t}$ and therefore will be a smoothly increasing function of $t$ starting from zero.

If on the other hand, there is a strong correlation between the pions, due to a resonance with an energy $E_R$, the spectral function will exhibit a maximum for $t = t_R = E_R^2$. This result was first shown by Frazer and Fulco$^5$ and is schematically illustrated in Fig. 1.

Therefore, in a model without a strong pion-pion correlation the spectral function $g(t)$ will be dominated by the large values of $t$ and $G(t)$ will have little dependence on $t$. The discrepancy of this model from the experimental data is discussed by Dreil.$^5$

On the other hand, a strong-correlation model leads to a rapid variation of $G(t)$ as suggested by experiment. Since for observable values of $t$ ($t \leq 0$) the dispersion denominator in Eq. (4) is always positive, if the resonance state has a reasonable width ($\frac{1}{2}m_{\pi}$), its effect can be well approximated by means of $\alpha/(t_R - t)$, where $t_R$ is the resonance position and $\alpha$ is the area under the curve.

Let us now discuss briefly the experimental results for the nucleon form factors at low momentum transfers. For $-t < 10m_{\pi}^2$ the nucleon form factors were roughly given as follows:

$$F_1^P(t) = F_2^P(t) = F_2^N(t) = F(t),$$
$$F_1^N(t) \approx 0.$$  \hspace{1cm} (5)

Many different analytic functions were proposed for $F(t)$ but a particularly good fit was obtained either with an exponential form or with the form proposed by Clementel and Villii$^6$:

$$F(t) = 0.2 + \frac{1.2}{1 - (t/22m_{\pi}^2)}.$$  \hspace{1cm} (6)

From our theoretical point of view, it is very difficult to understand an exponential form factor; on the other hand, the Clementel-Villii model can be naturally understood on the basis of a resonant state of energy $4.7m_{\pi}$. The constant term appearing in Eq. (6) represents the total contribution of the higher $t$ states which in the low-momentum-transfer region is approximately constant. Thus Eqs. (5) and (6) indicate that it is possible to interpret both isovector form factors $F_1^V$ and $F_2^V$ by means of the approximate form, which has a pole at $t_R \approx 22m_{\pi}^2$:

$$G_1^V \approx \frac{e}{2} \left( -0.2 + \frac{1.2}{1 - (t/22m_{\pi}^2)} \right),$$
$$G_2^V \approx \frac{e g_V}{2M} \left( -0.2 + \frac{1.2}{1 - (t/22m_{\pi}^2)} \right).$$  \hspace{1cm} (7)

FIG. 1. Schematic representations of $g(t)$ in arbitrary scale. (a) Uncorrelated pions; (b) strong pion-pion resonance.
By taking this attitude, the resonant state at \( E_R \approx 4.7 m_\pi \) will be attributed to a \( T=1, J=1 \) two-pion state.

Such a resonant state, at this energy, has been observed in different experiments\(^7\) on pion production by pions, and its parameters, deduced from Eq. (7), have been used\(^8\) to explain satisfactorily the low-energy behavior of the pion-nucleon scattering phase shifts.

The isoscalar charge form factor \( G_1^S(t) \) must be of the same order of magnitude of \( G_1^V(t) \), in order to give a vanishing neutron charge distribution. This means that we have to expect important low-\( t \) contributions to the isoscalar form factors. The isoscalar magnetic form factor is experimentally very small and little is known about its energy variation.

The recent\(^9\) experimental data on electron-proton scattering at larger values of the momentum transfer stress the need of having an important low-mass contribution to the isoscalar charge form factor. It is found that the charge form factor of the proton (Fig. 2) deviates strongly from Eq. (7) and it would be impossible to fit it with any reasonable form with only one pole.

A reasonable extension of the preceding ideas is to assume that \( G_1^V \) is still approximately given by Eq. (7) with the pole at \( t_V \approx 22 m_\pi^2 \), which is the value suggested by other experimental information. In this manner the flattening of \( G_1^p \) will be essentially due to the effect of \( G_1^S \). Thus \( G_1^S \) must be composed of an almost constant part and a part which goes to zero much faster than \( G_1^V \). This means that the charge form factor of the neutron will be positive at large values of \( t \). This result seems to be confirmed by recent experiments on the neutron.\(^10\)

If we tentatively attribute an average mass to the decreasing part of \( G_1^S \), we have something like \( 8 m_\pi^2 \), certainly less than \( 22 m_\pi^2 \). This shows that it would be very difficult to explain the rapid variation of \( G_1^S \) on the basis of a simple statistical formula for \( g_1^S(t) \), since the spectral integral will only start at \( 9 m_\pi^2 \). We thus expect the existence of a \( T=0, J=1 \) three-pion resonance (or bound state if \( t_S < 9 m_\pi^2 \)). The magnetic isoscalar part must also have a pole at the same position of \( t_S \), but the present experimental information about it is certainly not enough to allow detection of its effects.

From the preceding discussion we expect expressions for \( G_1^S \) which have similar form to those for \( G_1^V \). This leads to the following general form for the nucleon form factors:

\[
G_1^V = \frac{e}{2} \left[ (1-a_V) + \frac{a_V}{1-(t/t_V)} \right],
\]

\[
G_1^S = \frac{e}{2} \left[ (1-a_S) + \frac{a_S}{1-(t/t_S)} \right],
\]

\[
G_2^V = \frac{e g_V}{2M} \left[ (1-b_V) + \frac{b_V}{1-(t/t_V)} \right],
\]

\[
G_2^S = \frac{e g_S}{2M} \left[ (1-b_S) + \frac{b_S}{1-(t/t_S)} \right],
\]

where \( t_V \) and \( t_S \) represent the position of the isovector and isoscalar unstable particles, respectively.

The residues of the poles \( a_V, b_V, a_S, b_S \) are connected with the constants appearing in similar terms giving the effect of pion-pion interaction in \( \pi-N \) scattering, \( N-N \) scattering, and photo-production.\(^7,11\) The validity of the present model can thus be checked by trying a general fit of many different sets of experimental data using the same phenomenological parameters.
One of the best known experimental properties
of the form factors is that the mean charge radius
of the neutron is zero. This leads to
\[ a_S^n/t_S^{} = a_V/t_V^{} = a. \]
As a consequence we have
\[ G_1^p = e \left[ \frac{a}{2} \left( \frac{t_S}{t_S - t} + \frac{t_V}{t_V - t} \right) t \right] ; \]
\[ G_1^n = e \frac{a}{2} \left( \frac{t_V}{t_V - t} - \frac{t_S}{t_S - t} \right) t. \]
This means that we have in our model five
independent parameters \((a, t_V, t_S, b_S, b_V)\),
for one of which \((t_V)\) we know the approximate
value from independent experiments.
Some preliminary determinations\(^{10,12}\) of
the parameters contained in Eq. (8), using informa-
tion from the proton and neutron form factors,
confirm the validity of the present model. We
wish to stress a very important consequence of the
fact that \(t_S\) turns out to be smaller than \(t_V\).
From Eq. (10) one sees that the outer part of
the charge distribution of the neutron is positive
in contrast with what one would obtain on the
basis of a model without strong pion-pion inter-
action.\(^{13}\)
It is not surprising that, in a strong \(T = 1\)
model, the three-pion state has a mass lower than the two-pion state.
The existence of a resonance in a \(T = 1, J = 1\)
two-pion state forces the three-pion state to be in a \(T = 0, J = 1\) state in which all three the pions
are two-by-two resonating in a \(T = 1, J = 1\) state.\(^3\)
Moreover the \(T = 0, J = 1\) three-pion state is a
completely saturated unit because if we add an extra
pion it will be in a \(T = 0\) or \(2\) state with
respect to the others. So, if we believe that
there is a strong attraction in the \(T = 1\) state
only, we can expect that the four-, five-, etc.,
pion states are not strongly correlated at low
energy.
The possibility of detecting experimentally
the \(T = 0\) unstable particle (which we shall de-
note by \(\rho\)) is discussed in references 2 and 3.
A very interesting possibility is to identify it
with the \(T = 0\) particle observed by Abashian
\(\alpha\),\(^{14}\) who find \(t_S^{} = 5\). The choice \(J = 1\) for \(\rho\)
was suggested by Chew\(^{15}\) in order to explain the
small effect of this particle in the decay spec-
trum of the \(K\) meson.
The spin of \(\rho\) together with the small available

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\(^{4}\)This form is valid if no subtractions are needed; otherwise we have
\[ G(t) = G(0) + \frac{t}{\pi} \int_0^\infty \frac{g(t')}{t'(t'-t)} dt'. \]
\(^{5}\)S. D. Drell, Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics
PION-PION INTERACTION IN THE PHOTOPRODUCTION OF NEUTRAL PIONS WITH POLARIZED $\gamma$ RAYS

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The presence of a possible pion–pion interaction in a simple resonance state $T = 1, J = 1$ has already been investigated by several authors.\textsuperscript{1–3} In this Letter we shall see the effect of this interaction in the case of $\pi^0$ photoproduction from the reaction $\gamma + p \rightarrow \pi^0 + p$ with polarized $\gamma$. By using the Cini–Fubini\textsuperscript{4} approximate version of the Mandelstam representation, one can demonstrate\textsuperscript{5} that in order to take into account the pion-pion interaction in the photoproduction to a reasonable approximation, one should simply add to the expression of the scattering amplitude of CGLN\textsuperscript{6} a contribution of the form of the Born term that derives from the graph of Fig. 1. The contribution $S_{\beta'} (\beta)$ is the isotopic spin index) to the total $S$ matrix is found to be\textsuperscript{6}

$$S_{\beta'} = i(2\pi)^4 \delta_4 \left( p_1 + k - p_2 - q \right)$$

$$\times \frac{1}{(4k_0 \omega E_1 E_2 \nu_{\pi})} \nu_{\pi} \bar{u}(p_2) M u(p_1) \tau' \tau$$

(1)

with

$$M = \frac{8\pi A}{(p_2 - p_1)^2 + (4\mu)^2}$$

$$\times \left[ \frac{\mu_p' - \mu_n'}{2M} \left[ M_B(2q \cdot k + \mu^2) - M_B' \right] \right]$$

(2)

where $k$, $q$, $p_1$, and $p_2$ are the 4-momenta of the photon, pion, initial nucleon, and final nucleon, respectively; $k_0$, $\omega$, $E_1$, and $E_2$ are the corresponding energies; $\mu_p'$ and $\mu_n'$ are the anomalous magnetic moments of the proton and the neutron; $M =$ nucleon mass; $\mu =$ pion mass. The “mass” of the intermediate “particle” is assumed to be $-4\mu$. The constant $A$ is proportional to the “strength” of the photon–three-pion interaction and $M_A$, $M_B$, $M_C$, $M_D$ are the invariant terms defined by Chew et al.\textsuperscript{8}

In order to carry out the calculation, one simply has to add the coefficients of the invar-

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